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Report on the Doctoral Thesis by Vít Jelínek

This is an excellent thesis. The results are original, interesting, and definitely relevant to contemporary research in mathematics. The topic of the thesis, pattern avoidance in permutations and ordered combinatorial structures, is presently a central topic in combinatorics. The thesis is well written, well structured and with appropriate references. Most of the results have been published, or accepted for publication, in prestigious international journals or conferences.

Let me go into more detail into the contents of the thesis. The first four chapters deal with permutations.

Chapter 1 gives some necessary background. Chapter 2 contains the following result: given an $r \times s$ table T with fixed row and column sums $(x_1, \ldots, x_r)$ and $(y_1, \ldots, y_s)$, the number of ways of filling T with non-negative integers and avoiding a given permutation matrix P of order at most 3, depends only on the score-line multiset $\{x_1, \ldots, x_r, y_1, \ldots, y_s\}$ and on the order of P. This is an interesting and rather unexpected result, which cannot be extended to permutations of order greater than 3.

Two permutations σ and τ are Wilf-equivalent if for every n the number of permutations in S_n avoiding σ is the same as those avoiding τ. This concept can be generalized by looking at a permutation as a transversal of a square shape, and then considering more general kinds of shapes, like Ferrers diagrams, stack polyominos, and others. This is in fact the main theme in the thesis, to translate pattern containment into transversals or fillings of diagrams.

Chapter 3 contains a very interesting conjecture concerning the identity permutation I_k and its reversal J_k, namely: for every skew shape S, the number of I_k-avoiding transversals of S is at most the number of J_k-transversals. The author proves it for $k = 2$, which already implies non-trivial results. For $k > 2$ it remains an open problem.

Chapter 4 deals with pattern avoidance in involutions (permutations whose square is the identity). The main result involves again transversals of Ferrers diagrams, and its main consequence is to show that certain patterns

are Wilf-equivalent with respect to involutions. In particular this settles an open problem on patterns of length 5 and 6.

A recurrent technical tool in this first part of the thesis is what the author calls the 'red-green argument', which has to do with avoiding permutations whose associated matrix decomposes into two blocks.

Chapter 5 deals with patter avoidance in k-ary words. The main results here are the equivalence between the pattern 12^k and all its rearrangements, and similarly for the pattern 12^k3.

Chapter 6, the most substantial one, deals with patterns in set partitions. There are different ways of defining containment for set partitions. The one studied by the author is through the representation of a partition with k blocks as a word with restricted growth in the alphabet $\{1,\ldots,k\}$ (this is very different from other representations, like the graph-theoretic one encoding blocks as paths). The main tool in proving the numerous results in this chapter is to associate a 0/1 matrix to a partition so that there is a 1-cell in position (i,j) if the j-th element is in the i-th block.

The main result here is that the number of partitions with no k-crossing (represented by the pattern $12\cdots k12\cdots k$) is the same as the number of partitions with no k-nesting (pattern $12\cdots kk(k-1)\cdots 1$). In fact, this is a special case of a very general result giving the equivalence between infinitely many pairs of patterns. Many other results of a similar nature are proved. This is in contrast with former definitions of pattern avoidance, where the results obtained so far are not as comprehensive as those in the present thesis.

In summary, I consider the thesis submitted by Vít Jelínek to be of very high quality, showing his strong qualification to conduct independent and creative research in mathematics. I am happy to give a wholeheartedly recommendation for the approval of the thesis.

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