Serfling & Thompson Statistical Consulting
Plano, Texas USA

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Professor Zdeněk Doležal
Vice-Dean of Faculty, Mathematics and Physics
Charles University

Dear Professor Doležal:

I am writing in response to your request for an evaluation of the scientific quality of the Habilitation Dissertation of Dr. Stanislav Nagy, as an external examiner. I am very pleased to provide this.

My primary connection with Dr. Nagy's research has been a common interest in the area of statistical depth functions. This is a key topic in the challenging arena of multivariate data analysis (and more general settings such as functional data), and both Dr. Nagy and myself have worked quite broadly in it. Each of us has cited papers of the other, and we have corresponded at length on various issues. In these interactions, which I have greatly enjoyed, Dr. Nagy has shown a great mind and deep understanding and has corrected errors in my initial drafts. There are a few leading statistical depth functions, none dominating the others in appeal. Among these the halfspace (or Tukey) depth is perhaps the most intellectually challenging, in that a full appreciation of its properties requires mastery of a broad range of mathematics beyond statistics and probability. And yet a deep understanding of this depth is instrumental to a full use of its potential. Such an investigation has been carried off in the papers of Dr. Nagy's Habilitation Dissertation, and this represents a highly significant and very timely accomplishment, worthy of the highest recognition.

Dr. Nagy's Habilitation Dissertation consists of seven published papers treating the mathematics of the halfspace depth, along with summarizing comments including open questions. Paper (A) focuses on the connections between the halfspace depth and the notion of floating body in convex geometry. Under a mild assumption, the upper level sets of the halfspace depth coincide with the convex floating bodies of measures used in the definition of the affine surface area for convex bodies in Euclidean spaces. This connection leads to some specialized results, such as a bound of the halfspace depth in terms of the Mahalanobis depth, resolutions of open questions, and conditions under which the halfspace depth determines the underlying distribution. It also yields perspective suggesting new open questions of interest, for example how convex hulls of random samples relate to the convex hull of the support of the generating distribution. Further, the paper encompasses results on symmetry measures, going beyond just those relating to halfspace depth. Altogether, this paper is an incredibly erudite and productive survey of key connections between halfspace depth and floating bodies. One cannot fully do justice to it in a single paragraph.

Paper (B) provides a comprehensive review of results and questions regarding the halfspace characterization problem treated also in Section 8 of Paper A. This covers results of the author and of others, encompassing counterexamples, classes of distributions determined by their halfspace depth functions, reconstruction of the distribution from the halfspace depth, connections with floating bodies, and open conjectures. Papers (C) and (D) also treat the halfspace characterization problem, focusing on counterexamples and reconstruction, respectively.

Paper (E) introduces a variation of the sample halfspace depth, the *illumination depth*, designed to break ties and to extend the depth function in a nonzero way to points outside the convex hull of the data. This approach carries over a technique used in convex geometry with floating bodies. The illumination depth retains the affine invariance and robustness of the halfspace depth but introduces further computational challenge. Overall, it improves upon previous approaches by other authors and performs well in practical applications. The illumination depth appears to be a major new contribution.

Papers (F) and (G) treat the properties of approximate methods which reduce to computing the halfspace depth or projection depth on finitely many projections. Here it might be of interest to compare with some very competitive other methods in the literature using this kind of approach, for example Serfling and Mazumder, 2013, "Computationally easy outlier detection via projection pursuit with finitely many directions", *Journal of Nonparametric Statistics* and competitive methods using other approaches, for example Mazumder and Serfling, 2013, "A robust sample spatial outlyingness function", *Journal of Statistical Planning and Inference.*

An important measure of the professional development of a young academic, in my opinion, is whether the person also branches out to other research areas besides the dissertation project. Very nicely, Dr. Nagy has indeed fulfilled that criterion by this Habilitation Dissertation.

The papers of this Habilitation Dissertation are very interesting and favorably and significantly impact the direction of research on these topics. The papers are of extremely high quality and treat their topics with depth and thoroughness. Further, from a review of the originality check by the Turnitin system, there is no evidence whatsoever of plagiarism.

Overall, this Habilitation Dissertation is a truly meritorious and outstanding body of work, and it receives my highest commendation.

Very sincerely yours,