This thesis deals with two equivalent reformulations of the four color problem. The first of them shows the connection between graph coloring and the vector cross product; the second one expands on it and develops a relation to a formal grammar. These reformulations together with the proofs of their equivalence to the four color theorem are the results of two articles motivated by the effort to find a simpler proof of the famous problem, which was proved only by computer. They bring an interesting perspective on the problem of graph coloring and give the possibility to look on it in a new way, which could turn out to be more approachable. This thesis presents the proofs introduced in the source literature and adds the steps that are not elaborated in detail by the authors. It aims to formalize some of the ideas and to contribute to the comprehensibility of the proofs.