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Object : report on the habilitation thesis of Dr. Vít Průša

In his habilitation thesis, Dr. Vít Průša has presented four types of original works that correspond to the main four directions of his research career, generally dedicated to Partial Differential Equations (PDEs) that model the motions of *viscoelastic* fluids.

The Turnitin report indicates no significant indications lack of originality. The author naturally cites his own work. The scientific integrity and originality of the work seems clear.

In introduction (Chapter 1), Dr. Vít Průša motivates his studies of viscoelastic rate-type models as the second most used fluid models after Navier-Stokes, and he introduces the four research directions whose results are next detailed in four independent chapters. A few more explanations about specific applications of viscoelastic rate-type models (for engineering purposes typically) would have been appreciated here, illustrations of quantitative capabilities to forecast materials behaviours for instance. But in any case, the introduction is very well written, as the rest of the thesis. It contains interesting remarks about the history of the models, and a nice overview of the rest of the thesis, clearly articulated.

In Chapter 2, a first research direction is described. It aims at deriving viscoelastic rate-type models, the usual ones encountered in the literature in particular, from the first-principles of physics. The results obtained by Dr. Vít Průša, in collaboration with a leading researcher in thermodynamics, provide one with a clear link between fluid models. The result is very useful in theoretical mechanics for models comparison. For models comparison purposes, there are also other means, like comparing the properties of solutions (motions) produced by different models (PDEs) in similar configurations (geometry, initial conditions etc.). But this is arguably much more difficult (without analytical formula or numerical approximation) if not impossible sometimes (when solutions are not well defined). So the result of Dr. Vít Průša obtained with K.R. Rajagopal in reference [P7] and summarized here is definitely interesting.

A second research direction is described in Chapter 2, which aims at a precise mathematical formulation of a standard analysis of models in mechanics, with a view to comparing models too (in a configuration motivated by physical experiments). That part is concerned with more rigorous mathematics, rare in that mechanical game. Unfortunately, the configurations here analyzed, which are standard in mechanics, are « 0D » (without space dimension) and remain of very limited use. Those results were published in a good journal in mechanics [P8-P10] also in collaboration with K.R. Rajagopal.

In Chapter 3, Dr. Vít Průša summarizes more recent efforts in his career. Obtained in collaboration with various authors [P5, P6], the results concern the thermodynamical foundations of viscoelastic rate-type models, a very profound aspect regarding the physical justification of models. The subject is notoriously difficult. Dr. Vít Průša and his collaborators have obtained very appreciable results here, in this old but slowly-progressing field. The summary of the results, well-written, contains an interesting opening toward viscoelastic models with a molecular basis (and not necessarily with a rate-type closure) on the one hand. It would be interesting here to link the thermodynamics of the various model (micro-macro or macro) one another i.e. compare the thermodynamic potentials. Indeed, models can in fact be derived from postulated thermodynamics so the thermodynamic foundations are also a way to compare models. On the other hand, variations of existing models can also be proposed naturally here on varying the thermodynamics potentials. It is very appreciable that Dr. Vít Průša investigates how to model stress diffusion with a thermodynamical perspective. Hopefully, Dr. Vít Průša will continue investigating micro-macro modelling from a thermodynamical viewpoint, and will produce new models with a thermodynamical basis in the future, to give full power to his existing studies [P5, P6].

Chapter 4 covers the last direction of Dr. Vít Průša's researches, namely establishing a mathematical framework to explain the stability of some states of viscoelastic fluids governed by rate-type models. This is a natural continuation of Chapter 3, building on thermodynamics to construct Lyapunov functionals. These researches would have been advantageously introduced by experimental illustrations of physical states supposedly governed by rate-type viscoelastic models and indeed observed stable (or not). A loop could have been made with choices made in thermodynamical framework, whether the retained entropy production actually allows controlling large-time evolutions. In any case, the results from ref. [P1-P4] are very interesting insofar as they not only cover both micro-macro and macro models but also the challenging case of open systems, with physical states heterogeneous in space as targets. The approach leading to a candidate Lyapunov functional like (50) seems new and to deserve further studies – can it be systematized to any sort of admissible boundary conditions ? Of course, one has to assume a well-defined target state governed by the model and conditions required at open boundaries. But in any case, this seems a new and very interesting use of the old concept of « relative entropy methods ».

So finally, on considering the numerous aspects of viscoelastic rate-type models tackled by Dr. Vít Průša in his habilitation, covering a broad scope of techniques and ideas in theoretical mechanics and applied mathematics, I think Dr. Vít Průša is a well-established researcher in the field, who well deserves habilitation. He has shown ability to connect old relevant questions with interesting new ideas, that still promise fruitful results in a near future.

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