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"ULISSE DINI"

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Comments on the Habilitation Thesis: Mechanics and thermodynamics of viscoelastic fluids
by Vìt Průša

In his Habilitation Thesis the Author describes the results illustrated in the papers [P1]-[P10]. The Author essentially focuses on the theory of viscoelastic-rate-type fluids. In particular, the Author analyzes the physical and thermodynamic basis of mathematical models for viscoelastic-rate-type fluids. This analysis is applied to investigate the stability of particular flows. The stability analysis also makes use of general concepts developed by the Author in contexts broader than those related to viscoelastic models. The Author also reports results concerning the mathematical description of some phenomena often encountered in the experimental investigation of viscoelastic flows.

The Habilitation Thesis is well written and the subjects analyzed are of non-trivial interest. Actually, the topics illustrated in the Thesis are central in the field of theoretical mechanics and applied mathematics. The scientific language is clear and rigorous.

Here are some specific comments.

- In Chapter 2, the Author focuses on rate type models and proves that these models can be interpreted as approximations of implicit fading memory constitutive relations. This result is very interesting because it shows that the implicit constitutive approach provides a general framework including various classical models (Maxwell, Oldroyd-B, Rivlin-Ericksen and so on). The Author correctly states that fluids defined through implicit constitutive relations involving the stress and relative deformation gradient histories can generalize the notion of simple fluids and rate-type fluids.
- Still in Chapter 2 the Author investigates the responses of nonlinear viscoelastic materials in stress relaxation and creep tests. In these cases the mathematical models reduce to non-linear ODE's subject to step inputs. The Author, developing his theory in the framework of Colombeau algebra (an extension of the distributions theory into the nonlinear setting), provides an excellent theoretical framework for studying the response of such systems. Indeed, exploiting this formalism, the Author is able to derives an explicit formula describing the response of a non-linear viscoelastic fluid subject to step deformations in a lubricated squeeze flow setting.
- In Chapter 3 the Author derives a class of thermodynamically consistent variants of viscoelastic rate type models incorporating material coefficients that depend on temperature. The Author presents a model which is consistent with the laws of thermodynamics and the evolution equations for the mechanical quantities. The theory highlights that the temperature equation has to contain terms that are usually ignored in "practically oriented" models. Interestingly, the Author shows that the impact of these additional terms can be, in certain circumstances, non-negligible.
- Viscoelastic rate-type models with the stress diffusion term are investigated in Section 3.3. In particular, the Author analyzes the origin of the stress diffusion and shows that it can be a consequence of a new energy storage mechanism or of a new entropy production mechanism. The thermodynamic approach proposed by the Author paves the way for the analysis of the rest states (non-linear) stability.
- Chapter 4 is devoted to the analysis of stability issues in a thermodynamic setting. The Author starts from the well-known fact that thermodynamic arguments are useful in the identification of Lyapunov functionals. The latter are fundamental for the analysis of non-linear stability of spatially non-homogeneous equilibrium configurations of (thermodynamically) isolated systems. However, this approach fails when dealing with thermodynamically open systems. The Author shows that, even for open systems, a Lyapunov functional (physically well motivated and so very useful in the stability analysis) can be defined. The peculiarity of the proposed construction is its generality which makes it applicable even in complex thermo-mechanical contexts.
- The Author applies the general method illustrated in the first part of Chapter 4 to the simple case of a mechanically isolated fluid that reaches a spatially non-constant temperature when the walls are kept at a constant non-uniform temperature. In particular, regardless of the initial conditions on the velocity field, using the proposed thermodynamic Lyapunov functional, it is rigorously proved that the non-spatially homogeneous stationary temperature state is asymptotically stable.

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- Using again the same method (Lyapunov type functional constructed on the basis of thermodynamical arguments) the Author investigates the finite amplitude stability of steady flows of viscoelastic fluids described by various models (Oldroyd-B model, Giesekus model, FENE-P model, Johnson-Segalman model and Phan-Thien-Tanner model). The Author is thus able to rigorously show that the stationary state is stable (as physically expected).

In the first part of his Habilitation Thesis the Author reaches very interesting results considering with creativity and originality both mechanical and analytical aspects. His research is indeed driven by a rigorous approach to fundamental questions in scientific and technological applications. The Author essentially investigates two issues: the Oldroyd-B model as well as other viscoelastic rate-type models and the response of non-linear viscoelastic materials to discontinuous stimuli.

Concerning the viscoelastic rate-type models the Author shows that many models can be interpreted as approximations of simple fluids with fading memory in the sense of the classical Coleman and Noll theory. He reaches this result generalizing the theory of implicit constitutive relations recently introduced by Rajagopal. The results obtained are fundamental from a theoretical point of view because, for the first time, a unified theory for rate-type viscoelastic fluids is provided (a theory which, among other things, manages to generalize the classical simple fluids concept). The applications are numerous because, for instance, this approach allows to identify the model that best fits certain a priori required features.

Concerning the second issue, the Author analyzes the question of the response of nonlinear viscoelastic materials to jump-like stimuli. This task is really thorny since one faces the problem of multiplication of distributions (which is far beyond the classical theory of distributions). The Author obtains very interesting results both at an abstract level and at an applied level. Indeed, the viscoelastic fluids rheological parameters are experimentally determined by creep and relaxation tests based on discontinuous stimuli. The standard theory allows to calculate these parameters only for linear models but not for the non-linear ones. The Author's contribution is therefore extremely significant allowing to extend to non-linear models (for example very often used in applications involving biological materials) methods used only in the linear case.

The second part of the Habilitation Thesis is devoted to thermodynamic and stability issues. One of the main issue of continuum mechanics is the derivation of models consistent with the laws of thermodynamics, in particular with the second law of thermodynamics. In this framework the Author obtains interesting results for non-isothermal viscoelastic rate-type models. He also develops a thermodynamic consistent theory for stress diffusion models. These models have been recently introduced for the analysis of shear banding phenomena in complex fluids (a topic which is at the very cutting edge of non-Newtonian fluid mechanics). Typically shear bands are explained using non-monotone constitutive models. However, this approach alone is not enough to solve the problem, because there is typically a range of total shear stresses for which two shear rates are simultaneously stable. This resolution comes through the inclusion of nonlocal terms in the equations of motion for the microstructure, typically in the form of spatial gradient or "diffusion" terms. The Author's fundamental contribution in this field (well documented in the Thesis) is the development of a fundamental thermodynamic analysis which was totally lacking in this context. In particular, thanks to this analysis it is possible to identify the correct boundary conditions that must be imposed.

The Author analyzes also the nonlinear stability (finite amplitude) of spatially inhomogeneous steady states through thermodynamics "methods". With the help of thermodynamic arguments, he designs candidates for Lyapunov-type functionals. Though a rigorous analysis is still technically challenging, the Author's results about the Lyapunov functionals (constructed on the basis of thermodynamical arguments) are very important to address and understand some fundamental questions of stability in continuum mechanics paving the way to very promising future developments.

In conclusion, in his Habilitation Thesis the Author illustrates a series of original and high-level mathematical and physical results which have also numerous applications. The Thesis is undoubtedly a very good piece and reflects the remarkable Author's scientific career attested by the attached high-quality research papers.

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