ABSTRACT

This bachelor's thesis deals with Pell's equation, while clearly presenting structured information from studied domestic and foreign books, articles, and other sources. The goal of this thesis is to create study material primarily for university students but also for inquisitive high school students, and thus explain as intuitively as possible what Pell's equation is, how to find its solutions, and how it is related, for example, to continued fractions, approximations of irrational numbers, and invertible elements in $Z[\sqrt{n}]$. The main motivation for solving Pell's equation throughout the work is specifically that its solutions give best approximations of irrational square roots. Pell's equation is presented in a brief historical context. Further, it is proved that there is a non-trivial integer solution for every Pell equation, and the theory of continued fractions is used to find it. To make the creation of continued fractions easier, the so-called Tenner's algorithm is introduced. Specifically, the search for a solution to Pell's equation is derived using convergents and the periodicity of continued fractions of irrational roots. Subsequently, the structure of the solution is described: it is proved that there is a so-called minimal solution that generates all positive solutions, and a set of solutions that form an infinite cyclic group is described. At the end of the thesis, other uses and occurrences of the Pell equation are mentioned, e.g. the so-called Pell numbers and how they can be used to search for Pythagorean triples. Thus, comprehensive information is given covering the derivation of the equation, finding its solutions, and its use.