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Report for David Hartman's habilitation thesis entitled:

Properties of networks characterizing dynamical systems

The common topic of this habilitation thesis is the study of complex dynamic systems, with applications in biological, physical or social-economical systems.

After a short preface and its bibliography, the core of the habilitation thesis is described in a 28 page long chapter entitled *Complex networks analysis* with its extensive bibliography including more than 120 references. Reprints of 19 articles by D. Hartman and co-authors end the manuscript.

To study complex dynamic systems, D. Hartman proposes a pipeline starting with data preprocessing and ending with network analysis. Most of the steps in this pipeline are studied thoroughly and rigorously. The contributions are divided into four groups and are all published as articles in selective journals in the areas of expertise studied.

Complex systems are composed of smaller subsystems interconnected via particular synchronization patterns. Reducing dimensionality makes the network easier to process. A PCA method identifies a predefined number of principal components before selecting a smaller number. Non-random components are selected by comparing the eigenvalues of a specific covariance matrix of the original data. For time series covering long time intervals, D. Hartman has also studied the role of within-component and between-component links in the global network evolution.

D. Hartman has also studied how to determine a weight on the edges based on correlation between the input data (times series). Pearson's correlation can detect dependence reliably in the case of multivariate Gaussian probability distributions. D. Hartman has proposed an alternative in the case of complex non-Gaussian dependence patterns by using multivariate Fourier transform (FT) surrogate datasets. Applications to brain, climate and financial (stock exchange) complex networks validate the approach. D. Hartman has finally demonstrated biases caused by time series dynamics and proposed a correction done by replacing the absolute correlations with specific Z-scores.

Numerous analyzes can be carried out on such complex networks. D. Hartman has worked on the small-world character measuring to what extent the network is clustered and has a small average shortest path. A study provides arguments for the existence of potential bias of the small-world characteristics when working with functional networks. This bias has been analyzed for brain and climate datasets.

The other two groups of contributions are more independent of complex systems. Clear links are possible, and indeed mentioned, but experiments will have to validate the approaches on the problems studied, for example on cerebral and climatic complex networks. The theoretical results in these two groups of contributions are significant and demonstrate numerous scientific skills and a large scientific culture in two different domains: graphs and (interval) linear algebra.

In the third group of contributions, D. Hartmann has studied various symmetries applicable to large networks such as homomorphism-homogeneity and several generalizations. The contributions seem interesting although I did not reviewed this part into the details.

The fourth group of contributions deals with various forms of uncertainty represented by interval matrices. The contributions have been published in 6 papers co-authored with Milan Hladik, among which 5 appear in appendix of the manuscript. Most of the corresponding journals are selective and appear in the first Q1 quartile (or Q2), including *Applied Mathematics and Computation*, *Optimization Letters* and the *Electronic Journal of Linear Algebra*. The *World Congress on Global Optimization* and *SCAN* are relevant conferences for promoting the contributions. The plagiarism check (Turnitin system report) did not show a scientific error related to copying.

Works about the *regularity radius* of a matrix are very interesting. If a matrix expresses the result of series of measurements where the corresponding elements are subject to uncertainty, an interval matrix of radius δ can express the tolerance above which one matrix in the interval matrix becomes singular. Determining the maximum radius δ is NP-hard but D. Hartman follows a semi-definite programming relaxation already used for MAXCUT. This provides a randomized algorithm than can find bounds to δ with a high probability. Despite its probabilistic nature, I feel the result could be useful in practice, but it has not been tested on the brain, climate or financial networks. Comments on this point should be added in the manuscript, maybe an issue is raised by the inversion of the matrix?

Another topic is the computation of interval matrix powers for special classes of matrices. Again, beautiful theoretical complexity and algorithmic results are brought. The significance of the polynomial subclasses should be discussed. One significant polynomial complexity (up to a given accuracy) result applies to a fixed power coefficient and a special class of matrices comprising some tridiagonal interval matrices. It is based on the Tarski quantifier elimination procedure. D. Hartman has also studied parametric matrices and proved NP-hardness already for their squares. Another nice result goes through the well-known use of the spectral decomposition. It has been extended for the first time to interval matrices, which was far from being trivial.

The last contribution about interval methods concerns quadratic optimization: quadratic objective function and polyhedral convex feasible region. The problem is reformulated as an Euclidian norm maximization through a matrix factorization. Different vector norms bounding the Euclidian one (especially the maximum norm) and different objective matrix factorizations have been studied. By using a min–max inequality, a proposition can provide bounds on the optimum. This property seems to allow the computation of tight bounds for polyhedral feasible regions but not for box (interval) regions. The theoretical results are numerous and not trivial. However, the numerical results (shown in the World Congress on Global Optimization paper [54]) do not test the real complex networks previously studied and should also compare the approach to branch and bound algorithms.

Conclusion

D. Hartman has made numerous contributions to the study of complex systems as networks. Thanks to his skills in statistics, he has been able to propose a complete and relevant methodology for constructing a complex dynamic network that best reflects reality, with very interesting applications to neurological, climatic and financial systems, among others. Thanks to his skills in linear algebra and graphs, he was able to study symmetries in large networks. Thanks to his skills in interval algebra, he was able to design mathematical (interval) tools for taking into account uncertainty in models. In addition, D. Hartman has demonstrated his ability to collaborate with specialists in the fields studied, such as J. Nesetryl and M. Hladik.

The bibliography is extensive and the results are numerous, rigorous and elegant. With such a wealth of scientific knowledge and the promising avenues outlined, there is no doubt that D. Hartmann will be able to find thesis topics for his students, point them in interesting directions and mentor them with great success.

All in all, I warmly and unreservedly recommend that Dr. D. Hartman be awarded his habilitation thesis.

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