Abstract:

In four-dimensional general relativity, the algebraic classification has played an important role in study of spacetimes, including, but not limited to the search for new solutions to the Einstein equation. In the present work, we study its recent generalization to higher dimensions, based on categorizing Weyl tensor frame components by their transformation properties under boosts. Specifically, we concentrate on its connection to two well-established concepts.

Kaluza–Klein reduction can be regarded as a relation between spacetimes of different dimensionality. As such, it is desirable to be understood in the context of the higher-dimensional algebraic classification. We study algebraic properties of Weyl tensors related by Kaluza–Klein reduction. Specifically, we concentrate on reduction of vacuum spacetimes by one spatial Killing direction and investigate Weyl-alignment multiplicities of two related null directions that are parallel in a gauge where they are perpendicular to the Maxwell potential. We express relations of various quantities of the related spacetimes, such as Riemann and Weyl tensors, optical matrices and non-geodeticities, revealing some interesting consequences regarding reduction of Kundt spacetimes and of spacetimes admitting a geodetic null direction. Based on this, we formulate algebraic conditions necessary and sufficient for the Kaluza–Klein lift to preserve the Weyl alignment type. In the cases of multiplicities three and four with nonvanishing Maxwell field, where both spacetimes turn out to be Kundt, we provide an explicit form of the scalar potential demanded by the alignment preservation in six dimensions and greater, and discuss some qualitative differences from the four-dimensional case.

Goldberg–Sachs theorem is an interesting result concerning four-dimensional Einstein spacetimes that relates geometrical properties of null congruences with algebraic properties of the Weyl tensor. In an important application, it can be used as a constraint restricting form of the optical matrix of algebraically special Einstein spacetimes. We present a partial generalization of this constraint to six dimensions, by providing specific forms that an optical matrix must obey, assuming that it has rank four or three and the respective null direction is geodetic and aligned with the Weyl tensor, with alignment multiplicity of two. Doing so, we effectively reduce the number of free parameters of such optical matrices from ten to five.