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Review of Martin Balko's habilitation thesis

Let me state upfront that I strongly support Martin Balko's promotion to the associate professor position at Charles University in Prague. My support is based on his thesis and, moreover, on my personal experience: I know him personally and consider him a talented and promising young mathematician.

Martin Balko's main field of interest is combinatorics, linear algebra, and discrete geometry. In this direction he has reached several important results ten of those are included in his habilitation thesis. I will describe a few of them now.

A central topic of the thesis is Ramsey theory, its connection to discrete geometry, and in particular ordered Ramsey numbers of ordered graphs. This is a new topic, introduced and explored by Martin (with coauthors) in his PhD thesis (see [Bal+20]), and independently and about th same time by Conlon, Fox, Lee and Sudakov [Con+17]. Since then there has been a huge progress on ordered Ramsey numbers and this topic has become an exciting new and quickly developing area in extremal combinatorics. An important result (discovered in [Bal+20] and [Con+17]) is that the ordered Ramsey numbers may differ from ordinary Ramsey numbers for sparse graphs which is not the case for dense graphs.

Without giving the necessary definition we state another significant result from [Bal+20]: the ordered Ramsey number of an ordered graph G on n vertices is at most Cn128k where k is the bandwidth of G and C = C(k) is a constant that depends only on k. This answers a question from [Con+17] where they prove the same inequality in the special case when G is an ordered matching. More recently Martin proved (with coauthors) several important new results on ordered Ramsey numbers of oriented nested matchings in [BP21], and of random ordered matchings in [BJV19] that I cannot describe in detail in this short review. The latter paper [BJV19] also treats the minimum ordered Ramsey number $R(G)$ of an ordinary graph G and solves an open problem from [Con+17] by showing that $R(G)$ is a superlinear function of n for almost every 3-regular graph G where n is the number of vertices of the graph. Another important result from the same paper gives linear upper and lower bounds on the ordered Ramsey numbers of ordered alternating paths. They also determine the ordered Ramsey number of monotone cycles exactly, a remarkable result.

Ordered Ramsey numbers can be defined for ordered k-uniform hypergraphs as well (see [MS14] or [CFS11]). In [BV21] Balko and Vizer establish several bounds on these numbers including for instance an upper bound (Corollary 1.3.3) on the ordered Ramsey number of an ordered 3-uniform hypergraph on n vertices with fixed maximum degree. The precise statement of this important result is too long to be given in this review.

The paper "Drawing Graphs Using a Small Number of Obstacles" was awarded the "Best paper award" at the conference Graph Drawing 2015 and appeared in Discrete and Computational Geometry [BCV18]. It is a joint work with J. Cibulka and P. Valtr that studies a recently introduced graph parameter called obstacle number which takes its origins in geometric visibility problems. Martin and his coauthors make the so-far deepest contribution to the relatively new theory of obstacle representations by showing that the obstacle number of every n-vertex graph is at most $O(n \log n)$. This, in particular, refutes a conjecture of Mukkamala, Pach, and P´alv¨olgyi. The technique developed in this paper is elegant and also applicable to other graph drawing problems. For instance, it gives tight bounds on the complexity of a collection of faces in an arrangement of line segments.

Another problem, connected to visibility, is about is about the convexity ratio, $c(S)$, of a Lebesgue measurable set $S \subset Rd$, and about the Beer index of convexity, $b(S)$, of a Lebesgue measurable set $S \subset Rd$. The latter can be considered as the probability that for two points A, B chosen randomly, independently and uniformly from S the segment AB lies in S. The paper [Bal+17] gives several results on the relation between these two parameters. For instance they answer, in the affirmative, a conjecture of Cabello et al. by showing that $b(P) \leq 180c(P)$ for a simple polygon P (their result is in fact stronger and more general). Further, in an important new development, they extend the $b(P) \leq 180c(P)$ result for higher dimensional object. This requires a suitable (and unexpected) extension of the Beer index to higher dimensions.

Recently, Martin (with coauthors) has solved a long-standing conjecture about the minimum number of so-called 5-holes a set of n points in the plane can have. A k-hole in a set S of n points in the plane (in general position) is a k-element subset of S that is in convex position whose convex hull contains no further point from S; of course k ≥ 3 here. Let hk(n) denote the minimum number of k-holes in an n element set S ⊂ R2. It is known that hk(n) = 0 for k > 6. The determination of the order of magnitude of hk(n) for k = 3, 4, 5, 6 is close to my interests as I worked on it. In the paper [Aic+20] Martin (with coauthors) shows that h5(n) is superlinear in n. This is a positive answer to a well-known problem that has been around since the 80s and that is posed, for example, in the book of Brass, Moser, and Pach. This result is a very significant improvement over numerous previous estimates, which were all only linear in n. The proof is a remarkable combination of combinatorial arguments, geometric insight, and careful computer-aided calculations. This paper also contains the currently strongest lower bounds on h3(n) and h4(n). The same questions for randomly chosen ground set S ⊂ R2 are considered in [MSV21b].

I was asked to comment on the Turnitin page of the thesis. So I have checked it and the outcome is that the thesis contains no trace of plagiarism. The many coincidences are due to the fact that the thesis is formed by Martin Balko's papers that have appeared in print and Turnitin has detected those coincidences.

As his thesis shows Martin Balko is a talented and promising young mathematician. I'm sure he will continue to work very successfully. I support his promotion in the strongest possible terms.

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