Kraków, 4 IV 2023

Dr. hab. Sebastian Szybka, Prof. UJ
Department of Relativistic Astrophysics and Cosmology
Astronomical Observatory
Jagiellonian University

Referee's report on the habilitation thesis 'Beyond Symmetric Solutions in General Relativity' by Dr. Otakar Svítek

The habilitation thesis of Dr. Otakar Svítek describes his contribution to the field of general relativity (Einstein's gravity). Einstein field equations constitute a complicated system of partial differential equations, and simplifying assumptions are necessary to find exact solutions. After more than a hundred years of studies by several generations of physicists, not many physically relevant exact spacetimes have been found. Moreover, even the simplest solutions (like the spherically symmetric Lemaître–Tolman–Bondi spacetimes) exhibit non-trivial and counter-intuitive properties. Dr. Svítek's research, presented in the thesis, addresses the following problem: which properties of the simplified solutions represent generic behavior of the theory, and which are artifacts of simplification. In my opinion, the motivation to conduct this kind of research is pivotal to making progress in our understanding of gravity.

The habilitation thesis presented by Dr. Svítek consists of five chapters (including the introduction and the conclusions) and takes the form of a stapler thesis. It is based on nine papers co-authored by Dr. Svítek. The articles are included in the body of the thesis, and their list is provided in Appendix A. These articles can be separated into two groups, which are reflected in the structure of the thesis. The first group is related to black holes, and in particular, to the Robinson-Trautman solution. The second group is related to cosmology. There is also the article which provides a brdige between these two topics. The dissertation is well-written and carefully crafted. The author's contributions are clearly distinguished from the existing state of knowledge at the time the research began. The minor misprints do not alter my positive opinion about the thesis.

The importance of the problems studied by Dr. Svítek is underscored by the recent Nobel Prize awarded to Roger Penrose. He received the prize for demonstrating that black holes are a robust prediction of general relativity, not artifacts of symmetry assumptions. Penrose proposed symmetry-independent tools to study singularity formation. The approach followed by Dr. Svítek is more modest and not as general.

Chapter 1 serves as the introduction. In this chapter, Dr. Svítek outlines the structure of the thesis and presents his motivation for conducting the studies.

Chapter 2 is dedicated to the physically important class of solutions discovered by Robinson and Trautman in the early sixties. These solutions generalize non-rotating, spherically symmetric black holes to non-rotating black holes that, in general, do not possess additional symmetries. As such, the Robinson-Trautman spacetimes provide an interesting testbed for studying black holes without symmetries.

Two papers written by Dr. Svítek and his collaborators (the publications appended to the thesis as referenced therein [33], [34]) are devoted to the Robinson-Trautman spacetimes with a minimally coupled scalar field. In these papers, a non-trivial scalar field generalization of the Robinson-Trautman spacetime is presented and analyzed. The main outcomes of these studies can be summarized as follows: only scalar field solutions with a gradient field non-aligned with a principal null direction are admissible. For a dynamic solution, a scalar field is radiated away, consistent with the no-hair theorem. The global properties of scalar field solutions differ significantly from the simpler vacuum solutions. The presented results are of a technical nature. The studied spacetimes possess a singularity, which is naked at early times. This, combined with the remaining aspects of the setting (a scalar field), places the research within the category of mathematical relativity and moves it quite far from direct physical/astrophysical applications.

The further part of the research in which Dr. Svítek was involved, and which is related to Robinson-Trautman spacetimes, focuses on extensions to nonlinear electrodynamics (a hypothetical extension of Maxwell electrodynamics). In paper [41], it has been shown that one of the promising aspects of nonlinear electrodynamics — the removal of curvature singularity for spherically symmetric black holes — does not easily generalize beyond spherical symmetry. I find this result interesting, although its validity is restricted by the Robinson-Trautman class of spacetimes (only non-rotating solutions).

The remaining part of the research on Robinson-Trautman spacetimes is dedicated to clearly unphysical wormhole solutions that violate energy conditions [43]. In my opinion, the motivation to conduct these studies is rather limited. However, solutions derived from Robinson-Trautman spacetimes allow for stability analysis of spherically symmetric wormholes.

Chapter 3 illustrates Dr. Svítek's contributions to theoretical cosmology. Although the topic is quite different from that of Chapter 2, which was primarily devoted to black holes, the prevailing theme remains consistent: the study of relativistic effects that are hindered in idealized models or by approximate treatment. This chapter is naturally divided into two parts.

The first part is dedicated to the properties of inhomogeneous cosmological spacetimes. Dr. Svítek is interested in sets of initial data for the Szekeres spacetime that do not lead to shell-crossing singularities [52]. Such sets are not of direct physical interest because, in the real universe, high-density contrast pressure would intervene and, very likely, prevent collapse (pressure is absent by definition in Szekeres models, so shell-crossing singularities signal a failure of the matter model). However, such sets are interesting if one wants to construct simple toy-models of structure formation. The second cosmological paper [54] deals with the existence and properties of horizons in inhomogeneous perfect fluid cosmological models. In particular, Dr. Svítek and his collaborator are interested in the existence and properties of horizons in generalizations of the Lemaître–Tolman–Bondi metric to the spherically symmetric case with pressure and to the case with pressure but without any symmetries (the Szekeres-Szafron spacetime).

In the second part of Chapter 3, Dr. Svítek describes his contribution to the problem of averaging in cosmology. This problem was first clearly formulated as a fitting problem by George Ellis in 1987. The standard cosmological model is based on the assumptions of perfect spatial isotropy and perfect spatial homogeneity. However, it is well-known that in the real universe, perfect isotropy and homogeneity hold only in a statistical sense on large scales. The universe on a small scale is highly inhomogeneous. Thus, the essential question is how to describe a statistically homogeneous but locally inhomogeneous spacetime by an effective homogeneous model. This problem is known under many names, which have very similar meanings: the fitting problem, the problem of averaging, and the cosmological backreaction problem. The discovery of dark energy at the dawn of our century renewed interest in this topic. It has been suggested that dark energy could be an artifact of the perfect homogeneity of the cosmological model. The heated debate lasted almost two decades.

Dr. Svítek and his collaborator contributed to this debate in the article [61], where an innovative method for averaging spacetimes was proposed. This method was inspired by the works of other authors and is based on averaging of Cartan scalars. The authors applied their method to particular examples and concluded that, under some assumptions, the inhomogeneity effect may be identified with accelerated expansion (a positive cosmological constant). The subsequent work on this topic, in which Dr. Svítek was involved, utilized a different approach to averaging [68]. This alternative method, suggested by the authors, is applicable to a specific class of spacetimes that are locally rotationally symmetric. The main outcome of the study underlines the backreaction effect for shear of the fluid flow, which has been overlooked in other approaches.

I find Dr. Svítek's contribution to the problem of averaging in cosmology very interesting. However, I have also a critical remark. The section 'Recent developments in averaging methods' does not contain the most recent update on the status of the averaging problem, which I think is an obvious omission. In my opinion, the problem has been at least partially settled by numerical studies, as has been clearly indicated in the article co-authored by Coley and Ellis (Class. Quant. Grav. 37(1), 013001, 2020) — in cosmology, the observable differences caused by backreaction effects will be of the order of 1%.

Chapter 4 is dedicated to the limiting procedure in general relativity as a generation method. In this chapter, Dr. Svítek describes his contribution to the problem that links Robinson-Trautman spacetimes to dust cosmological models. In other words, these two different geometries (studied in Chapters 2 and 3) could, in some cases, be related to each other. It has been shown in the article [73] for a non-zero cosmological constant that the null limit of Szekeres spacetimes corresponds to Robinson-Trautman spacetime without gravitational radiation. This result is interesting and generalizes the work of other authors.

Chapter 5 serves as a summary of the thesis.

The articles that constitute the core of the thesis have been published in reputable journals, like Physical Review D and Classical and Quantum Gravity. All of them are co-authored by two researches, so the results described in the thesis were obtained in collaboration with other people. Using the data available via the Web of Science service, one may find that some of Dr. Svítek's co-authors had to be students. It is natural to assume that Dr. Svítek contributed significantly to the presented research.

The scientific achievements presented in the thesis should be evaluated in the context of the field to which they have contributed. General relativity is an established theory, and all the easy problems were solved long ago. As a result, modern relativists must build on the work of previous generations and tackle problems that have remained unsolved for decades. Therefore, it is not surprising that progress in mathematical relativity is incremental. Nevertheless, general relativity is a working theory, confirmed by countless observations, that reveals how nature operates. The results presented by Dr. Svítek demonstrate definite progress in our quest to understand gravity.

I have acquainted myself with the results of the anti-plagiarism Turnitin system, and I confirm that the system did not find any plagiarism in Dr. Svítek's habilitation thesis (only a negligible overlap with the existing literature written by the author). The presented results are original.

Conclusion

In summary, the material presented in the thesis clearly demonstrates valuable contributions of Dr. Svítek to general relativity. The presented findings have enriched our knowledge of exact solutions to Einstein's equations. **Therefore, I conclude that the presented dissertation meets the formal requirements for a habilitation thesis, and I recommend admitting the candidate to the subsequent stages of the procedure.**

Dr. hab. Sebastian Szybka, Prof. UJ