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Faculty of Science
Department of Mathematical Sciences
Division of Applied Mathematics

28 June 2022

Subject: Report on Michal Pavelka's habilitation thesis

To the habilitation committee:

I provide below my report on the habilitation thesis of Michael Pavelka entitled "Multiscale geometric approach to continuum mechanics and thermodynamics". My report takes into account both the summary of the work provided in Chapters 1-4 of the thesis and the 6 articles that are part of the habilitation.

The research articles are overall of good quality: they are fairly technical for the field, including many elements of abstract algebra and differential geometry, but do touch on important physical applications, such as non-Newtonian fluids and charged systems. The candidate's own contributions and ideas for future research are well summarised in the thesis: they are substantial and interesting, and leave me in no doubt that the candidate carries important research in the field of continuum mechanics and thermodynamics. For this reason, I recommend acceptance of the habilitation.

Please do not hesitate to contact me if you require more information.

Regards,

[redacted]

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Report on
Multiscale geometric approach to continuum mechanics and thermodynamics
by Michal Pavelka

Summary

Dr. Pavelka's research centers on partial differential equations (PDEs) arising in physics and chemistry and on studying these equations within the GENERIC framework, developed in the 1990s. Many PDEs have a GENERIC form, as has been shown over the years, composed of two parts: a conservative part, determined by a Poisson bracket and an energy function, and a dissipative, gradient-like part, determined by an entropy function and a dissipation function.

In the articles submitted as part of his habilitation thesis, Dr. Pavelka studies various aspects of the GENERIC formalism, including its consistency with respect to thermodynamics, how some PDEs fall into this formalism, and how dissipative evolutions can arise from conservative evolutions by means of averaging or coarse-graining. In the introductory chapters, the thesis summarises the theory of conservative evolutions, as embodied in Hamiltonian dynamics and described by Poisson brackets, and then moves on to summarise how GENERIC describes this type of evolutions in addition to dissipative evolutions. The 3rd chapter discusses the contributions of the candidate contained in the 6 articles submitted, which I summarise next.

Main contributions

In the order in which the articles are listed:

Article 1: Shows that a class of PDEs known as symmetric hyperbolic thermodynamically compatible equations (SHTC) are compatible with the GENERIC formalism. Examples of such equations include the Navier-Stokes equation of fluid dynamics, as well as PDEs used to describe elastic solids and non-Newtonian fluids.

Article 2: Studies the Lie algebra structure of PDEs arising in continuum mechanics in the Lagrangian and Eulerian frames.

Article 3: Studies the Poisson bracket structure of PDEs arising in the continuum description of systems of charged particles, such as electromagnetic fields advected by charged fluids. Various physical phenomena related to magnetization, polarization, and dissipation are discussed mathematically.

Article 4: Discusses the mathematical implications of having non-convex dissipation potentials in the GENERIC formalism, considering the case of shear transition in non-Newtonian fluids as a potential physical application.

Article 5: Shows how dissipative evolutions can arise in conservative dynamical systems when degrees of freedom are averaged out. The technique is referred to as Ehrenfest regularisation and is illustrated with simple examples related to rotating bodies, fluid mechanics, and kinetic theory.

Article 6: Shows how a theory of coarse-grained (or state-reduced) evolutions, put forward by Turkington in 2012 for nonequilibrium systems, can be cast in the form of GENERIC equations.

The common thread that I see in these articles is an interest in understanding the properties of PDEs from point of view of the GENERIC formalism, in particular, in understanding the physical sources of the conservative and dissipative parts of these PDEs. Of these, I would consider the last two articles to be more fundamental from a physical point of view, as they try to put forward mechanisms whereby dissipative evolutions can arise from conservative evolutions. This goes at the heart of one of the main challenges of statistical mechanics, namely, to understand how dissipative evolutions arise at the macroscopic level from conservative evolutions at the microscopic level, and at justifying ultimately the GENERIC formalism from first principles. Some of the open questions listed in the concluding section of the thesis touch on these issues.

Recommendation

The research articles covered by the habilitation are overall of good quality: they are relatively technical for the field, including many elements of abstract algebra and differential geometry, and are fairly complete, amounting to about 160 pages in total. However, they do relate to concrete physical applications, such as non-Newtonian fluids and charged systems, and are for this reason important, in my opinion.

Whether or not the GENERIC formalism is valuable as a theory of physical PDEs is yet to be determined, but I feel that it is important to study its structure and to interpret known PDEs without this formalism, as Dr. Pavelka does, so as to get a sense of its predictive power. The candidate's own contributions in this direction are well summarised in the thesis: they are substantial and interesting, and leave me in no doubt that the candidate carries important research in the field of continuum mechanics and thermodynamics. This is also confirmed by the recent publication of a monograph with de Gruyter in 2018, summarising his recent work with his collaborators Klika and Grmela.

Given all these contributions, I would recommend that the habilitation be accepted. I do not have specific questions for the candidate, as such, nor corrections, except maybe for the english throughout, which could benefit from having more articles ('the' especially). The candidate will see fit, I'm sure, to proofread and correct the thesis before the final submission.

Turnitin report

The final overlap of 34% listed in the Turnitin report seems high, but is, in effect, just a consequence of the work summarised in the thesis being already published in the 6 papers submitted, as well as in the de Gruyter monograph. Most items listed in the Turnitin report are 1% or below, so I am entirely satisfied by the report.