Université de Paris

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Report on the habilitation thesis of Sebastian Schwarzacher

The manuscript submitted by Sebastian Schwarzacher for his habilitation thesis goes back to May 2021, and covers essentially the contents of five papers :

- an article with B. Muha (ref. [142], now in Annales de l'IHP),
- an article with Dominique Breit (ref. [23], now in Annali della Scuola Normale di Pisa),
- an article with M. Sroczinski (ref. [169], now in SIMA),
- a preprint with B. Benesova and M. Kampschulte (ref. [13]),
- an article with G. Gravina, K. Tuma and O. Souchek (ref. [91], now in Journal of Fluid Mechanics).

These papers contain entirely original works, as confirmed by the plagiarism check of the Turnitin system. The general theme of all these works is the interaction between a fluid and an elastic structure. Two kinds of situations are encountered:

i) a fluid interacting with an elastic surface: either an elastic plate (that is a horizontal wall with allowed normal deformation), or an elastic shell (that is an arbitrary surface with allowed normal deformation).

ii) an elastic solid immersed in a fluid.

Actually, these two generic situations cover themselves different mathematical models, both for the fluid (from incompressible Stokes and Navier-Stokes to compressible Navier-Stokes with temperature) and for the elastic structure (from linear elastic plate to nonlinear Koyter shell models, generalized standard materials ...). A pleasant aspect of this manuscript is the effort made in presenting these various models, and the will to study them mathematically.

While the mathematical theory of the interaction between fluids and rigid structures is now well-developed, a lot remains to be done about elastic structures, regarding the well-posedness theory, the development of numerical codes, and the qualitative description of the elastic surface/body dynamics. Contributing to these aspects is the purpose of the manuscript. After a nice introduction, notably an enlightening description of variational strategies for fluid-structure interactions, the work is declined in several chapters.

Chapter 2 is about the existence of weak solutions for 3d incompressible Navier-Stokes equations coupled with a nonlinear Koyter model for the elastic shell. This model includes a bending energy term, which results in a nonlinear fourth order equation of beam type for the displacement. Roughly, the authors prove existence of a global weak solution, until the loss of coercivity of the elastic energy, or until the touching between the shell and the whole boundary. It is a generalization of the work [148], where the base shell is a cylinder. The main input of the paper is an improved regularity estimate for the displacement η, that is shown to be in $L^2(H^{2+s}) \cap W^{1,2}(H^s)$ for any $0 < s < \frac{1}{2}$. The regularity of $\partial_t \eta$ can be formally understood taking into account the impermeability condition $\partial_t \eta = u(t, \cdot)|_\eta$ and the regularity of the trace of $u \in L^2(H^1)$. But the derivation of the regularity estimates on η itself, in this nonlinear context, is not obvious, and established here through the use of fractional derivatives and Nikolskii spaces. This is a nice and robust approach. Afterwards, this extra regularity helps to establish compactness, combined with an Aubin-Lions lemma in moving domains.

Chapter 3 extends the results of Chapter 2 to the case of the compressible Navier-Stokes-Fourier system. One key feature is the derivation of an energy equality, as established in the purely hydrodynamic case thanks to the works of Feireisl and Novotny. Again, the extra regularity on the displacement from Chapter 2 is an important point here. The construction of approximate solutions and the proof of compactness are quite technical, relying on the construction of appropriate divergence-free extensions and again Aubin-Lions lemma in moving domains. I do not know the literature enough to distinguish these technical tools from those already existing (see for instance ref. [127], or the article *A generalization of the Aubin-Lions-Simon compactness Lemma for problems on moving domains* by Muha and Canic).

Chapter 4 provides a weak-strong uniqueness result for the solution of the incompressible Navier-Stokes equation below an elastic plate. Assumptions for uniqueness are on the velocity field v of the strong solution, and read

$$v \in L^r(W^{1,s}), \quad \partial_t v \in L^2(W^{-1,r})$$

with $r > 2$ and $s > 3$ in 3d, and $r = s = 2$ in 2d. Although it is probably far from optimal, notably with regards to the usual Prodi-Serrin condition for Navier-Stokes (which only involve L^p bounds in space for the velocity field), this result is a valuable contribution to the largely open question of weak-strong uniqueness for this kind of fluid-structure interaction problems.

Chapters 5 to 7 provide existence results for various models of interaction between a fluid and an elastic body. Construction of solutions is based on the method of minimizing movements of De Giorgi. Note that this method is naturally designed for first order systems of gradient flow type. In the introduction of the thesis, S. Schwarzacher explains very well the way he generalizes it to second order systems, natural in the context of elasticity with inertia. To my knowledge, the application of minimizing movements to fluid-structure interaction problems is very new. As emphasized by S. Schwarzacher, beyond existence results, this kind of approach could probably be used for the design of numerical schemes, which makes it in my opinion appealing. Concretely, this method is applied in Chapter 5 to the Stokes equation coupled with inertialess elasticity, described through a Piola-Kirchoff stress-tensor σ. This stress-tensor is described in terms of both an energy functional E and dissipation functional R:

$$\operatorname{div} \sigma = DE(\eta) + D_2 R(\eta, \partial_t \eta)$$

for which quite general assumptions are made. Note that in the simple (but unphysical case) where $R(\eta, \partial_t \eta) = |\partial_t \eta|^2$, we recover an equation of gradient flow type. In subsequent chapters 6 and 7, the inertia of the solid and then of the fluid are added.

Chapter 8 is of a more qualitative nature, as it investigates theoretically and numerically the possible mechanisms for rebounds of an elastic body close to a wall. Inspiration is taken from the reduced dynamics of a rigid body near a wall in a Stokes flow, which under simplifying symmetry assumptions obeys an ODE of the form

$$\ddot{h} = -F_\mu(h)\dot{h}$$

where h is the distance between the body and the wall, and $F_\mu(h)$ is the drag exerted by the viscous fluid on the body. This drag is singular in h, and depends on the flatness of the body near the wall : $F_\mu(h) \sim \mu h^{-1}$ for a spherical body, $F(h) \sim \mu h^{-\alpha}$, $\alpha > 1$ for flatter body surface. In particular, this prevents collision between the body and the wall. To try to explain the rebound of elastic bodies, S. Schwarzacher and his co-authors introduce a simple toy model where elasticity is described by a pendulum inside the body, whose deformation ξ influences the shape of the drag term: typically, they consider an ODE model of the form

$$M\ddot{h} = -\mu F_\xi(h) - k\xi, \quad m(\ddot{h} - \ddot{\xi}) = k\xi, \quad F_\xi(h) = h^{-\max(\xi,0)-1}$$

and show that a rebound is possible for this model, and persists in the limit of vanishing μ. Moreover, it is shown to fit nicely with numerical simulations of a much more complete fluid-solid system. This is a very intriguing fact: as no clue is given on how this toy model could stem from more realistic modeling, this suggests to complete the study with further investigations.

In summary, this habilitation thesis unifies and puts in context a nice set of mathematical studies related to fluid-structure interaction problems, in the two frameworks of elastic boundaries and immersed elastic bodies. The first part of the thesis (chapters 2 to 4), dedicated to the existence of weak solutions and to weak-strong uniqueness results, shows the technical skills of S. Schwarzacher, as well as his refined understanding of the underlying models. The second part of the thesis (chapters 5 to 8) is in my opinion the strongest one, developing innovating ideas. The use of minimizing movements to discretize fluid-structure interaction problems is original and promising for the design of future numerical schemes. The qualitative study of rebounds of elastic bodies, although at an early stage, is in my opinion a landmark that opens the road to a very nice research program. Considering this scientific record, the promotion of S. Schwarzacher to a *Professeur* position in France would be natural, and I give my warm support to the defence of the habilitation thesis.

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