The object of study of the present thesis are evolutionary problems satisfying volume preservation condition, i.e., problems whose solution have a constant value of the integral of their graph. In particular, the following types of problems with volume constraint are dealt with: parabolic problem (heat-type), hyperbolic problem (wave-type), parabolic free-boundary problem (heat-type with obstacle) and hyperbolic free-boundary problem (degenerate wave-type with obstacle). The key points are design of equations, proof of existence of weak solutions to them and development of numerical methods and algorithms for such problems. The main tool in both the theoretical analysis and the numerical computation is the discrete Morse flow, a variational method consisting in discretizing time and stating a minimization problem on each time-level. The volume constraint appears in the equation as a nonlocal nonlinear Lagrange multiplier but it can be handled elegantly in discrete Morse flow method by restraining the set of admissible functions for minimization. The theory is illustrated with results of numerical experiments.