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Report on the thesis "Easton's theorem and large cardinals"

I have read Mgr. Honzik's thesis "Easton's theorem and large cardinals", and I find it to be a very interesting and potentially important piece of work in the area of forcing and large cardinals. The thesis amply shows that Mgr. Honzik has mastered the important existing techniques in the area, and is capable of original and creative work. The main original results are Theorem 5.7 and 5.17 (on violating GCH at large cardinals) and Theorem 6.6, 6.21 and 6.28 (on violating SCH).

To explain the significance of the results in Section 5, a little history is in order. A celebrated theorem by Easton states that by doing a cardinal and cofinality preserving class forcing over a model of GCH, the continuum function can be forced to take any reasonable[1] values on the class of regular cardinals. Another celebrated theorem by Scott states that if GCH fails at a measurable cardinal then it fails at many smaller regular cardinals: in particular forcing a la Easton can destroy measurable cardinals from the ground model.

Mgr. Honzik's work in Chapter 5 reconciles Easton's work with Scott's by finding conditions on a ground model function $F : REG \to CARD$ that (roughly speaking) permit F to be the continuum function of an extension with measurable cardinals. This is by no means a trivial matter, and requires a new style of argument for changing the values of the continuum function. Chapter 5 also contains an extension of the argument in which measurable cardinals are replaced by strong cardinals; this is harder because a variation on Scott's argument shows that strong cardinals enjoy even stronger reflection properties in relation to the continuum function.

The historical background for Chapter 6 is the long story of work on the Singular Cardinals Hypothesis. The SCH says roughly that if κ is singular then 2^κ is as small as possible; it is known that SCH holds in any of

[1] "Reasonable" means satisfying two easy theorems of ZFC, namely that $\kappa < \lambda \implies 2^\kappa \leq 2^\lambda$ and $cf(2^\kappa) > \kappa$

Easton's models, and that in fact to violate SCH requires the existence of large cardinals. There are several way of violating SCH, but they all go back to Prikry's seminal result that if κ is measurable then there is a cardinal preserving extension in which κ is a singular cardinal.

In Chapter 6 Mgr. Honzik proves several interesting global results about the failure of SCH, in the same spirit as Easton's theorem and the results of Chapter 5. Again this is no easy matter; the idea is to iterate forcings of the Prikry type, arguing at each stage that the needed large cardinals have been preserved so that the next step of the iteration can be defined. I discussed these problems with Mgr. Honzik in 2007 and expressed some skepticism that this kind of construction could work, so I am particularly happy that he has proved me wrong.

Several potential applications come to mind. For one the method of iterated κ-Sacks forcing used in Chapter 5 should be applicable to problems about the cardinal invariants of ${}^{\kappa}\kappa$ and $P(\kappa)$, and their interaction with large cardinals, For another the methods of Chapter 6 may prove useful in a research program which has been pursued by Gitik and his students, which aims at classifying all the possible behaviours of the continuum function and the consistency strength needed to realise them.

In conclusion Mgr. Honzik's thesis is a fine piece of work, and I recommend that it should be passed with the highest honours.

Yours truly,

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