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RE: Opponent Review of Habilitation Thesis "Topological Drawings of Graphs" by Jan Kynčl, Charles University, Prague.

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Kynčl is a well-known and well-respected researcher, particularly in the area of topological graph theory, which is concerned with visual representations (drawings) of graphs. He has done important work in this area, and his habilitation thesis collects six journal papers and one conference paper on topological graph theory. The papers very naturally form three groups, so let me discuss each group by itself.

HANANI-TUTTE THEOREM

The Hanani-Tutte theorem states that a graph is planar if and only if it can be drawn in the plane so that every pair of independent edges crosses an even number of times. This theorem, contained in a 1934 paper by Hanani, was rediscovered several times, but it was only in the 2010s that graph drawing researchers started having a deeper look at the result. It became apparent that the result applies not just to planarity but to many variants of planarity. Clustered planarity was particularly interesting, since the complexity of clustered planarity testing was still open at the time (it was only settled last year by Fulek and Tóth), and Hanani-Tutte theorems lead to efficient algorithms. In P4 the authors showed that several special cases of clustered planarity do have a Hanani-Tutte theorem, but they also showed that there cannot be a Hanani-Tutte theorem for general clustered planarity. This was unexpected (even though, as is pointed out in the thesis, the counterexample had been found earlier in algebraic topology).

The most impressive results in this area are P6 and P7. There had been some evidence that Hanani-Tutte theorems may not just be true for the plane but for arbitrary surfaces. In P6, the authors destroyed this conjecture by showing that it is false for orientable surfaces of genus at least 4 (the non-orientable case, and genus 2, 3, 4 are still open). This was again unexpected, and required a clever, insightful construction. In P7 Fulek and Kynčl gave a complementary result in which they showed that there is a function f so that if a graph has a Hanani-Tutte drawing on a surface of genus γ, then it has an embedding on a surface of genus $f(\gamma)$. This is a beautiful and difficult result, relying on some (unpublished) graph minor theory.

The Hanani-Tutte theorem comes in two forms (weak and strong), P5 shows that there is a unified form of the theorem that implies both forms.

DRAWINGS OF THE COMPLETE GRAPH

While I personally like Kynčl's results on the Hanani-Tutte theorem, my favorite contributions of his are P1 and P2. These two papers contain the technically most difficult and strongest results contained in the thesis. We are here concerned with drawings of the complete graph K_n, so we are entering the world of crossing numbers, and approaches to proving Hill's conjecture on the crossing number of K_n. This is a crowded field of research, but even in that field P1 and P2 stand out. Extending techniques developed in the early 2000s, P1 shows that Hill's conjecture is true for monotone drawings of K_n. The same result was obtained by a different group of researchers at the same time, but the results in P1 are quite a bit stronger in that they apply to a much weaker notion of crossing number, than the standard crossing number. P1 marked an important step forward in our understanding of drawings of the complete graph.

A rotation system is an ordering of the edges leaving each vertex of a graph. Can we tell whether a particular rotation system of a complete graph can be realized by a simple drawing (that is no two edges intersect more than once)? Kynčl answered that question with a (somewhat complicated) polynomial-time algorithm in a 2011 paper. P2 shows that there is a very simple criterion with which we can test realizability: the rotation is realizable if and only if all its induced rotations on graphs of at most six vertices are realizable. While this is an easy, and polynomial-time, criterion, proving it correct is a technical tour-de-force. And as if that was not enough, P2 also gives a homological version of the result. This is a beautiful, difficult, and important result, and a major contribution to the literature.

SIMPLE DRAWINGS

As mentioned above, simple drawings are drawing in which every two edges intersect at most once (counting a common endpoint). Crossing-minimal drawings are always simple, so the study of simple drawings has drawn attention because of that (e.g. the characterization of realizable rotations of simple drawings of the complete graph in the previous section can be used to filter out unrealizable drawings, speeding up computational attacks on crossing number problems). In P3, the authors take a different approach, looking at extending existing simple drawings with additional edges. It is initially a bit surprising that this cannot always be done (showing how different simple drawings are from pseudolinear or rectilinear drawings). P3 establishes properties of saturated drawings, that is drawings which cannot be extended. This is a nice contribution to the literature.

Kynčl's thesis contains some gems of the literature (in P2, P6 and P1), and all papers are solid contributions to topological graph drawing, showing Kynčl to be a technical master of his field, who abounds in original and insightful ideas.

The thesis was accompanied by a turn-it-in report, which I have been asked to comment on. The report shows overlap with the already published versions of the papers contained in the thesis, which is to be expected; it offered no indications that any of the material in the thesis is not original.

I strongly recommend that the habilitation thesis by Jan Kynčl be accepted, and support his appointment as associate professor.

Please let me know if I can be of any more help.

Sincerely,

[redacted]

Marcus Schaefer
Professor

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