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Electric Vehicle Manufacturers and Battery Raw Materials Market

Bachelor's Thesis

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Declaration

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2. I hereby declare that my thesis has not been used to gain any other academic title.
3. I fully agree to my work being used for study and scientific purposes.
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In Prague on

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References

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Abstract

This thesis investigates the impact of volatility in the EV battery raw materials market on the stock returns of individual EV producers. The study uses the GARCH-ARJI model, which captures the jump component of volatility, and the Bayesian method to estimate the parameters and predict daily stock returns. The results suggest that jumps exist in the EV battery raw materials market, and the GARCH-ARJI model fits the data better than a benchmark GARCH(1,1) model. That lags of both jump intensity and size may affect the mean equation of EV producers' stock returns.

Abstrakt

Tato práce zkoumá dopad volatility na trhu surovin pro EV baterie na výnosy akcií jednotlivých výrobců EV. Studie využívá model GARCH-ARJI, který zachycuje skokovou složku volatility, a Bayesovu metodu k odhadu parametrů a predikci denních výnosů akcií. Výsledky naznačují, že na trhu surovin pro baterie pro elektromobily existují skoky a model GARCH-ARJI odpovídá údajům lépe než srovnávací model GARCH(1,1). Toto zpoždění v intenzitě a velikosti skoku může ovlivnit střední rovnici výnosů akcií výrobců EV.

Keywords

EVs, volatility, GARCH, jump intensity, jump size, GARCH-ARJI, Bayesian

Klíčová slova

EV, volatilita, GARCH, intenzita skoku, velikost skoku, GARCH-ARJI, Bayesian

Title

Electric Vehicle Manufacturers and Battery Raw Materials Market

Název práce

Vyrobci elektrickych vozidel a trh materialu pro vyrobu baterii

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1. Introduction

The electric vehicle (EV) industry has had rapid growth in recent years and has seen wide media coverage. The push to reduce carbon emissions driven by policymakers promotes significant transformation in the automobile market. By the end of 2023, the sales of electric vehicles are projected to increase nearly ten times from 2012. Although the efficiency of batteries used in EV production has also grown significantly, the media coverage suggests that the supply chain is still vulnerable to price changes occurring in the battery raw materials market. In this light, it is worth studying how the volatility in the relevant commodity market affects the stock returns of individual EV producers.

While there has been significant research on the car makers industry and its relationship with oil prices, relatively little attention has been paid to the volatility in the EV battery raw materials market and its implications for EV producers. In this thesis, I aim to address this research gap by examining the volatility in the EV battery raw materials market and its impact on the stock returns of EV producers.

Based on previous research for oil and stock returns, I hypothesize that there is a jump component in the EV battery raw materials market volatility. Traditional GARCH models aim to capture smooth volatility, whereas other specifications should be used to capture the jump component. Specifically, I construct the GARCH-ARJI model, which uses mixture distribution to model the number of jumps occurring between two periods. Jump intensity and size are not assumed to be constant but rather have time-varying specifications that improve model fit, according to previous research.

The estimates of parameters related to jump intensity and size are used to predict the daily stock returns of individual EV manufacturers. The results show that lags of both variables might have an effect when introduced into the mean equation of stock returns.

Moreover, the study applies the Bayesian method rather than the maximum likelihood estimation. It is not argued that one is better than another, but rather another point of view on the model estimation process is presented. The advantage of the Bayesian method for time series is that one does not have to bother with robust standard errors and the assumptions used to make them such. The result of the estimation is the whole distribution of parameters under investigation. Also, the simulation process from the model, posterior predictive distribution of data, allows to account for uncertainty in parameter estimates and check if the observed data should look plausible.

The thesis is structured as follows. In the second section, the literature review of the car manufacturing industry and commodity markets is provided. In the third section, the methodology and data sources are described. In the fourth section, the results are presented. The last section is the conclusion.

2. Literature Review

2.1 Determinants of automobile manufacturers stock returns

Despite the increasing adoption of electric vehicles (EVs) across the globe, policy reforms intended to facilitate and smooth the transition from fuel-consuming cars, and widespread media coverage, there is a lack of related literature studying the stock returns of EV manufacturers. It can be explained by the fact that until the mid of 2010s, the industry was in its infancy, with less than 2 million newly sold electric vehicles in 2015[1]. Nevertheless, more public EV producers have appeared since then, providing us with the necessary data. Hence, methods traditionally applied to analyze car manufacturers' stock returns can be utilized for EV ones.

Hall (2001) suggests that the growth of cash flows is a crucial factor in comprehending stock market movements. Then, if prices of commodities, such as oil or lithium, can affect the cash flows, we may expect a relationship between equities and commodities.

Cameron and Schnusenberg (2009) investigated the relationship between oil prices and stock prices of traditional automobile manufacturers. The authors created an oil price factor, which was added to the Fama–French three-factor model, and the hypothesis of the negative relationship between stock returns and the oil price factor was studied. Moreover, they divided the sample into two parts, pre- and post-Iraq invasion, studying the hypothesis that there was a structural change in the relationship at that point. Also, the authors performed a qualitative assessment of traditional car producers and divided them into two subsamples, depending on the prevalence of SUVs[2] in their total sales.

[1] Taplin, N. (2021). Electric Vehicles Are Shattering the Barrier to Adoption That Could Matter Most. The Wall Street Journal. Retrieved from https://www.wsj.com/articles/electric-vehicles-are-shattering-the-barrier-to-adoption-that-could-matter-most-a30d154e?mod=Searchresults_pos6&page=1

[2] Sport utility vehicles

Manufacturers focusing on high marginal and less fuel-efficient cars – SUVs – were hypothesized to suffer more from oil price increases. The idea behind is that higher oil prices, through the channel of higher gasoline prices, weaken consumer demand for full-size cars, or consumers opt for alternative means of transportation. That, in turn, affects the future cash flows leading to lower stock returns.

The authors measured the oil price factor in two ways: using WTI crude oil prices and energy ETF. The ETF provided more support for the hypothesis of the negative relationship between stock returns and the oil price factor. Using WTI oil prices resulted in the regression coefficient for the oil price factor being insignificant at a 5% level for both pre-and post-Iraq invasion periods. However, when using ETF, the coefficient was significant at a 1% level for the post-Iraq invasion period. Moreover, the different ways of measurement brought the same results for the hypothesis that the oil price factor is more significant and has higher explanatory power for SUV car manufacturers than non-SUV manufacturers: the authors did not manage to reject it.

Baur and Todorova (2018) noted that in the academic literature, there was limited focus on the relationships between the stock prices of individual car manufacturers and the oil market. They argue that the developments in the oil market may trigger both demand-side and supply-side effects regarding the business of companies. On the supply side, the automotive industry could be impacted by the effects on their production input costs. As for the demand side, changes in oil prices might prompt individuals to reduce their driving frequency or modify their consumption patterns by preferring more fuel-efficient vehicles. The authors also mentioned the rise of electric cars and declining input production costs for these: the price of lithium-ion battery packs for EVs experienced a 65% decline from 2010 until 2016. Against this backdrop, Baur and Todorova assessed how much oil price developments impact electric vehicle manufacturers and included the US electric car maker Tesla in the analysis.

They hypothesized that Tesla is positively exposed to oil price shocks through the substitution effect between combustion-engine cars and electric cars.

Baur and Todorova (2018) acknowledged that Cameron and Schnusenberg (2009) accounted for the possibility of a structural break related to the Iraq invasion in 2003 but argue that the potentially time-varying nature of the oil price factor for individual car manufacturers has not been discussed extensively. In order to fill the gap, the authors applied the adjusted Fama-French three-factor model with a 24-month forward rolling window. Hence, they could investigate the time-varying nature of oil price sensitivities. Furthermore, citing previous studies that found significant asymmetric effects of oil price shocks on financial markets (e.g., Broadstock et al., 2016), Baur and Todorova examined a potential dependence of the relationship on the oil price level utilizing a threshold model version. Moreover, they applied another version of the adjusted Fama-French model, with additional separation of oil returns into two parts, positive and negative. The rationale is that the sensitivity strength might differ for positive and negative returns. Finally, this paper examines a broader scope than previous research by analyzing the performance of 16 automobile manufacturers for 25 years.

On the full sample, Baur and Todorova reported that the oil price sensitivities of individual carmakers exhibit varying significance. The oil price factor betas' estimated coefficients are mainly in the negative range, especially for daily and weekly returns, and to a lesser degree for monthly returns. The standout company is Tesla, the only producer of electric cars in the sample, with a weekly oil price sensitivity of 0.27 and a monthly sensitivity of 0.48, which is considerably higher than for the other companies examined. Hence, the hypothesis of the substitution effect between combustion-engine cars and electric cars was not rejected using the full sample.

Analyzing time-varying oil price betas, the authors noted the change in betas signs observed between 2008 and 2016. Betas became negative for most companies after the Global Financial Crisis. Baur and Todorova highlighted that during a certain period, even leading manufacturers of hybrid vehicles such as Toyota and Nissan, known for being more eco-friendly, behaved similarly to manufacturers of heavier vehicles like Ford and Fiat Chrysler, with oil price betas ranging from -0.15 to -0.2. It indicates that the impact of oil prices on the high-profit margin segment of their product portfolio – SUVs – is stronger than the trend towards smaller and more environmentally friendly cars. The negative sensitivity is consistent with the findings for SUVs of Cameron and Schnusenberg (2009). However, they did not find this for producers with a lesser share of SUVs in their portfolio.

Furthermore, applying the model to detect different sensitivity strengths for positive and negative oil returns, the authors identified car manufacturers with insignificant betas on average but significant ones if positive and negative oil returns are considered separately. It demonstrates the importance of separating (unsigned) oil price factor betas into positive oil price factor betas and negative oil price factor betas.

Speaking of the threshold model version, the authors tried different price thresholds. For prices above 100 US dollars, Baur and Todorova found a significant increase in the number of oil price factor betas that are statistically significant. However, the authors claimed that this result seems to be driven by the extreme oil price movements observed before and after the start of the Global Financial Crisis in the second half of 2008 only.

The literature review on determinants of stock prices of traditional automobile manufacturers showed the main methods usually applied to analyze the individual companies' stock returns. The extensions of the adjusted Fama–French three-factor model, such as time-varying and threshold models, were presented that can help deepen the analysis. The results of the reviewed papers were presented with main conclusions and insights. The oil price factor can explain the stock returns of traditional car manufacturers, although the effect changes over time and might differ for positive and negative returns. Also, more eco-friendly car manufacturers may be affected by the oil price factor to the same extent as those that produce heavier and less fuel-efficient cars. Tesla's stock returns have a positive relationship with the oil price factor, and this can be explained by the substitution effect between combustion-engine cars and electric cars.

2.2 EV manufacturers and the battery raw materials market

Baur and Todorova (2018) mentioned that the price of lithium-ion battery packs for EVs experienced a 65% decline from 2010 until 2016. More recent data shows battery costs drop 90% from 2010 to 2020[3]. The continuing declining price of battery packs is a crucial factor for consideration since it is the single most expensive component in EVs and the primary reason they typically cost more than conventional vehicles. According to a recent analysis[4], the cost of a battery pack for an electric vehicle increased by 6.9% in 2022 compared to the previous year. The significant increase in prices was mainly attributed to the rising costs of essential components used in the batteries of most EVs, such as lithium, nickel, and cobalt.

Technological advances affect the production of battery packs, and iron-based batteries, known as LFP, do not use nickel and cobalt, which are increasingly supply-constrained and expensive. Experts estimate that iron-based batteries now represent nearly a third of all batteries in electric vehicles worldwide, and that share may continue to grow[4]. Specifically, such batteries now power the majority of EVs in China and are an option on some Tesla Model 3s in the U.S. These developments give rise to an intuition that the effect of the

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[3] Neil, D. (2021). Electric Cars Are Struggling to Meet Two Key Needs: Speed and Range. The Wall Street Journal. Retrieved from https://www.wsj.com/articles/evs-batteries-range-electric-vehicles-tesla-kia-porsche-mercedes-11671232024?mod=article_inline

[4] Barr, A. (2021). How Tesla Opening Its Superchargers Alters the EV Charging Map. The Wall Street Journal. Retrieved from https://www.wsj.com/articles/how-tesla-opening-its-superchargers-alters-the-ev-charging-map-c9398c90?mod=Searchresults_pos3&page=1

metals prices on EV manufacturers might be decreasing over time. However, there is a lack of related literature studying the stock returns of EV manufacturers.

Baur and Gan (2018) analyzed the sensitivity of EV producers' stock returns to the price of lithium and discussed possible production cost and demand effects. The former asserts that when lithium prices increase, it would be difficult for automotive manufacturers to pass on the increased cost to customers unless the economic benefit of owning an EV significantly outweighed not owning one. The authors asserted this is true since EVs are already more expensive than traditional combustion engine vehicles. From this follows that the share price of EV manufacturers is negatively related to the lithium price.

Alternatively, the increased demand for battery components is driving the price of lithium up. This is reflected in the global sales of EVs, which have risen by over 340% between 2014 and 2018. As the demand for lithium increases due to its use as a key component for EVs, the price of lithium has increased by 265% during the same period. Despite the potential increase in the cost of goods sold for EV manufacturers due to rising lithium prices, this is offset by the greater demand for EVs. Hence, it follows that the share price of EV manufacturers might positively related to the lithium price.

This seemingly contradictive setup seems to require the application of previously described methods, such as time-varying and threshold models. EV manufacturers might be able to pass the increased cost of inputs on customers till a certain threshold, and the market can reflect this in different sensitivities of stock returns to metal prices above and below the threshold. In this case, even assuming the demand effect holds, the coefficient estimates will have a negative sign above the threshold. However, the authors did not examine the possibility of such a relationship. The application of threshold models can help to resolve the issue.

Moreover, Baur and Gan examined only the lithium prices, whereas the battery pack price also depends on nickel and cobalt, although their effect might have decreased. As was shown in previous studies, time-varying models can be applied to study the sensitivities if they change in time. The authors left a gap here as well.

The authors revealed a statistically significant relationship between stock prices and lithium prices for six Chinese manufacturers and a single German producer, Volkswagen. The findings suggest that the positive coefficients observed among the Chinese manufacturers signify an EV demand effect. The authors related this to China's dominant and rapidly expanding EV market, combined with the extensive government subsidies and investment in charging infrastructure. Conversely, the negative coefficient for Volkswagen is explained by a production-cost effect since the company has undertaken substantial investments in EV production without comparable government support accessible to Chinese counterparts. Given that the sample contained 24 companies and the fact that no other stocks had a statistically significant negative association with lithium prices, it might be the case that the gaps in the methodology described above could have affected the results.

2.3 Short-term fluctuations in commodity markets and automobile manufacturers: dynamic perspective

The above-mentioned works studied the expected returns of automobile manufacturers from a longer-term point of view. Authors mostly used monthly or weekly data applied with asset-pricing models, which could not investigate the different types and degrees of fluctuations in commodity markets and the short-term effect of those. Nevertheless, sudden movements in commodity markets happen, and other models are typically used to understand the implications.

Gronwald (2012) defined the extremely sudden fluctuations in global crude oil prices caused by international emergencies as oil price jumps. Zhang and Shang (2023) argued that since the traditional GARCH model cannot describe oil price jumps, a new model framework is needed to represent the fluctuation dynamics of the global crude oil market. They postulated that there is a lack of studies on the influence of oil price fluctuations on automobile producers and focused their work on the Chinese ones.

Zhang and Shang found that the expected and unexpected shocks have an asymmetrical influence on automobile markets. An expected increase in oil prices leads to a decline in returns, while an unexpected one has the opposite effect. Moreover, the results showed 'U-shaped' responses toward global oil price jumps. The jumps increase the fluctuations in returns at the moment, relieve them in the lagging period, and then intensify again in the second lag period. The authors explain it through the irrational behavior of agents. The framework the authors applied to estimate the expected and unexpected global oil price shocks and price jumps is taken from earlier works studying jumps and volatility in individual stock prices. Zhang and Shang defined the jump intensity of oil return for the present and previous periods. Then, they introduced it to the mean equation of the EGARCH model of automobile markets in the first case and to the variance equation of the model in the second case.

Chan and Maheu (2002) and Maheu and McCurdy (2004) contributed significantly to the framework studying jumps in financial time series. They argued that news about anticipated cash flows and the appropriate discount rate is particularly relevant for stock prices. Then, instead of relating the volatility of stock returns to the flow of information to the market directly, they proposed models of the conditional variance of returns implied by the impact of different types of news.

Maheu and McCurdy analyzed the innovation to returns, which they measured directly from price data. They viewed the latent news process to consist of two distinct components: normal news and unusual news events. The authors assumed these components have different effects on returns and the expected volatility of individual stocks. They postulated that normal news innovations cause gradual changes in the conditional variance of returns, while the second component of the latent news process leads to infrequent moves in returns, which they referred to as jumps. Therefore, the news process induces two components in the equation for returns, which are identified by their volatility dynamics and higher-order moments. Maheu and McCurdy attempted to model these components as normal innovations, and abnormal or jump innovations.

The authors proposed the following interpretation of the described decomposition of the conditional variance of returns into two components. The GARCH component captures the normal time-variation of volatility associated with the predictable decay of the impact from past news innovations to returns, whereas the jump component captures events when significant news occurs that can cause an unusual change in returns.

This framework allows for studying volatility in the EV battery raw materials market. As was already stated, the price of lithium has undergone a significant increase of 265% between 2014 and 2018. Moreover, during the economic recovery after the Covid-19 pandemic, it achieved new record highs[5]. These abrupt changes in prices of raw materials used to produce EV battery packs can cause fluctuation in the daily stock returns of EV producers. Consequently, the methodology of the described works can be applied to define the expected and unexpected price shocks and jumps in the battery raw materials markets. Then, these estimates can be used in the models of individual stock returns of EV producers.

[5] Trading Economics. (n.d.). Lithium 2010-2023 Data | 2024-2025 Forecast | Price | Quote | Chart | Historical. Retrieved May 3, 2023, from https://tradingeconomics.com/commodity/lithium

2.5 Estimation methods in volatility modeling

Maheu and McCurdy (2004) specified that they used maximum likelihood estimation, which is commonly used for this type of problem. In practice, the negative likelihood function is minimized by using an algorithm called an optimizer. One has to choose a particular method for the algorithm and specify starting values of the parameters being estimated. However, there might be issues in the estimation process that can cause an algorithm to fail to find the right solution. Such practical subtleties are not frequently discussed, especially in applied research, but Danielsson (2011) provides a review of issues one may face.

Some likelihood functions are not well-behaved and can have several peaks. Because of that, the algorithm may not be successful in finding the global maximum. To deal with the issue, Danielsson advises trying various random starting values. However, he argues that the disadvantage of MLE is that parameter estimates can be very sensitive to the choice of starting values. Moreover, MLE can become unstable when there are many parameters to estimate.

MLE provides a point estimate of parameters under investigation. However, one can use a specified likelihood function with the Bayesian method, which estimates whole distributions of the parameters. Then, one can summarize his uncertainty about the values of the parameters with both means and variances or multiple quantiles. Importantly, with Bayesian inference, one does not need robust standard errors, as the result of calculations is a distribution of each parameter. Also, interpretation of confidence intervals differs between the two approaches. In the non-Bayesian inference, one should say that as n goes to infinity, meaning that the study repeats, and point estimates and standard errors are calculated an infinite number of times, then 95% of the computed confidence intervals would contain the true parameter value. In the Bayesian inference, which does not rely on asymptotics, the interpretation is more straightforward as one calculates the probability that the parameter value lies in some interval (Richard McElreath, 2020). Applying the Bayesian method, one has to specify the prior distribution of the parameters, or, in other words, to make subjective judgments about their value before seeing the data. However, it can be considered an advantage because one can utilize knowledge from other studies and even simulate the data.

The appearance of powerful sampling algorithms helps the dissemination of Bayesian inference. One of these, Hamiltonian Monte Carlo, has proven a remarkable empirical success (Betancourt, 2014). It is argued that the method is built upon a rich theoretical foundation and is uniquely suited to the high-dimensional problems of applied interest. The purpose of the work is not to go into details of the algorithm mechanics, so the Hamiltonian Monte Carlo is not discussed in depth, as applied researchers do not discuss the Newton-Raphson optimizer, which is usual for MLE.

Although in the 1980s, the Bayesian method was criticized, and there were philosophical and mathematical arguments about the superiority of MLE (Higgins, 1977), the debates seem to end, and it has become more common in applied research. What is of particular interest to this thesis is the work of Dellaportas and Politis (2000), who made a complete Bayesian analysis of GARCH and EGARCH models. They highlighted that with efficient sampling algorithms, local maxima do not present a problem, and posterior densities of functions of the parameters are easily available. However, in the model specification they did not fully utilize the possibility to assign informative priors to the parameters of interest, i.e., assign distributions to parameters that carry some knowledge of the data-generating process. Instead, they preferred assigning uniform priors, i.e., putting equal weight on parameter values. On the other hand, it could be explained by the fact that the posterior distribution converges to the MLE as the number of observations goes to infinity, so priors affect the estimation less if there is plenty of data. Nevertheless, engineers of modern statistical software for Bayesian inference, such as STAN, strongly advise assigning prior distributions to parameters of interest because it makes estimation more efficient[6].

2.6 The contribution of this thesis to the existing literature

This thesis expands the work of Dellaportas and Politis (2000) by applying the Bayesian method to estimate a version of the GARCH-ARJI model with daily data on the EV battery raw materials market. The obtained estimates of jumps in that returns are used in the models of individual stock returns of EV manufacturers. The work of Zhang and Shang (2023) is extended in the sense that the Bayesian method is applied rather than the maximum likelihood estimation, and jumps in the EV battery raw materials market are analyzed rather than in oil returns.

[6] Stan Development Team. (2021). Stan User's Guide. Retrieved May 3, 2023, from https://mc-stan.org/docs/stan-users-guide/index.html

3. Hypotheses and Methodology

3.1 Hypotheses

Following the studies analyzed in the literature review section, the thesis investigates several hypotheses related to returns and jumps in the EV battery raw materials market.

For the first hypothesis, the distributions of jump intensity and jump size parameters are studied. In line with previous research for other commodity markets, the existence of jumps is hypothesized with varying jump intensity.

For the second hypothesis, jump intensity and the mean value of jump size distribution are introduced into the mean equation of individual stock returns of EV manufacturers, studying if jumps in the EV battery raw materials market affect individual stock returns directly.

Formally, the hypotheses are formulated as follows:

1. Jumps in the EV battery raw materials market have varying intensity and size and explain the volatility in the time series of returns.

2. The intensity of jumps in the EV battery raw materials market affects the mean value of EV manufacturers' stock returns. The mean value of the jump size distribution is associated with the mean value of EV manufacturers' stock returns.

This work does not assume any direction of the relationship since the results of previous studies show their unexpected nature that is possibly related to the behavior of market participants.

Important to note that the previously described studies did not investigate the possible effect of the jump size distribution on volatility or the mean value of stock returns.

3.2 Methodology

As was stated above, the traditional GARCH and EGARCH models cannot describe the jumps of financial time series. To include both smooth movements and jumps in returns of the EV battery raw materials market GARCH-ARJI model is used in this paper. The detailed setting of the model is as follows.

$$r_t = u + a_t \tag{1}$$

$$a_t = \varepsilon_{1,t} + \varepsilon_{2,t} \tag{2}$$

where r_t represents the returns of the EV battery raw materials market in period t. As was discussed in the literature review section, the disturbance term is divided into two parts. The first component, $\varepsilon_{1,t}$, is intended to capture the normal time-variation of volatility associated with the predictable decay of the impact from past news innovations to returns. The second component, $\varepsilon_{2,t}$, captures events when significant news occurs that can cause an unusual change in returns. The former is assumed to be a standard GARCH component:

$$\varepsilon_{1,t} = \sqrt{h_t} Z_t; \;\; Z_t \sim NID(0,1) \tag{3}$$

$$h_t = \omega_0 + g_t a_{t-1}^2 + \beta h_{t-1} \tag{4}$$

where g_t is a positive-valued function that will be specified later, and the constraints are $\omega_0 > 0$, $0 > \beta < 1$ to ensure positivity and stationarity.

Specifying $\varepsilon_{2,t}$ refers to the works of Chan and Maheu (2002) and Maheu and McCurdy (2004). Firstly, information set at time $t-1$ consists of the history of returns $\Phi_{t-1} = \{r_{t-1}, \dots, r_1\}$. Also, let $Y_{j,t}$ be jump size where j indicates a jump's number. Then, the sum of jump size from one to N_t and its conditional expectation given the information of the previous period define $\varepsilon_{2,t}$:

$$\varepsilon_{2,t} = J_t - E[J_t|\Phi_{t-1}] \tag{5}$$

$$J_t = \sum_{j=1}^{N_t} Y_{j,t}\,; \; Y_{j,t} \sim NID(\theta_t, \delta) \tag{6}$$

$$\theta_t = \vartheta + \varphi a_{t-1} \tag{7}$$

Thus, the conditional expectation of $\varepsilon_{2,t}$ is zero and the first moment of the jump size distribution can respond to the last period's market unexpected return. This variant of mean specification follows the work of Zhang and Shang (2023).

Meanwhile, N_t is a random variable and has a Poisson distribution:

$$P\,[N_t = j \mid \Phi_{t-1}] = \frac{e^{-\lambda_t}\lambda_t^j}{j!}, j = 0, 1, 2 \ldots \tag{8}$$

$$\lambda_t = \lambda_0 + \rho\lambda_{t-1} + \gamma\xi_{t-1} \tag{9}$$

In words, λ_t is jump intensity and follows an autoregressive process, so the constrains are $|\rho| < 1$ and $\rho \geq \gamma \geq 0, \lambda_0 > 0$ to ensure positivity. ξ_{t-1} is defined as the change in the conditional forecast of N_{t-1} as the information set is updated:

$$\xi_{t-1} = E[N_{t-1} \mid \Phi_{t-1}] - E[N_{t-2} \mid \Phi_{t-1}] = \sum_{j=0}^{\infty} jP[N_{t-1} = j \mid \Phi_{t-1}] - \lambda_{t-1} \tag{10}$$

That means that for each time $t - 1$ one has to update its expectations based on new arrived information in order to use this to estimate λ_t. $P[N_{t-1} = j \mid \Phi_{t-1}]$ is often called filter or posterior probability of the current jump frequency j. Bayes rule is applied to get a formula:

$$P[N_t = j \mid \Phi_t] = \frac{f(r_t \mid N_t = j, \Phi_{t-1})\; P[N_t = j \mid \Phi_{t-1}]}{P[r_t \mid \Phi_{t-1}]} \tag{11}$$

Using the conditional density of returns given that a number of jumps occur, the denominator of (11) is obtained through the summation of the numerator term for $j = 0, 1, 2 \ldots$. In practice, one cannot sum up till infinity, so the summation has to be constrained at some reasonable j assuming that the probability of more jumps than that is zero. Following the work of Maheu and McCurdy (2004), where they used 20 jumps, the same bound is chosen.

The conditional density of returns given that a number of jumps occur requires the calculation of the mean and variance of returns given the same condition. For this, we need to take the expectation of (5). In order to do that, the first two moments of the left-hand side part should be calculated. Standard calculations show:

$$E[J_t^i|\Phi_{t-1}] = \sum_{j=0}^{\infty} E[J_t^i|N_t = j, \Phi_{t-1}] \times P[N_t = j\,|\Phi_{t-1}], i > 0 \quad (12)$$

$$E[\varepsilon_{2,t}|\, N_t = j, \Phi_{t-1}] = E[J_t|N_t = j, \Phi_{t-1}] - \theta_t\lambda_t = \theta_t(j - \lambda_t) \quad (13)$$

$$Var(\varepsilon_{2,t}|\, N_t = j, \Phi_{t-1}) = Var(J_t|N_t = j, \Phi_{t-1}) = (\delta^2 + \theta_t^2)\lambda_t \quad (14)$$

With these calculations at hand, one can integrate out the discrete-valued variable N_t, governing the number of jumps to get the denominator of (11):

$$P[r_t\,|\Phi_{t-1}] = \sum_{j=0}^{\infty} f(r_t|N_t = j, \Phi_{t-1})\, P[N_t = j\,|\Phi_{t-1}] \quad (15)$$

$$f(r_t|N_t = j, \Phi_{t-1}) = \frac{1}{\sqrt{2\pi(h_t + j\theta_t)}} \exp\left(-\frac{(r_t - u + \lambda_t\theta_t - j\theta_t)^2}{2(h_t + j\theta_t)}\right) \quad (16)$$

The final step before the likelihood function is to specify g_t from equation (4). Maheu and McCurdy (2004) argued that important news events that result in a jump may be quickly incorporated into prices and do not affect future volatility much. Or otherwise, news that causes jumps may cause future volatility. To investigate this, they included the estimated number of jumps in the GARCH equation. Although they tried to account for possible asymmetric effects, this work adopts simpler specification:

$$g_t = \exp(\alpha_0 + a_1 E[\lambda_{t-1}|\, \Phi_{t-1}]) \quad (17)$$

$E[\lambda_{t-1}|\, \Phi_{t-1}]$ is the inference regarding average number of jumps from (11).

Then, the log-likelihood function is:

$$L(\Psi) = \sum_{t=1}^{T} \ln(P[r_t\,|\Phi_{t-1}, \Psi]) \quad (18)$$

where Ψ is a set of parameters to be estimated. As was already stated, this thesis adopts the Bayesian method rather than the MLE. In general terms, posterior distribution of parameters is defined by the following formula:

$$p(\Theta|D) = \frac{p(D|\Theta)\, p(\Theta)}{p(D)} \tag{19}$$

where D denotes data and Θ is a set of parameters.

The main difference between the Bayesian method and the MLE is that the MLE treats $\frac{p(\Theta)}{p(D)}$ as a constant whereas the Bayesian method requires specifying prior distributions of parameters. This allows for applying the knowledge obtained either through pure mathematics or the empirical results of previous studies. The former provides us with constraints on parameters according to which the list of potential priors is narrowed. The latter communicates the values of coefficients which can function as a starting point.

There is general advice to regularize priors (Richard McElreath, 2020), i.e. to use ones that are skeptical of extreme parameter values. The regularization is designed to yield smoother, more stable inferences than would be obtained from maximum likelihood estimation or Bayesian inference with a flat prior. However, there is a risk of underfitting the data since narrower priors put lower weight on some parameter values. Because of that, it is argued that the fundamental tool for validating the model after data have been collected is the posterior predictive distribution (Gelman et al., 2017). If the model fits, replicated data generated under the model should look similar to observed data. To put it another way, the observed data should look plausible under the posterior predictive distribution (Gelman et al. 2013).

For the GARCH component of overall volatility, h_t, priors can be defined according to the traditional results of previous studies: ω_0 is often close to 0, whereas β is close to 0.9, and the constraints help define the appropriate variability.

The defining of priors for the jump component requires more attention. Interestingly, the values of coefficients for λ_0 in (9) differ a lot between studies.

Maheu and McCurdy reported values from 0.012 to 0.26 for different stocks. Zhang and Shang obtained 0.0021 for WTI futures, although their model specification slightly differs. Based on these results, one should put more weight on lower values in the prior distribution of λ_0. However, there has been a lot of volatility on the financial markets in recent years, so values of λ_0 more than 0.26 can be assumed to be possible. Further, the prior distributions for ρ and γ have to be set with caution. The model inherently assumes $\rho \geq \gamma \geq 0$, so prior distribution of γ with a lot of weight on higher values might imply ρ close to 1, even though this could be suboptimal in the sense of a worse fit. Even though the degree to which an option for higher values of $\lambda_0$would be left and variance of priors set for ρ and γ are at the mercy of the author of this thesis, the posterior predictive distribution and sensitivity checks are performed to ensure the robustness.[7]

Then, the estimated jump intensity and jump is introduced into models of EV producers stock returns. For the second hypothesis:

$$R_t = u_2 + a_{2_t} + \sum_{l=1}^{L} d_l \lambda_{t-l+1} + \sum_{l=1}^{L} k_l \theta_{t-l+1} \tag{20}$$

$$a_{2_t} = \sqrt{h_{2_t}} X_t; \;\; X_t \sim NID(0,1) \tag{21}$$

$$h_{2_t} = \omega_2 + \alpha_2 a_{2_{t-1}}^2 + \beta_2 h_{2_{t-1}} \tag{22}$$

where λ_{t-l+1} is jump intensity and θ_{t-l+1} is the mean value of jump size distribution for different periods. Here, jump intensity and the mean value of jump size distribution are introduced into the mean equation of stock returns testing the hypothesis that jumps in EV battery raw materials market predict stock returns of EV manufacturers.

Model comparison is accomplished via WAIC (Wannabe, 2010; Vehtari et al., 2017). Its construction allows one to get the standard error, which makes it

[7] Full list of prior distributions is provided in the Appendix

more powerful than traditional methods such as AIC and BIC. Hence, one can say whether the difference in WAIC between the two models is significant in the statistical sense.

4. Data and Empirical Results

4.1 Data sources

As it was already discussed in the literature review section, the key metal for the EV battery raw materials market is lithium. However, its prices are not easily available on the internet. To the knowledge of the author of this thesis, even Eikon database does not contain sufficiently long history of lithium prices that could be used to model returns in the EV battery raw materials market. Hence, one has to come up with approximation.

The rapid proliferation of the exchange-traded funds' industry in recent years gave rise to several ETFs focused on EVs. One of them, Lithium & Battery Tech ETF (LIT), claims to invest in companies throughout the lithium cycle, including mining, refinement, and battery production. Hence, it might be an appropriate proxy for the EV battery raw materials market. However, a thorough examination of its holdings revealed that electric vehicle companies constitute a substantial percentage of net assets. For example, the ETF holds such producers of electric vehicles as Tesla and BYD. Although these companies started to produce EV batteries, they have not been significant players in the mining and refining lithium industries. Hence, jumps modeled for this ETF may be partially influenced by company-specific news related to EV producers. Nevertheless, given the fact that the ETF is diversified enough, and there are no better alternatives available, it is decided to use Lithium & Battery Tech ETF as a proxy for the EV battery raw materials market. Also, given the fact that returns of EV producers affect contemporaneous returns of this ETF by definition, the predictive nature of the models is assessed. In other words, information from the period of EV producers' stock returns is not used in the models.

Stock returns of the following EV producers are used: Tesla (TSLA), BYD Company (BYD). These particular EV producers are chosen with the consideration of a sufficiently long history of stock returns. Other major EV producers, such as NIO, Rivian, XPeng, and Lucid Motors, are left out of the scope of the analysis due to a lack of stock return observations.

The time range of the sample for LIT is from January 1, 2015, to March 31, 2023. All of the above data come from Yahoo Finance database. The formula for asset returns is as follows:

$$r_t = 100 * \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{23}$$

4.2 Empirical Results

4.2.1 Fitting the returns of the EV battery raw materials market

Table 1 shows the results of the GARCH-ARJI model fitting. The distribution for β has a mean value of 0.91, in line with expectations. The distribution of a_1 has a mean value very close to 0, indicating that jumps do not affect the GARCH component of volatility. The mean value of λ_0 distribution is similar to the observed in other studies. The mean values of ρ and γ distributions are sufficiently above 0 meaning that jump intensity follows a process similar to ARMA. The mean value of the parameter for the intercept of jump size, ϑ, is negative, suggesting that negative jumps prevail in the EV battery raw materials market. The mean value of φ is positive, although the probability of being lower than zero is substantial. Nevertheless, this parameter is important since it allows for the variation of jump size over time. The distribution of jump size variance, δ^2, has a mean value of 2.82, suggesting that jumps can be rather big.

Table 1: GARCH-ARJI for LIT

parameter	mean	sd	2.5%	97.5%
μ	0.06	0.03	0.00	0.13
ω	0.01	0.01	0.00	0.03
α_0	-2.65	0.29	-3.22	-2.13
α_1	-0.02	0.29	-0.61	0.50
β	0.91	0.02	0.87	0.93
λ_0	0.12	0.04	0.05	0.22
γ	0.23	0.09	0.05	0.41
ρ	0.43	0.09	0.26	0.62
ϑ	-0.36	-0.72	-0.81	-0.01
ϕ	0.14	0.13	-0.08	0.40
σ^2	2.82	0.84	1.53	4.82
WAIC	7812.9			
dWAIC	-58.8			
dseWAIC	21.7			

Table 1 presents the estimated parameter coefficients for GARCH-ARJI model. The reported mean, standard deviation, and 95% credible intervals (2.5% and 97.5% percentiles) provide information on the distribution of each parameter in the posterior samples. The model fit is compared with a benchmark GARCH(1,1) model. dseWAIC is the standard error of the difference in WAIC.

Figure 1 shows the predictive posterior distribution of data under these two models. It demonstrates 0.5 and 99.5 percentiles of the predicted return distribution for each period for each model. It can be seen that the observed data look more plausible under the model with jumps. GARCH(1,1) model predicts returns that are wider than observed ones, suggesting that it often overestimates volatility, whereas the GARCH-ARJI model fits volatility better. It can be seen especially in times of high volatility, such as jumps in 2020 related to the Covid-19 pandemic.

Figure 1. Posterior predictive distribution of the EV battery raw market material returns under two models

Figure 2 shows the mean of λ_t posterior distributions for different periods. Jump intensity is low most of the time with occasional spikes and subsequent rapid declines, which is in accordance with relatively low mean values of ρ and γ distributions.

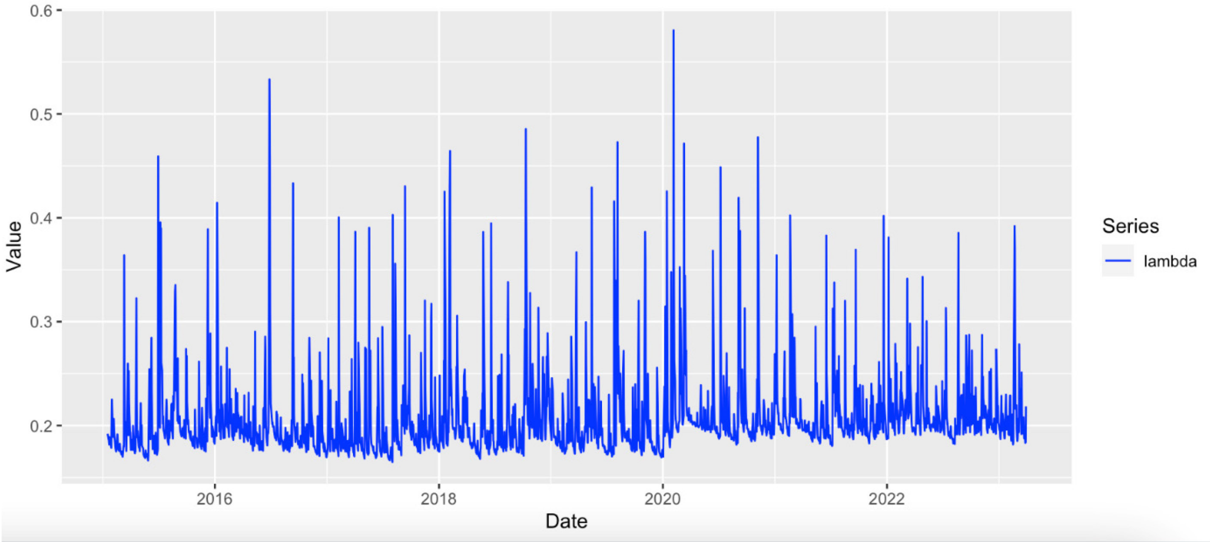

Figure 2. The mean of λ_t posterior distributions

The highest spike in the studied time range happened in 2020 at a time of market stress due to the Covid-19 pandemic. Nevertheless, lower spikes between 0.4-0.5 occur often, suggesting that the model might be useful to capture this type of volatility in the EV battery raw materials market.

Figure 3 shows the mean of θ_t posterior distributions for different periods. The variance of the time series changes in the second part of the time-range. This suggests that θ_t might be volatile from time to time and more sophisticated models might be useful to model the parameters of jump size distribution.

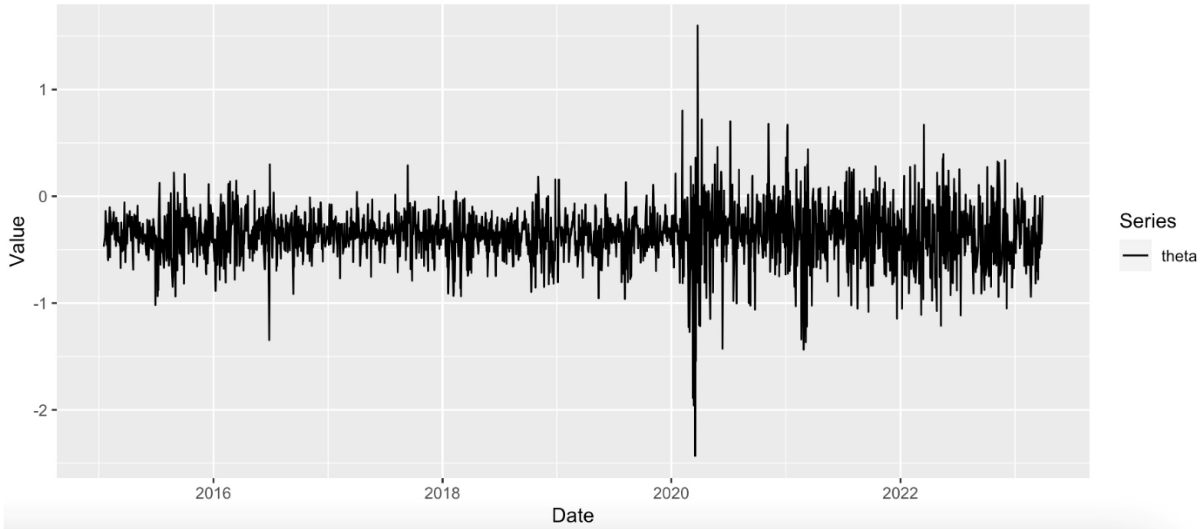

Figure 3. The mean of θ_t posterior distributions

This GARCH-ARJI model is compared to the benchmark GARCH(1,1) model. WAIC of the GARCH-ARJI model is lower, and the standard error of the difference in WAIC is sufficiently low to confirm the statistical significance of the difference. Hence, one can conclude that the GARCH-ARJI fits data better.

4.2.2 Empirical results of the impacts of jumps in the EV battery raw materials market on mean of EV producers stock returns

Table 2 and Table 3 show the results of the inclusion of jump intensity and jump size into mean equations of individual stock returns.

For BYD, the parameter for the first lag of jump size, k_1, has a distribution that does not include 0 in the 95% credible interval. In other words, the standard deviation of the coefficient's distribution is sufficiently low to exclude 0 from possible values of the parameter. The mean value of the parameter is positive, meaning that the higher the mean value of jump size in the previous period, the higher the stock return in the current period. The parameter for the second lag of jump intensity, d_2, has a distribution that does not include zero in the 95% credible interval. The mean value of the parameter is negative, indicating that a higher jump intensity two periods earlier leads to a lower return in the current period. This model for BYD returns is compared to the benchmark GARCH(1,1) model. WAIC of this model is lower, and the standard error of the difference in WAIC is sufficiently low to confirm the statistical significance of the difference in the conventional sense.

Table 2: Hypothesis 2, BYD

parameter	mean	sd	2.5%	97.5%
μ_2	0.79	0.15	0.49	1.09
ω_2	0.24	0.07	0.11	0.39
α_2	0.12	0.02	0.09	0.17
β_2	0.84	0.03	0.78	0.88
κ_1	0.60	0.21	0.20	1.01
κ_2	0.07	0.21	-0.33	0.49
δ_1	-0.67	0.43	-1.50	0.18
δ_2	-0.89	0.42	-1.68	-0.06
WAIC	9286.9			
dWAIC	-26.1			
dseWAIC	11.23			

Table 2 presents the estimated parameter coefficients for a model that includes both jump intensity and jump size into the mean equation of BYD stock returns. The model incorporates two lags of each. The reported mean, standard deviation, and 95% credible intervals (2.5% and 97.5% percentiles) provide information on the distribution of each parameter in the posterior samples. The model fit is compared with a benchmark GARCH(1,1) model. dseWAIC is the standard error of the difference in WAIC.

For TSLA, the results show that only the parameter for the first lag of jump size, k_1, has a distribution that does not include 0 in the 95% credible interval. In other words, the standard deviation of the coefficient's distribution is sufficiently low to exclude 0 from possible values of the parameter. Other parameters have distributions that do include 0 with sufficient probability, which can be seen from the parameters' 95% credible intervals. The standard error of the difference in WAIC is not sufficiently low to confirm that the model has lower WAIC than a benchmark GARCH(1,1) model.

Table 3: Hypothesis 2, TSLA

parameter	mean	sd	2.5%	97.5%
μ_2	0.35	0.21	-0.04	0.78
ω_2	0.57	0.18	0.27	0.98
α_2	0.07	0.01	0.05	0.10
β_2	0.89	0.02	0.84	0.93
κ_1	0.63	0.28	0.07	1.18
κ_2	0.23	0.29	-0.35	0.79
δ_1	0.07	0.47	-0.86	1.04
δ_2	0.10	0.46	-0.81	1.02
WAIC	10799.2			
dWAIC	-3.1			
dseWAIC	6.03			

Table 3 presents the estimated parameter coefficients for a model that includes both jump intensity and jump size into the mean equation of TSLA stock returns. The model incorporates two lags of each. The reported mean, standard deviation, and 95% credible intervals (2.5% and 97.5% percentiles) provide information on the distribution of each parameter in the posterior samples. The model fit is compared with a benchmark GARCH(1,1) model. dseWAIC is the standard error of the difference in WAIC.

Overall, the returns of both stocks seem to react positively to the mean value of jump size distribution. Also, the mean values of parameter coefficients are similar, suggesting that the effect might be persistent across the cross-section of stocks. This finding is particularly interesting given the fact that the model uses only lagged variables, so the information is known before the current trading day.

The studies discussed in the literature review section did not investigate the effect of jump size in a commodity market on stock returns. Hence, the finding might be relatively unknown across market participants. Also, the parametrization of jump size is simple and further improvements are possible. For example, one can assume that the variance of jump size might not be constant and changes over time.

5. Conclusion

This thesis examined the volatility in the EV battery raw materials market through the GARCH-ARJI model and used jump intensity and jump size estimated from the model to predict the daily returns of EV producers. The EV battery raw materials market was defined through the ETF specializing in lithium miners and battery producers. Two EV producers were chosen to predict returns: Tesla and BYD Company. Data regarding daily prices of the ETF and the companies were collected from January 1, 2015, to March 31, 2023.

The model for stock returns of individual EV producers was built using only lagged variables. This feature is partially related to the fact that the ETF, which approximates the EV battery raw materials market, held stocks of the EV producers. Hence, returns of these affect the return and volatility of the ETF.

The hypothesis of jumps' existence in the EV battery raw materials market was not rejected. For each period, the posterior predictive distribution of returns was compared with that of a benchmark GARCH(1,1) model. The GARCH-ARJI model fitted volatility better. Also, this fact was confirmed through lower WIAC, and the difference was statistically significant, at least on the 5% level. The highest jump spike over the time range under investigation happened during the market stress related to the Covid-19 pandemic.

The hypothesis of the effect of jumps on individual EV producers' stock returns was not rejected completely. Jump intensity seems to affect the stock returns of only one EV producer, whereas jump size affects the stock returns of both under investigation. The result extends the work of other authors who studied only the effect of jump intensity. Furthermore, the jump size distribution might change over time, and further work could extend this thesis by different specifications of jump size distribution. Hence, the predictive power of jump size for daily stock returns might increase because jump size would be estimated more precisely.

Overall, the results showed that the GARCH-ARJI model can be used to model volatility in commodity markets, and one can utilize the estimates of the parameters to study stock returns of relevant stocks. Also, the thesis demonstrated that the Bayesian method can be successfully applied to model time-series data.

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List of Appendices

Appendix no. 1: Full list of prior distributions used for the final models' estimation

GARCH-ARJI model:

$u \sim N(0, 0.1)$

$\omega_0 \sim N(0, 0.5)$

$\alpha_0 \sim N(-2.5, 1)$

$a_1 \sim N(0, 1)$

$\beta \sim Beta(2.5, 1)$

$\lambda_0 \sim N(0, 0.15)$

$\rho \sim N(0.4, 0.15)$

$\gamma \sim N(0.3, 0.15)$

$\vartheta \sim N(0, 0.5)$

$\varphi \sim N(0, 0.5)$

$\delta \sim exponential(1)$

Stock return model:

$u_2 \sim N(0, 0.1)$

$\omega_2 \sim N(0, 0.5)$

$\alpha_2 \sim Beta(1, 2.5)$

$\beta_2 \sim Beta(2.5, 1)$

$d_1 \sim N(0, 0.5)$

$d_2 \sim N(0, 0.5)$

$k_1 \sim N(0, 0.5)$

$k_2 \sim N(0, 0.5)$