O‘ZBEKISTON FANLAR AKADEMIYASI
ULUG‘BEK NOMIDAGI
ASTRONOMIYA INSTITUTI

100052, Toshkent sh.
Astronomiya ko‘ch., 33-uy
Tel: (+99871) 2358102
Faks: (+99871) 2344867
E-manzil: info@astrin.uz
Veb: http://www.astrin.uz

ULUGH BEG ASTRONOMICAL
INSTITUTE OF THE UZBEKISTAN
ACADEMY OF SCIENCES

33, Astronomy St.,
Tashkent, 100052, Uzbekistan
Tel: (+99871) 2358102
Fax: (+99871) 2344867
E-mail: info@astrin.uz
Web: http://www.astrin.uz

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R E P O R T

of the opponent on the Habilitation Dissertation of Dr Tomas Ledvinka
entitled ‘Dynamic Effects in Weak and Strong Gravitation’,
presented for the scientific title on Relativistic Astrophysics and Theoretical Physics

The gravitational interaction invented by Einstein as a dynamic interplay of the spacetime geometry and the fields spreading in this curved canvas led to enormous theoretical discoveries and surprising explanations of otherwise puzzling astronomical observations. The mathematical methods developed in the framework of general relativity include new approaches to differential geometry, classical field theory, analytic approximation methods, and the powerful numerical methods.

Taking into account that the signatures of the general relativity should be strongest in the astrophysical processes occurring in the close vicinity of massive black holes and can be decoded from the observations of the related phenomena the author of the habilitation thesis presents the results of his extensive theoretical studies related to mathematical issues of gravity and the various phenomena connected to astrophysical processes arising around and by black holes governed by general relativity.

The Thesis by Dr Tomas Ledvinka presents several topics contained in six selected original papers [P1-P6] utilizing these methods. The first topic is related to Hamiltonian description of gravitational interaction. Studies of the dragging effects of gravitational waves are devoted to Machian effects in general relativity. Research devoted to the gravitational collapse of gravitational waves belongs to numerical relativity. The topics discussed and related to the dynamics of fields and bodies play an important role both in the weak field limit and extremely strong gravitation fields, all in the framework of classical general relativity in 3+1 dimensions.

The habilitation dissertation of Dr Tomas Ledvinka starts from the “**Overview of studied problems”** where he underlines the results of his important research on the imprints of the strong gravitational fields of black holes constructed in the framework of general relativity (GR). The author summarizes that in the habilitation thesis main attention is focused on the properties of the black holes including the treatment of Hamiltonian description of gravitational interaction. The chapter 1 consists of Hamiltonian for many-body gravitating system in the post-Minkowskian approximation, Hamiltonian for a spinning test particle in Kerr spacetime, Dragging effects of gravitational waves, Gravitational collapse of gravitational waves.

The chapter 2 entitled “**Selected original papers**” contains list of selected original papers
[P1] Relativistic Closed-Form Hamiltonian for Many-Body Gravitating Systems in the Post-Minkowskian Approximation;
[P2] Comparing Hamiltonians of a spinning test particle for different tetrad fields;
[P3] Effects of rotating gravitational waves;
[P4] Rotation of inertial frames by angular momentum of matter and waves;
[P5] Slicing conditions for axisymmetric gravitational collapse of Brill waves;
[P6] Universality of Curvature Invariants in Critical Vacuum Gravitational Collapse.

Due to the nonlinearity of general relativity, even analytical treatment of the two-body problem is not possible. However, various analytic approximation techniques can be applied, usually in the form of an expansion in a small parameter, such as velocity of motion, mass ratio of the interacting bodies, or the gravitational constant. In the latter approach, gravitating bodies can move with arbitrary velocity and it is thus called post-Minkowskian (PM) approximation. If no other interaction of the bodies is assumed (which is true for the black holes), the only source of their acceleration is the gravitational interaction, so in the first PM approximation the bodies "feel" only that part of the gravitational field of the other bodies which corresponds to their uniform, unaccelerated motion. It has been justified by the author in [P1] that this approximation allows removing off the field degrees of freedom from the problem and finding a closed-form Hamiltonian for a system of gravitationally interacting bodies which is a **fundamental result**.

Astronomical observations indicate that magnetized neutron stars and black holes are rapidly rotating, and due to this reason, the Hamiltonian description of the motion of gravitating spinning bodies plays an essential role in relativistic astrophysics. In a certain (the so-called pole-dipole) approximation, a spinning body is represented by a particle endowed with position, momentum, and spin satisfying the respective Poisson-Dirac brackets for such canonical quantities. It has been justified by the author in [P2] that a certain choice of local coordinate systems leads to a simpler Hamiltonian description of the motion of a spinning test particle in the vicinity of the Kerr black hole which is an **important result**.

According to the general relativity, the frame-dragging effect is induced in spacetime, which is due to the rotation of the gravitational source with stationary distributions of mass energy. The frame-dragging effects are caused by mass-energy currents and so-called gravitoelectromagnetism, which is analogous to the magnetism of classical electromagnetism. While the influence of the rotating matter on the gyroscope it surrounds is thoroughly studied starting 1918, the case of gravitational waves is more subtle. The author explores weak gravitational waves in a Minkowski spacetime and studies their influence on the inertial frame of an observer surrounded even though not exposed to these waves. Explicit formulas for the second-order quantities describing the dragging effects have been obtained by the author in [P3,P4] when the amplitude of the gravitational wave is taken as an expansion parameter, for a particular shape of the rotating gravitational wave packet describing the first-order approximation of a gravitational wave around a Minkowski background. It is a really **original and pioneering result**.

For stronger gravitational waves, the nonlinear effects open the possibility that huge energy can be concentrated into a small enough volume that a black hole is formed. This problem must be treated numerically and despite great breakthroughs in numerical relativity, certain aspects of this process are still not understood. To perform such computations, the obvious feature of general relativity (which is a gauge theory) turns out to be a serious obstacle as the strong dynamic gravitational field cannot be decoupled from the strong dynamics of the coordinates used in the description of the spacetime. From one side, the nonlinearities of the general relativity imply the existence of black holes containing the physical singularities, and on the other side, these nonlinearities may lead to singular coordinates. Apart from an obvious increase of computing power, the breakthroughs made in the 2000s involved understanding how to choose coordinates in highly dynamic spacetimes with collapsing matter or orbiting and merging black holes. Unfortunately, the developed moving puncture method does not work for the collapse of gravitational waves. This makes the formation of black holes by the collapse of gravitational waves an important, yet a purely theoretical problem. In [P5], the author has found a recipe for the coordinates numerically constructed during the gravitational wave collapse, which enables to follow this process during near-critical gravitational collapse, i.e., in situations where fine-tuned initial data lead to the creation of extremely small black holes. A detailed analysis of these simulations performed by the author in [P6] revealed surprising features of a phenomenon sought for almost three decades on the self-similarity in near-critical spacetimes arising in the collapse of gravitational waves. These are **important results** having astrophysical applications.

I kindly comment the originality check of the habilitation dissertation of Dr Tomas Ledvinka done by Turnitin system. I kindly note that the high percentage of coincidence in this plagiarism check has a text overlap nature and is due to the fact that Dr. Ledvinka´s dissertation is very specific and made as a collection of reprints of the six most important published papers co-authored by Dr. Ledvinka, and accompanied by his commentary. All presented results are **original** ones and there is no doubt about it.

The habilitation dissertation of Dr Tomas Ledvinka is a complete survey. The scientific results presented for the habilitation defense are completely new and original. The main results of the dissertation have been widely presented at seminars and international conferences and have been extensively published in the leading international refereed journals. Numerous research papers were published by the author and 6 of them which are associated with the presented habilitation thesis correctly reflect the content of the dissertation.

Reliability of the research results is provided by use of modern methods of general relativity, field theory of gravity and the theoretical physics and highly effective analytical and numerical methods and algorithms. It has been performed careful comparison of consistency of the obtained theoretical results with observational data and results of other authors. The obtained results and conclusions are well consistent with the main provisions of the field theory of gravity.

The disadvantage of the dissertation is the presence of a few stylistic errors and typos. There is now a wide exploration of the obtained results for astrophysical applications. In general, they can not affect the assessment of the high scientific value of the results presented.

The scientific and practical significance of the work is determined by the ability of the developed formalism in the dissertation to analyze the properties of gravitational compact objects and gravitational waves produced by them. Moreover, the results of research can be used to obtain estimates of different parameters of black holes such as rotation and other parameters, as well as the parameters that appear due to the higher-order corrections in the theory of gravity. Results can also be useful for the analysis of the nature and dynamics of the gravitational field, in the development of observational experiments and criteria for the detection and identification of the compact gravitational objects etc. Thus, the habilitation dissertation is performed at a high scientific level and the results can be described as a **big scientific achievement**.

The habilitation dissertation 'Dynamic Effects in Weak and Strong Gravitation' meets all the requirements of the regulations on the procedure for the award of scientific titles in the Czech Republic and may be admitted to the defence at the Institute of Theoretical Physics of Charles University, Faculty of Mathematics and Physics. The author, Dr Tomas Ledvinka certainly deserves the award of the scientific title of Docent ("venium docendi" or Associate Professor).

ЎзР ФА Астрономия институтининг
булим мудири ф.-м.ф.д., проф. [redacted] Б.Ж. Ахмедов

The referee report has been composed by
Professor, Dr Sc Bobomurat Ahmedov
Head, Theoretical Astrophysics Department
Ulugh Beg Astronomical Institute, Astronomy St. 33
Tashkent 100052, Uzbekistan
E-mail: ahmedov@astrin.uz
Phone: +998-71-2358102(home)/2355786(office)/+998977088068 (mob)