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Examiner's Report on "Completion of semi-uniform spaces and quasi-topological groups"

PhD Thesis submitted by Barbora Batikova, at the Charles University in Prague.

Contents: A t-semi-uniform space is a semi-uniform space whose closure is topological. They are the main topic of this thesis. The author studies completions of t-semi-uniform spaces. A t-semi-uniform space is called complete provided that every Cauchy filter converges. She examines several types of Cauchy-like properties for filters, which are equivalent in uniform spaces. In all cases the category of Hausdorff complete t-semi-uniform spaces is shown to be an epireflective subcategory of the category of Hausdorff t-semi-uniform spaces. But there is no reflective subcategory of the category of Hausdorff t-semi-uniform spaces, in which all reflection arrows are embeddings.

However it is shown that the category of all complete t-semi-uniform spaces is an almost reflective subcategory of the category of t-semi-uniform spaces, where for some of the studied Cauchy-like properties all the almost reflection maps can be chosen embeddings.

In the last section of her thesis Mrs Batikova studies quasi-topological groups. They generalize topological groups. The topologies of quasi-topological groups are induced by semi-uniformities. She shows that those Hausdorff quasi-topological groups which are complete in their two-sided semi-uniformity form an epireflective subcategory of the category of Hausdorff quasi-topological groups. For several types of Cauchy-like properties examples are given of quasi-topological groups that cannot be embedded into a group that is complete in its two-sided semi-uniformity.

Comments: The English is understandable, but often far from perfect.
The thesis also contains relatively many misprints (see below). Nevertheless it is carefully written and the results seem to be correct. While in this thesis it is difficult to single out a major result, the candidate has creatively constructed an impressive collection of new and interesting examples. The main arguments are clearly stated and explained.

She evidently knows the pertinent results from the literature and combines well-known methods with her own original ideas. The research is well motivated. Several technically delicate arguments are presented. The results will be useful to all those mathematicians who are interested in generalizations of uniform spaces and topological groups.

We recommend that the thesis is accepted and the PhD degree awarded.

However we would like to suggest that some minor corrections are previously made in the final version. Indeed, as stated above, we found relatively many minor misprints, which can be readily corrected.

Corrections: No serious attempt is made to improve the English.

p. 2: Acknowledgement: "suggestions"

p. 4, line 1: Most of the

p. 4, line 5: generalizing

p. 4, line 10: are known(?)

p. 4, line -9: see ?

p. 5, line 13: completion

p. 5, line 19: neighborhood

p. 5, line 20: quasi-uniforms

p. 5, line -1: autors

p. 6, line 2: add something like "related to quasi-uniformities" Otherwise the statement is not true.

p. 6, line 21: research was

p. 6, line -13: an epireflective

p. 6, line -7: the used

p. 11, line 20: U1\cap U2 ; something is wrong here.

p. 14, line 5: the (misprint)

p. 15, line 21: ansver

p. 16, line 10: every of its neighborhoods (similar mistake at other places, e.g. p. 23, line -1)

p. 17, line 2: Euclidean

p. 17, line -1: Something is missing.

p. 19, line 12: because

p. 21, line 8: delete "a".

p. 23, line 22: converges to a...

p. 23, line -3, and p. 26, line -4: {\mathcal U}_0 should be explained.

p. 24, line -13: has any of our Cauchy-like properties.

p. 25, line 5: contains

p. 26, line -2: is

p. 27: Definition 3.21: Is there any reason that total boundedness and precompactness were interchanged (compared with what seems to be the standard definition in the theory of quasi-uniform spaces)?

p. 28, line -15: show

p. 28, line -13: $(\omega+1)$

p. 28, line -2: here and (elsewhere!) the reader expects "contradiction"

p. 29, line 1: has a

p. 29, line -23: more general situation

p. 30, line -8: is a ${\mathcal B}$-closed

p. 31, line 19: delete "a" finitely

p. 32, line -6: Hausdorff

p. 33, line 18: subobjects

p. 33, line 19 and line 21: substitute

p. 33, line 24: group(s).

p. 33, line -21: add "an" emreflective

p. 35, line 17: in the next paragraph (?)

p. 36, line 1: this sentence is not correct.

p. 36, line 6: of its points

p. 37, line 20: y^{-1}

p. 37, Def. 5.6: The reader here (and also elsewhere) wonders whether there is really a reason why sometimes "semi-uniformity" and then again "t-semi-uniformity" is written. Please check throughout the thesis whether you always write what you mean.

p. 38, line 20: coincides

p. 39, line 7: classic

p. 40, line 10: previous

p. 40, line 11: the there constructed (?)

p. 40, line 16: subgroup

p. 40, line -5: of the t-...

p. 41, line 5: the there described

p. 41, line 14: forms

p. 41, line 25: replace "numeration" by "well-ordering"

p. 41, line 26: such that (not "that"); also elsewhere this mistake can be found.

p. 41, line -15: inverse

p. 42, line 15: finally

p. 43, references [2], [6]: and

p. 43, reference [16]: Gen.

p. 43. reference [20]: the accent in Penalver does not look Spanish.

Rondebosch, 6 June 2008
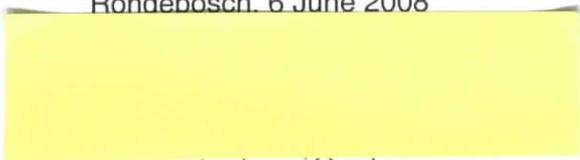
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