Review of Habilitation Thesis of Dr. Otakar Svítek (ITP, Charles U.) *Beyond symmetric solutions in General Relativity*

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This thesis is based on nine recent papers co-authored by Dr. Otakar Svítek, all of which have been published in prestigious international peer-reviewed journals such as Physical Review D, European Physical Journal C, and Classical and Quantum Gravity. The common theme of these papers is an interest in finding solutions to the Einstein equations in cases where there is little or no symmetry, making the task quite ambitious and technically challenging. However, these works are also united by their mathematical rigor and detailed analysis.

While the absence of symmetry makes the study more realistic and closer to the real world, sometimes readers may wonder whether the assumptions that allow the study of these particular classes of solutions are general enough to be realized in physically relevant situations. It is not always clear whether there are physical principles responsible for selecting these classes of solutions since symmetry is usually what simplifies and singles out solutions. However, the situation here seems to be much more subtle.

Typically, analytic solutions are very rare for nonlinear partial differential equations (PDE), and their presence indicates an interesting phenomenon. Below, I provide more detailed comments on each of the chapters in the thesis

Introduction In the concise introduction the author offers the reader a glimpse into to the essence of the work. Here I have a very minor comment: when author is writing that "The Penrose theorem [3] thus proved that black holes are indeed occurring generically..." I would say that Penrose proved that there is necessarily a singularity for at least one geodesic after the trapped surface is formed. For most applications, the assumed formation of the trapped surface already implies the existence of a black hole. As it is stated in [1] "After realizing the power of the idea of trapped surfaces, Penrose proceeded to prove that once a trapped surface had formed, it is impossible, within the theory of general relativity and with a positive energy density, to prevent the collapse towards a singularity". By the way, it's worth noting that the Hawking effect invalidates one of the assumptions of the Penrose theorem, namely, the classical Null Energy Condition, due to the violation of this condition by the Hawking radiation near the horizon. However, it should be acknowledged that the authors' reference to this result is quite standard.

Black holes without symmetries Here the author introduces the Robinson–Trautman (RT) family of geometries that is characterized by the properties of the principle null geodesic congruence that should be shear-free, twist-free and expanding. Clearly these properties make these solutions so special as to be naively of zero measure. Indeed, two out of three optical scalars are fine-tuned to vanish for the principle null geodesic congruence. It would be very interesting to analyze linear perturbations around these solutions. The main point is that these solutions do not require spherical symmetry. In this chapter the author first discusses his works [33] and [34] where an explicit family of solutions of RT class with free massless minimally coupled scalar field was derived and analyzed. Two major properties found are: a singularity is formed earlier than a quasilocal horizon to cover it, and that the scalar field disappears outside of black hole in the late-time asymptotic. The results are interesting and relevant for a better understanding of limitations of the Penrose's cosmic censorship hypothesis and of the no hair theorems in dynamical setups.

Then the author proceeds to his work [41] deriving and analyzing the RT solution in GR with different reincarnations of nonlinear electrodynamics. Here the main result is that, contrary to solutions with exact spherical symmetry, one cannot avoid the gravitational singularity in RT solutions. This is definitely an important result showing the generic character of the gravitational singularities. However, for this part I would notice that nonlinear electrodynamics Lagrangians used to model such spacetimes, say (31), (34) and especially (37) from [41] are not that well physically motivated from the point of view of QFT or effective field theory approach. Though, one should not blame the author for this, as these theories were introduced before and are not his inventions. Moreover, I think in 2.1.2 one could better explain the origin of asymptotic expansion (2.10), as this vanishing of all "Fourier" coefficients for $i \leq 14$ is far from obvious, while in 2.1.3 one could already there define the notion of a "quasilocal horizon".

Then the author discusses his [43] where a wormhole was constructed inside of the RT class of solutions. Surprisingly the throat of the wormhole can be supported by two dust streams with negative energy densities. Moreover, this configuration shows a nonlinear stability (within the RT class) of the late-time spherically symmetric wormhole. Of course, dust with negative energy density is a bit non-standard matter. But it is much simpler than many other proposed models.

Inhomogeneous cosmologies and averaging First, the authors discusses general inhomogeneous models like the spherically-symmetric Lemaître–Tolman–Bondi (LTB) solution involving dust, and the spherically-symmetric Lemaître model using a perfect fluid. Then, the author describes even less known solutions without symmetries: the Szekeres solution for dust and its generalization to perfect fluids due to Szafron. Here I have a couple of minor comments. When the author writes that "One can imagine this cosmological model as being composed of concentric spherical shells of dust with varying density that evolve under their own gravity. This makes it obviously ideal starting point for models of a star interior as well." - this may sound confusing as for many readers the very existence of stars is only possible because of the pressure preventing the gravitational collapse. Around the sentence "The function $M(r)$ can be interpreted as a gravitational mass contained within the comoving spherical shell at any given r." one could mention that this is a manifestation of the gravitational mass defect. The author mentioned that "...collision of neighboring dust shells leading to the name shell-crossing singularity. This type of singularity is highly undesirable because density additionally changes sign in this location. Ensuring the absence of shell-crossing singularities constitutes crucial part of deriving useful inhomogeneous models for cosmology." and then later "Therefore when using Szekeres solution in cosmology the avoidance of shell-crossing singularities represents important aspect which is harder to ensure than in spherically symmetric LTB solution due to increased complexity." Here, I think it is crucial to stress that in the usual cold dark matter (CDM) the formation of the virtualized structure is directly connected to formation of caustics and shell-crossings. Of course, it is important to avoid shell-crossing singularities when dealing with smooth PDEs and fluid-like dust / DM, as it is unclear how to continue evolution across the caustic. However, for non-interacting dust particles it is not a problem, but rather a desired behavior. Regarding the Szekeres solution I think it would be useful to clarify whether dust should be irrotational one or not.

Then the author discusses [52] where the conditions for density extremes that are necessary for avoiding the shell-crossing singularity were derived. Moreover, it was showed that in the special case of a trivial curvature function, the conditions are preserved by evolution. These are interesting and nontrivial results.

After that the author proceeds to [54] where the existence and properties of horizons in inhomogeneous perfect fluid cosmologies have been studied. Most interesting for me was to see that, under simple assumptions, in the spherically symmetric Lemaître model the horizons are null-hypersurfaces, while the matter on the horizons is of special character—a perfect fluid with negative pressure.

Then the author moves to discuss averaging techniques and in particular the averaging of the Cartan scalars proposed in [61]. The most impressive result, in my opinion, was that when applying this procedure to two different LTB models, it yields a correction to the usual Friedmann equations in the form of either a positive cosmological constant or, in the second example, spatial curvature.

Then the author discusses the Buchert approach to averaging problem in cosmology and the generalization of this technique to class II dust-filled locally rotationally symmetric space-times proposed in authors paper [68]. The proposed full set of averaged equations is not closed, but an infinite hierarchy of equations that can be truncated when only finite precision is sufficient or when invoking additional relations for higher order correlation terms. This resembles the so-called BBGKY (Bogoliubov–Born–Green–Kirkwood–Yvon(hierarchy in statistical physics and I think this is an important finding. The averaging method is then applied to approximate LTB (so-called onion) model showing that backreaction in the equation for the shear of the fluid flow is dominant over the backreaction in the equation for expansion. This seems to suggest that the popular Buchert approach might omit a significant effect. This is clearly important for a better understanding of the averaging problem.

The relation between black hole solutions and cosmological models Here the author discusses how the "null dust" black hole solutions can be obtained by pushing the speed of the timelike dust flow towards the speed of light, and vice versa, in a well-defined limiting procedure. Namely, in [73] a systematic procedure to retrieve the timelike-dust metric from the null-dust case is presented. I think this is interesting, but more relevant for mathematical physics.

To conclude: This work covers timely research directions in general relativity and is interesting to read. While reading, I learned a lot of new material that is often underrepresented in even advanced textbooks on general relativity. Furthermore, I have reviewed the originality check of the Habilitation Thesis conducted by the "Turnitin" system, and it is evident that the thesis is the original work written by the author, with a completely appropriate overlap with his previously published works. Overall, my impression is very positive, and I am certain that this work deserves to be accepted and acknowledged as a completely legitimate Habilitation Thesis.

References

[1] Scientific Background on the Nobel Prize in Physics 2020, *Theoretical foundation for black holes and the supermassive compact object at the galactic centre*, The Nobel Committee for Physics https://www.nobelprize.org/uploads/2020/10/advanced-physicsprize2020.pdf

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