Assessment of the habilitation thesis of Stanislav Nagy, Ph.D. with title

Halfspace depth: Theory and computation

Stanislav Nagy's work is a summary of seven publications and at first glance it is a very compact work studying one topic in great depth and from different sides. However, on the other hand, the habilitation thesis links also other works of Stanislav Nagy related to the topic studied in the habilitation thesis. Which proves Stanislav Nagy's wide scope. This fact is also evident from the work itself, in which results from statistics are connected together with geometry, theoretical results are derived and innovative statistical procedures are proposed along with their implementation in R with a core in C.

The work itself in the first chapter summarizes the results of the article, which connects halfspace depth together with convex geometry, especially with the concept of floating bodies. The floating bodies are further used in the paper to solve the problem of characterization and reconstruction of the halfspace depth for larger set of measures than in previous literature.

The second part summarizes a paper which deals again with the problem of characterization and clarifies some older results which proves the characterization for various sets of measures.

The third part summarizes a paper which shows an example of random measures that are not characterized by halfspace depth, i.e. refutes the old assumption that halfspace depth characterizes all probability measures.

The fourth part summarizes a paper which simplifies the reconstruction procedure of finite atomic measure from its halfspace depth.

The fifth part summarizes a paper which introduces a refinement of the halfspace depth called illumination depth. It again uses the term from convex geometry, the illumination sets, in order to define the depth also for the data which are outside the domain of the studied data. During reading this part I was rather curious how the illumination sets are connected to the main mass of the data. It was said that illumination sets adjust to covariance structure of the data set only. Also, I am curious, regarding the illumination depth, for how big dimensions it is computationally tractable. And I would like to point out here the continuous measure which does the similar job for functional/multivariate data sets, that is based on pointwise ranks (Mrkvička T., Myllymäki M., Kuronen M., Narisetty N. (2021). New methods for multiple testing in permutation inference for the general linear model, *Statistics in Medicine* **41/2**, 276-297).

The last two parts deal with computation of the halfspace depth. The six one shows how the usual procedure for calculation of it is insufficient in the convergence to the true halfspace depth. And the last paper tries to find better algorithm for its computation. It shows that the projection based algorithms are less effective than those based on Nelder-Mead algorithm. It has to be highlighted that the algorithm had to be adapted in order to optimize on sphere instead on Euclidean space as it is necessary for halfspace depth computation. My question here is again how big dimensions are still tractable with this algorithm.

As can be seen from the overview, Stanislav Nagy has done a significant amount of work in the important field of statistics. I think that the submitted thesis fully meets the requirements for a habilitation thesis.
The work is written carefully in a formal way, logically and clearly laid out. The agreement with other works detected by the Turnitin program was detected only with the works of Stanislav Nagy, therefore I consider it irrelevant in the work summarizing the previous results of Stanislav Nagy.

I therefore recommend recognition of the submitted work as the habilitation thesis of the author.

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