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Faculty of Mathematics and Physics
Charles University
Professor Ondřej Kalenda
Chair of the Habilitation Commission
For Dr. Dušan Pokorný

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Evaluation of the Habilitation Dissertation and of the output of the plagiarism check by the *Turnitin* system

Dear Professor Ondřej Kalenda,

As requested I am happy to serve as an opponent (external examiner) of the Habilitation Dissertation of Dr. Pokorný. Let me first declare that I have met Dr. Pokorný on several occasions, but as far as I can see I do not have any conflict of interest in fulfilling this task.

I have gone through the check of originality of the thesis done by the system Turnitin and it is clear that the thesis represents an original work with the negligible overlap with the existing literature written by the author.

The habilitation thesis is of cumulative nature, that is, it is a collection of five already published scientific or scholarly papers or technical papers supplemented with a brief and very instructive introduction. Four of these papers have been published in peer reviewed journals and are written jointly with coauthors, one single authored paper is published on arXiv (available since September 2019). In addition to the work submitted for the habilitation, Dr. Pokorný has made other significant and original mathematical contributions to geometric measure theory and the theory of fractal sets.

The overall theme of the habilitation thesis is the integral geometry of very general classes of subsets of Euclidean space, the so-called WDC sets which are defined as weakly regular sublevel sets of differences of convex functions, and the investigation of fundamental properties of these classes of sets. Main roots of this line of research are the classical theories of submanifolds and convex subsets of Euclidean space and their integral-geometric properties. Initial contributions came from Herbert Federer (for sets with positive reach) and Rolf Schneider (for convex sets), more recent developments have their origins in various contributions and ideas by Martina Zähle, Joseph Fu, Jan Rataj and Luděk Zajíček. The starting point for the current thesis is a paper by Pokorný-Rataj (2013), which in the sequel opened the way to the study of integral-geometric properties of WDC sets (or unions thereof). There a notoriously difficult problem left open by Fu (from the late 1980s) is re-

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solved by showing that every d.c. function (that is, difference of convex functions) can be nicely approximated by smooth functions with control over second derivatives (and therefore is a Monge--Ampère function), hence it admits a normal cycle. As a consequence, a Crofton-formula could already be derived, but for technical reasons a principal kinematic formula could not yet be established in Pokorný-Rataj (2013).

In a major breakthrough paper by Fu-Pokorný-Rataj (2017), the integral-geometric theory of WDC sets has been advanced in several directions. A crucial ingredient was the proof of the new measure geometric fact that for a d.c. function the set of pairs of certain support elements of the graph of f has the right Minkowski content. The argument refines and applies a technique developed by Ewald-Larman-Rogers, similar ideas have been used by Pavlica-Zajíček and Pokorný-Rataj (2013). On the basis of such a result, which is of independent interest, the paper states and proves the principal kinematic formula in a very general framework. The result is stated for WDC sets in a Riemannian isotropic space and thus provides a far reaching generalization of (almost) all results available in the literature in this context. In particular, the case of finite unions of sets with positive reach having generic intersections is covered. This impressive contribution efficiently uses and develops further a combination of tools and methods from convexity, differential geometry, geometric measure theory and non-smooth analysis. The paper closes with a list of stimulating open questions and conjectures.

The definition of a WDC set is natural, but it is based on an auxiliary function called DC aura which is not uniquely determined. Thus the question arises whether or (better) how WDC sets can be characterized geometrically. Moreover, it is interesting to study the structural properties of WDC sets. These tasks are even more compelling in view of the above mentioned applications to integral geometry, which demonstrate the relevenace of WDC sets. Although much more is known in the special case of sets with positive reach, a paper by Rataj-Zajíček (2017) indicates that a complete structural understanding and classification of sets with positive reach is not yet available (and may remain a real challenge for quite some time), in spite of recent progress (the two-dimensional case is much better understood though). In their 2019 paper, Pokorný-Rataj-Zajíček provide a first detailed analysis in the case of WDC sets, corresponding to and extending some of the results proved in Rataj-Zajíček (2017) for sets with positive reach. For instance, it is shown that the boundary of a compact WDC set is contained in a union of finitely many suitably chosen DC surfaces. There are some partial results towards a stratification of general WDC sets, but a complete classification is available (and due to the authors work) only in the planar case so far. The analysis is based on the fine differentiability properties of convex and DC functions (and on their singularities). The paper ends with the description of several related open problems.

A natural question that has been asked in the preceding papers is whether the distance function of a WDC set is always a DC aura (as is the case for sets with positive reach). The paper Pokorný-Zajíček (2021) provides a positive resolution for dimension two, but the general case remains open. It follows that a locally WDC set is automatically a WDC set. The positive answer in the plane uses the classification derived in Pokorný-Rataj-Zajíček (2019) for the planar case and the fact, derived in Pokorný-Zajíček (2020), that the distance function of a set is a DC function if the set is the graph of a DC function. Hence any such

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set is a WDC set. Moreover, the complexity of the system of planar, compact WDC sets is described by taking iterated countable unions/intersections/unions in the space of planar compact sets (with the Hausdorff distance), in particular the class of WDC sets is a Borel set. The question remains to strengthen the corresponding weaker result in higher higher dimensions where the system is shown to be an analytic set. The same is true for the main result used from Pokorný-Zajíček (2020), which remains open in dimensions three and higher. In particular, in the Euclidean plane this paper contains also other (interesting partial) results on the relationship between sets which are unions of locally finite systems of DC graphs and closed sets whose distance function is DC.

The aim of the single author paper by Pokorný (arXiv 2019) is to extend the framework outlined above even further by considering certain finite unions of WDC sets. A corresponding extension of sets with positive reach to finite unions of such sets has been the subject of various previous papers by Rataj and Zähle. In view of applications to Stochastic Geometry, such extensions are of utmost importance. It is shown (and based on topological preparations) that compact sets which are finite unions of WDC sets admit a normal cycle and then it is deduced that also a local kinematic formula holds. In addition, in the plane a complete characterization of such sets is provided and, for instance, it is proved that compact subanalytic sets or compact Lipschitz manifolds with bounded curvature belong to the class. These findings support the conjecture that at least in the planar case, compact sets that are finite unions of WDC sets constitute the maximal integral geometric class of sets.

The habilitation thesis contains substantial and highly relevant research in integral and metric geometry. Dr. Pokorný demonstrates that he is a young researcher of great potential who is able to make important contributions in various fields of mathematics. The papers related to the thesis have led to deep new insights on problems which for quite some time seemed to leave little hope for further progress. It is probably relevant to point out that Dr. Dušan Pokorný has made further contributions which are not subject of the habilitation thesis and this evaluation and which were written with different coauthors. In summary, Dr. Pokorný has a distinguished research program and internationally highly appreciated achievements. For these reasons and by international standards, I strongly recommend to accept the Habilitation Dissertation from a scientific point of view.

Yours faithfully,