

Motivated by recent investigation of several particular situations, we study various quantum graphs equipped with circulant vertex couplings and characterize their spectral properties. The case of a star graph is analyzed in full generality, and the same applies to the condition determining the spectrum of periodic rectangular lattices. Special attention is paid to permutation-invariant vertex conditions on a rectangular lattice, as well as to a coupling interpolating between the δ and 'rotational' coupling on a quantum chain, with the focus on low- and high-energy bands and the discrete spectrum. We describe not only their dependence on the topology and the vertex condition, but also provide detail of their behaviour with respect to the parameters involved.