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# Opponent's review of the habilitation thesis of Tomáš Ledvinka

I have been asked to provide a report on the habilitation thesis of Dr. Ledvinka, and I am happy to do this. I do not know him personally; what I am writing here is entirely based on the documentation which has been sent to me and what I know of his reputation.

As indicated by the thesis title, his research has been focussed on "Dynamical Effects in Weak and Strong Gravitation" including effects in many-body systems having fairly weak gravity; ones for spinning test particles in the vicinity of a rotating black hole; dragging of space-times by gravitational waves, and gravitational collapse of gravitational waves due to their self-interaction. In addition to standard analytical calculations, extensive use has been made of state-of-the-art computing, utilising computer algebra systems for facilitating long and complicated analytic calculations as well as using the techniques of numerical relativity for the collapse calculations.

The style of the thesis is that Chapter 1 presents an extended commentary on six selected original papers, placing them in a wider context and giving further details, while Chapter 2 consists of reprints of the six papers concerned. In this report, I will comment on the six papers together with the associated commentary on them. I note that the candidate also has ten other refereed papers on related subjects listed in the NASA ADS Astrophysics Data System.

Section 1.1 of Chapter 1 "Hamiltonian description of gravitational interaction" starts by dealing with the post-Minkowskian approximation for N-body systems in general relativity, in which the approximation consists of considering the gravitational terms as being small quantities whereas velocities are only limited by the speed of light (a notable difference from the commonly-used post-Newtonian approximation). The related highlighted paper [P1] is very clearly written and outlines the steps taken in reaching a closed form for the Hamiltonian representing a system of gravitationally interacting particles within this approximation. Getting this closed form was a considerable achievement at the time of publication of the paper (2008) and this is the most cited of the papers highlighted here. In the thesis, it is noted that some more recent work has now gone beyond it, but I think that the paper remains a very notable one. Following this, Section 1.1 continues by considering the Hamiltonian for a spinning test particle within Kerr spacetime (Paper [P2]), extending earlier work on this. A defect was found in the earlier work in that the formulation was found not to give conservation of angular momentum for orbital motion around a non-rotating (Schwarzschild) black hole. This was traced to neglect of higher-order terms in the previous Hamiltonian and steps were taken to correct it. The main astrophysical application of this work is to extreme-mass-ratio-inspirals (EMRIs), in which a small black hole spirals into a large one, located in a galactic centre, due to emission of gravitational waves. These are expected to be among the most important sources for the forthcoming LISA gravitational-wave detector. Another application of the methodology is to analysis of possible chaotic behaviour. The paper is extremely thorough and I think makes a very important contribution.

Section 1.2 of Chapter 1 “Dragging effects of gravitational waves” addresses issues connected with Mach’s Principle concerning how local inertial frames are determined by the influence of distant gravitating objects. Those objects were traditionally thought of as being just stars and galaxies but now one needs to include other constituents, particularly dark energy and dark matter. Since gravitational waves also carry energy and angular momentum, it is relevant to consider whether they can exert significant Machian influences as well. Paper [P3] studies effects of rotating gravitational waves in vacuum asymptotically flat spacetimes, using the model of an ingoing rotating shell of radiation, which later transforms into an outgoing one. It is demonstrated that this does give rise to dragging of inertial frames despite the fact that energy and angular momentum of gravitational waves cannot be localised in general, unlike the situation for ordinary matter (an issue which has been controversial in the past). It is also found that dragging of inertial frames at the origin occurs instantaneously, with no time delay depending on the position of the shell (a very surprising result although a convincing explanation is given). The apparent motions of “fixed stars” on the “celestial sphere” as seen from the local inertial frame at the centre, looking through the rotating gravitational waves, are calculated and displayed. Further aspects of frame dragging by material shells and gravitational waves are examined in Paper [P4]. While I am not surprised by the similarity in behaviour between gravitational waves and matter in this respect, I do find all of this quite interesting.

Section 1.3 of Chapter 1 “Gravitational collapse of gravitational waves” reports on numerical investigations of critical collapse behaviour for black hole formation by means of collapse of gravitational waves due to their self-interaction. Paper [P5] deals with finding a suitable slicing condition for this, given that previous work had encountered problems due to the standard moving puncture method not producing sufficiently smooth slices when evolving near-critical Brill wave data. Maximal slicing does not have this problem but is too computationally expensive for using here. It is shown however that a less expensive approximate form of it (referred to as ‘quasi-maximal’ slicing) can be successfully used here without spoiling well-posedness or leading to constraint violations. Using this methodology, it is shown how the created black holes settle down to being Schwarzschild ones, something which it was not possible to show before. Paper [P6] carries this further. For different families of near-critical initial data, universal “echoes” are observed similar to those seen previously for scalar-field collapse. I think that this work shows considerable inventiveness and ingenuity.

I have been asked to comment on the result of the Turnitin plagiarism check. I can see that this methodology could be useful in some circumstances but think that results from it need to be treated with care. I am told that the score of 15% given for this Habilitation thesis might be viewed as rather problematic but having some substantial overlap is inevitable for a thesis which is largely a commentary on published papers written by the author and others. I am completely confident that there is no issue whatever of plagiarism here.

In summary, from what I have read in the habilitation thesis and attached documents, together with what I know about the candidate’s reputation and overall scientific production, I am pleased to support granting of the habilitation to Dr. Ledvinka.

[redacted]

Oxford, 12th June 2022