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Lockdown Policies and Firms‘ Investments in a Two-Period Macroeconomic Model

Bachelor’s thesis

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Declaration of Authorship

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Abstract

Throughout the COVID-19 pandemic, the implementation of lockdown policies was often accompanied by some level of commitment toward its development or termination. This thesis introduces multiple gradually improving two-period macroeconomic models which induce a time inconsistency problem regarding the lockdown policy choice. The effect of the policymaker's commitment is studied and analyzed as a possible solution to this problem. Aside from thoughts concerning directly the construction of the models, for instance, the reasoning behind its short-term nature or the importance of the shape of input functions, this thesis evaluates the policy commitment as an efficient tool. It further emphasizes the importance of the commitment's credibility.

Keywords time inconsistency, COVID-19 pandemics, two-period model, lockdown policies, firms' investments, strategic interaction

Title Lockdown Policies and Firms‘ Investments in a Two-Period Macroeconomic Model

Abstrakt

Zavádění lockdown opatření v důsledku pandemie COVID-19 bylo doprovázeno určitou úrovní závazku směrem k jejich vývoji nebo ukončení. Tato práce zavádí několik postupně zpřesňujících se dvouobdobových makroekonomických modelů, které ve volbě lockdown politik vyvolávají problém časové nekonzistence. Vliv závazku tvůrce těchto politik je studován a analyzován jako možné řešení tohoto problému. Vedle úvah týkajících se přímo konstrukce modelů, například zdůvodnění jejich krátkodobého charakteru nebo důležitosti tvaru vstupních funkcí, tato práce hodnotí závazek jako účinný nástroj. Dále zdůrazňuje význam jeho spolehlivosti.

Klíčová slova časová nekonzistence, pandemie COVID-19, dvouobdobový model, lockdown politiky, investice firem, strategická interakce

Název práce Lockdown Politiky a Investice Firem ve Dvouobdobovém Makroekonomickém Modelu

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Contents

List of Tables

List of Figures

Chapter 1

Introduction

During the ongoing pandemic of COVID-19 disease, almost four million cases have been reported in the Czech Republic, according to Dong *et al.* (2020). The first three disease cases were reported on 1 March 2020. Consequently, the state of emergency declared on 12 March by the Government of the Czech Republic (2022) was followed by multiple restrictions, some of which were kept in place until 2022. The number of new daily cases based on the data from Dong *et al.* (2020) shows that the disease was occurring in multiple waves, and the Government of the Czech Republic (2022) responded by adjusting the severity of the measures. The government has attempted to commit to the restrictions' end date. However, these were often prolonged way past the initial deadline, as is apparent from the timeline published by the Government of the Czech Republic (2022); hence the government's commitment was losing credibility. The effect of a commitment and the importance of its credibility are issues abundantly studied in the literature, often regarding one of the strongest instruments used to slow the pandemic outbreak - lockdown policies.

Lockdown policies affect the economy in a similar manner as capital or labor taxes; therefore, even if lockdowns are primarily regulations, they may be interpreted as temporally imposed implicit steep tax schedules. Economics is traditionally interested in whether large taxes work as expected. Typically, tax avoidance and tax evasion are the first concerns with the effect of taxation. In the case of lockdown policies, this is not the main concern because lockdown policies are not easy to avoid or evade, especially in the case of strict lockdowns. The second concern is about distortions to factor supply and consumption associated with taxation. For instance, labor taxation motivates overconsumer leisure. Again, in the case of lockdowns, this is less of a concern if the aim is to reduce the supply of implicitly taxed labor.

An important concern associated with and studied predominantly in the context of capital taxation is time inconsistency. It was first identified by Kydland & Prescott (1977) and arises when the policymaker has the incentive and opportunity to deviate from a previously announced plan. As the tax rates are set while the investments into capital have already been made by the private sector, it is rational for the government to impose a large tax. Nevertheless, in a dynamic environment, this leads to underinvestment in the future as the investors anticipate the large taxation. According to Persson & Tabellini (1999), the loss of control over expectations of the private sector and lack of credibility caused by the sequential decision-making and a lack of policy instruments traps the economy in a third-best equilibrium with excessive government's reliance on a highly distorting policy instrument. Moreover, Persson & Tabellini (1999) state that the "credibility problems are not confined to capital taxation: they are the norm rather than the exception in a dynamic economy" and have been later studied in the context of other government policies, especially monetary and fiscal policies as in Alesina & Tabellini (1987) or Persson *et al.* (1987).

The COVID-19 pandemic has led to an extensive study of policy responses as the lockdown policies are yet another source of uncertainty in addition to the pandemic development. The lack of policymaker's credibility is present since the decision-making has remained sequential, and the policy instruments are insufficient because lockdown of only the infected individuals is unattainable, just as the perfect income redistribution in taxation. Chang & Velasco (2020) suggest that the time inconsistency and self-fulfilling pessimistic expectations problems can be ruled out by the credibility of announced policies. Moser & Yared (2021) then approach the time inconsistency problem through a model with binding policy commitment. Their pandemic-hit infinite-horizon model of the economy was a major inspiration for this thesis' model in the characteristics inducing the time inconsistency, i.e. the timeline, variable interactions and some function definitions. In Moser & Yared (2021), the pandemic evolves according to a SIRD model, where the health state depends on the share of susceptible, infected, recovered and deceased individuals. They show that the optimal government policy is not time-consistent, and that the government may benefit from limiting future lockdown policy discretion. They also provide a calibration of the model.

The aim of this thesis is to illustrate the time inconsistency problem in the context of lockdown policies by constructing a two-period model based on the dynamic model introduced by Moser & Yared (2021). The problem of time inconsistency in this thesis arises when firms have to make irreversible investments into intermediate inputs, for instance, raw materials or rent, prior to the policy choice. Lockdown policy caps the labor supply; hence the firms have to anticipate these policies in order to properly set the investments to maximize profit, as labor and intermediate inputs are complements. On the other hand, the government focuses on the societal welfare, which consists of not only the disposable income of the workers but also the health state, whose development depends on the implemented lockdown policy. However, this thesis concentrates more on the incorporation process of these features into the model. It enters the strategic interaction with a simple model and gradually describes the development of necessary environment characteristics, variable interactions and function definitions needed.

A major difference from the model of Moser & Yared (2021) is the reduction to two periods. As suggested by Persson & Tabellini (1999), characterizing the equilibrium in a two-period model is relatively easy, which streamlines the logic demonstration. Moreover, this modification should at least partially reflect the nature of COVID-19, which has occurred in multiple pandemic waves. The approach to the health state evolution is consequently changed. It can only decrease in line with the amount of supplied labor throughout the model and returns to the initial state after the model's termination. The objective is to use this model to demonstrate the effect of a commitment to the lockdown policy on the outcome. Such commitment is not credible as long as the government makes policy choices after the investments are made. Hence, the model enables the policymaker to create a binding lockdown plan in advance and then describes the strategic interactions and their equilibria with and without the commitment. The differences between these situations are then analyzed, and conclusions regarding the optimality of the government policy commitment are carried out.

The thesis is structured as follows. Chapter 2 defines an environment for the model by describing the timeline, variables, and agents with their motivations, establishes the commitment and provides an overview of models. Chapter 3 then introduces the models with linear input functions, explains the role of taxation and reveals the main limitation of this model, which lead to the improvements in function definitions for Chapter 4. This chapter has a similar outline; however, its results differ, and the final models reveal the time inconsistency problem. Chapter 5 briefly debates other alternations of input functions and some assumptions. Chapter 6 concludes.

Chapter 2

Environment

In this chapter, the environment for the model will be introduced. The first section presents the outline of periods, their stages and the actions made in them. The following section describes the mentioned variables and defines their mathematical properties and particular relationships. In the third section, the agents and their objectives are determined. Fourth section shows how the timeline can be alternated by the government commitment and the last section provides an overview of models introduced in this thesis.

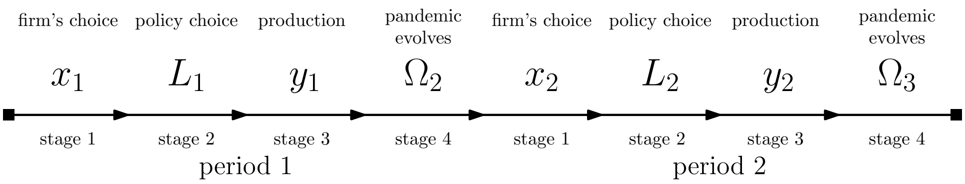
2.1 Timeline

This thesis works with a model restricted to two periods. Therefore, the situation simulates a pandemic wave, where fewer inputs are treated as endogenous in comparison to the dynamic model of Moser & Yared (2021). This follows the intuition that firms can have contracted wages with workers in advance or that the tax rates can not be changed immediately. Also, the initial health state is given, and all the agents are aware that the health state will after the model's termination return to its natural level no matter its final status.

Each of the two periods will be divided into four stages. In the first stage, firms make irreversible investments for future production. These are intermediate inputs, such as raw materials, rent, labor or marketing. Then, in stage two, the government implements the lockdown policy, i.e. the level of lockdown. Therefore firms have to anticipate the government's actions and, in contrast, the government has to consider the impact of its decisions on future periods. The sequence of the first two stages is one of the conditions for the time inconsistency optimization problem.

The production takes place in stage three, and it is based on the investments made in stage one and the labor supply, mitigated by the combination of the current lockdown policy from stage two and the prevailing health state. Then the pandemic evolves in stage four. In this stage, the labor supply is perceived as the number of susceptible individuals, which, combined with the current health state, determines the health state for the next period. In stages three and four, active agents, firms and the government make no actions. Their previous decisions are reflected in the production level and limited dynamics of a pandemic, whose evaluation is described further in societal welfare etc. The flow of periods, stages and choices is visualized in Figure 2.1.

Figure 2.1: Timeline of variable choice, without commitment



2.2 Variables

Let $i = 1, 2$ denote the period. The firms make investments x_i during the first stage and represent the irreversible input into production. This input can not be, from its definition, negative ($x_i \geq 0$) and has some exogenously given cap $\bar{x}$ so that $\forall x_i : x_i \leq \bar{x}$. Ω_i represents the current health state. It is defined as a share of healthy individuals, hence $\Omega_i \in \langle 0, 1 \rangle$ where zero denotes no healthy individuals and one no sick individuals. The government restricts the economy by implementing a level of lockdown $L_i \in \langle 0, 1 \rangle$, and this level is zero for no lockdown, one for full lockdown, as it reflects the share of restrained labor. The level of lockdown L_i in this thesis affects only the labor supply l_i, it does not directly enter any other variable definition functions.

Supply of labor l_i is defined as $l_i(L_i, \Omega_i) = (1 - L_i)\bar{l}(\Omega_i)$. Assume that the labor supply l_i is inexhaustible in the sense that the supply of qualified workers has no other cap than the lockdown level and the health state of the population. The labor supply denotes the share from its maximal level, achieved under no lockdown ($L_i = 0$) and maximal health ($\Omega_i = 1$), therefore $l_i \in \langle 0, 1 \rangle$. The potential labor supply function $\bar{l}(\Omega_i)$ represents the possible lower performance of infected workers, $\bar{l}(\Omega_i) \in \langle 0, 1 \rangle$, and is increasing.

As mentioned above, the health state Ω_i evolves as a function of the previous period's health state and labor supply, $\Omega_{i+1} = g(l_i, \Omega_i)$. The intuition is that the health state Ω_i is decreased as more labor l_i is supplied since more individuals meet and potentially spread the infection, which further affects the health state of the next period, Ω_{i+1}. Thus, the function $g(l_i, \Omega_i)$ is decreasing in l_i. Recall that the health state for the first period Ω_1 is taken as given.

The production y_i is based on the level of investments x_i and the labor supply l_i. The price of output is fixed throughout the model, and it can be assumed equal to one. Therefore, the production y_i equals the revenue of production. The production function $f(x_i, l_i)$ is increasing in both input factors. The consumption is also increasing in both input factors, $c_i(w, l_i) = w\ l_i$. However, the wages w in this model are set at some inalterable level $w > 0$. This is because neither the government nor firms can dramatically change the wage level in a short time horizon. Moreover, giving this mandate to one of the agents would strengthen their position and might lead to their absolute advantage in decision-making, which is not desirable. The same holds for a cost of an investment $r > 0$ and the tax rate $t \in \langle 0, 1 \rangle$. These three variables, especially their attainable values, will be elaborated on further.

2.3 Agents

The model consists of two active agents, a representative company, further referred to as firms, and the government. Each of these agents has a different objective, firms maximize profit π, see (2.1), while the government sets its policy to maximize societal welfare W, defined in (2.2).

Workers are treated as passive agents, unable to choose between labor and leisure, save their income or borrow for consumption. Enabling them to do so would make the interaction even more complex, as consumption is fundamental for the societal welfare, which the government maximizes. The results would depend on the objective of the workers, whether they try to achieve equal consumption in both periods or whether they have other goals. However, an inspection of these interactions is not the scope of this thesis. Therefore workers supply labor up to the level l_i, which, together with the predetermined wage w, dictates the payment received. All of this pay is spent on consumption in the same period, c_i.

The total profit of the firms equals the sum of profits in both periods, $\pi = \pi_1 + \pi_2$. Profit earned in a given period i equals the revenue of production

y_i minus the costs, which consist of investment costs $r\ x_i$ and labor costs $w\ l_i$.

$$\pi_i = y_i - r\ x_i - w\ l_i \tag{2.1}$$

Societal welfare W is a sum of the worker's welfare utility functions, defined as $u(c_i + t\ \pi_i, \Omega_i) = c_i + t\ \pi_i + \Omega_i$ for all periods i. The utility is dependent on the health by Ω_i and the disposable income $c_i + t\ \pi_i$, where the exogenous inalterable tax rate t denotes the portion of profit π_i that will be given to the societal welfare. It should be stressed that although the literature investigates time inconsistency particularly in the context of capital taxation, this will not be the case in this thesis, as the problem arises from the policy choice. The ultimate objective of the government and workers is to have a healthy society with a high disposable income; hence this function is growing in all inputs.

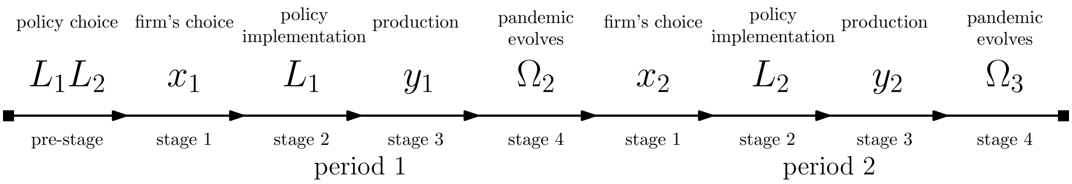
$$W = u(c_1 + t\ \pi_1, \Omega_1) + u(c_2 + t\ \pi_2, \Omega_2) = c_1 + c_2 + t\ \pi + \Omega_1 + \Omega_2 \tag{2.2}$$

The variables described in (2.1) and (2.2) will be used precisely in these functional forms in all the following chapters and relevant models.

2.4 Commitment

The timeline presented in Section 2.1 can alter when the commitment is introduced. In the case without commitment, the government chooses its policy L_i after the firms' investments x_i. Nevertheless, it has the opportunity to commit to its policy choice for future periods. It is necessary for this commitment to be credible; thus, the binding policy choice is made already before period one, in the pre-stage. The adjusted timeline is shown in Figure 2.2.

Figure 2.2: Timeline of variable choice, with commitment



What is the motivation for the commitment? In the initial timeline, firms are aware that the government might want to implement a stricter lockdown policy L_i in order to maximize societal welfare W, as it would not have to face the consequences of decreased investments x_i while they are already sunk. The firms rationally anticipate this behaviour and adjust the investments x_i. This can drive the economy away from the optimum and could be avoided if the government commits to the lockdown policies L_i in advance.

2.5 Model Overview

This section serves as a summarization of all the models introduced in this thesis, together with their basic characterizations. It should help the reader distinguish between the cases and provide an overview of the differences.

Table 2.1 is divided as follows. Name of the model, in the first column, will be used in the particular section or subsection titles. Columns two to four contain the functions of potential labor supply ($\bar{l}(\Omega_i)$), health state evolution ($g(l_i, \Omega_i)$) and production ($f(x_i, l_i)$), respectively. The fifth and sixth column provides information on whether the investments (x_i) or lockdown policy (L_i) are set endogenously by the firms or the government (end.) or exogenously (exo.). Column seven describes the interval for the tax rate (t), and the last column (com.) defines whether the government in the model commits to the lockdown policy or no. These attributes uniquely define any model introduced further in this thesis.

<table>
<tr><th>name</th><th>$\bar{l}(\Omega_i)$</th><th>$g(l_i, \Omega_i)$</th><th>$f(x_i, l_i)$</th><th>x_i</th><th>L_i</th><th>t</th><th>com.</th></tr>
<tr><td>Model 1</td><td rowspan="10">Ω_i</td><td rowspan="10">$\Omega_i - l_i$</td><td rowspan="6">$x_i + l_i$</td><td>end.</td><td>exo.</td><td rowspan="2">$t \in \langle 0, 1 \rangle$</td><td rowspan="2">-</td></tr>
<tr><td>Model 2</td><td>exo.</td><td>end.</td></tr>
<tr><td>Model 3</td><td rowspan="4">end.</td><td rowspan="4">end.</td><td rowspan="2">$t = 0$</td><td>no</td></tr>
<tr><td>Model 4</td><td>yes</td></tr>
<tr><td>Model 5</td><td rowspan="10">$t \in (0, 1 \rangle$</td><td>no</td></tr>
<tr><td>Model 6</td><td>yes</td></tr>
<tr><td>Model 7</td><td rowspan="8">$\sqrt{x_i\ l_i}$</td><td>end.</td><td>exo.</td><td rowspan="2">-</td></tr>
<tr><td>Model 8</td><td>exo.</td><td>end.</td></tr>
<tr><td>Model 9</td><td rowspan="2">end.</td><td rowspan="2">end.</td><td>no</td></tr>
<tr><td>Model 10</td><td>yes</td></tr>
<tr><td>Model 11</td><td rowspan="4">$\sqrt{\Omega_i}$</td><td rowspan="4">$(-l_i^2 + 1)\Omega_i$</td><td>end.</td><td>exo.</td><td rowspan="2">-</td></tr>
<tr><td>Model 12</td><td>exo.</td><td>end.</td></tr>
<tr><td>Model 13</td><td rowspan="2">end.</td><td rowspan="2">end.</td><td>no</td></tr>
<tr><td>Model 14</td><td>yes</td></tr>
</table>

Table 2.1: Model Overview

Chapter 3

Linear models

The previous chapter has specified some basic requirements for the shape of functions of potential labor supply $\bar{l}(\Omega_i)$, health state evolution $g(l_i, \Omega_i)$ and production function $f(x_i, l_i)$. This chapter will attempt to meet these requirements with an elementary function form, the linear relationship. It also introduces the solution process and carries out some important observations.

3.1 Definition of functions

For the potential labor supply $\bar{l}(\Omega_i)$, an increasing function which correctly projects the definition interval of health state Ω_i on potential labor supply $\bar{l}$ is needed. These conditions are satisfied by the following function.

$$\bar{l}(\Omega_i) = \Omega_i \tag{3.1}$$

The function describing the evolution of the health state $g(l_i, \Omega_i)$ has to be decreasing in labor supply l_i. Moreover, for $l_i = 0$, the health state for the next period Ω_{i+1} should remain unchanged from the current health state Ω_i. Again, the interval projection has to be preserved, which is ensured by the definition interval of health state Ω_i and labor supply l_i.

$$\Omega_{i+1} = g(l_i, \Omega_i) = \Omega_i - l_i = L_i \; \Omega_i \tag{3.2}$$

Under this definition, the only stable state in terms of health is achieved when no individual is working. Otherwise, the pandemic only strengthens and the health state decreases. Hence the tradeoff between slowing down the pandemic and non-negative production is modelled.

The production function $f(x_i, l_i)$ has only one condition; it should be increasing in both arguments, investment x_i and labor supply l_i.

$$f(x_i, l_i) = x_i + l_i \tag{3.3}$$

3.2 Models without strategic interaction

Throughout the thesis, the actions and responses of the firms and the government will be studied. The model has multiple inputs; however, they can be, one by one, treated as exogenous, which allows to focus on certain sections of the complex problem. Moreover, variables wage w, price of an investment r or the tax rate t are not only exogenous but also set constant due to operating only in two periods of a pandemic wave. First, the extreme cases with one or two directly connected variables will be described. Solving this problem provides a close look into the motivation and decision-making of the active agents, firms and the government.

3.2.1 Model 1: Firms' decision

The first case treats only the investments x_i as endogenous. Thus, stages two and four are entirely exogenous; the level of x_i does not enter them at all. Therefore, the problem that has to be solved reduces only to stages one and three. The firms choose the optimal x_i in order to maximize profit π_i from production y_i, which takes place in stage three. Nevertheless, r and w are given, and the labor supply l_i is purely exogenous.

The conditions ensuring a non-negativity of profit have to be imposed. The marginal effect of investments x_i and labor supply l_i on the production y_i has to be greater or equal to the marginal effect on the cost of production, i.e. the investment and labor costs $r\ x_i + w\ l_i$. The restriction for a price of an investment $r \in (0, 1\rangle$ follows from (3.4). For $r > 1$, the firms would invest $x_i = 0$, as any positive investment x_i would yield a negative payoff. Wages $w \in (0, 1\rangle$ as calculated in (3.5). In the event when wages $w > 1$, lower labor supply l_i, caused by decreased health state Ω_i or some positive lockdown L_i, would make the firm better off, as it has no option to choose the employment level. The situations where firms do not invest or benefit from pandemics are counter-intuitive and rather unrealistic. Therefore, this model and whole Chapter 3 proceeds with the restricted price of an investment r and wages w. It is important to note that this does not contradict the predetermination of these variables, as this intuition holds even without the existence of a pandemic.

$$\begin{aligned}\frac{\partial y_i}{\partial x_i} &\geq \frac{\partial (r\ x_i + w\ l_i)}{\partial x_i} \\ 1 &\geq r\end{aligned} \tag{3.4}$$

$$\begin{aligned}\frac{\partial y_i}{\partial l_i} &\geq \frac{\partial (r\ x_i + w\ l_i)}{\partial l_i} \\ 1 &\geq w\end{aligned} \tag{3.5}$$

Under the abovementioned conditions for the price of an investment r and wages w, the investments $x_i \in \langle 0, \bar{x} \rangle$. The (3.6) shows the non-negative effect of investments x_i on the profit π_i. What are the implications of this result for the particular zero or positive profit generating types on the investment market? For the price of an investment $r = 1$, firms are making profit $\pi = 0$ no matter the investments x_i. Thus, there is no optimal choice for the firms. On the other hand, for the price of an investment $r \in (0, 1)$, investments x_i have a positive effect on the profit π_i. Firms will choose to invest the maximal amount possible $x_i^* = \bar{x}$. The existence of a particular optimal choice better serves this thesis' objective, and therefore, this chapter advances considering the case where firms generate a strictly positive profit. However, an analogous argumentation would be functional in the firms' profits equal zero. For labor market, it is important to realize that under any wage w, the firms have to utilize all available workers l_i, and the restriction on $w \in (0, 1)$ is imposed for logical purposes, not as the one for the price of an investment r.

$$\frac{d\pi_i}{dx_i} = \frac{\partial (x_i(1-r) + l_i(1-w))}{\partial x_i} \geq 0 \tag{3.6}$$

Even though the firms do not need to know the level of labor supply l_i in order to set the investment level x_i, the solutions of models further in this chapter still evaluate the information that firms have about the policy choice L_i and consequently about the labor supply l_i. These findings might be beneficial for intuition explanation and the comparison with advanced models in Chapter 4 and Chapter 5 with non-linear input functions.

Model 1 summary: Firms generate maximal profits through choosing the investments $x_1^* = x_2^* = \bar{x}$. The lockdown policies L_1 and L_2 are exogenous.

3.2.2 Model 2: Policy choice

This subsection has set the lockdown policy L_i and health state Ω_i endogenous. With only the health state Ω_i endogenous, there would be no optimization, as it would not be possible to influence it with the exogenous lockdown L_i. On the other hand, with only lockdown L_i endogenous, it would make no sense to implement any lockdown if there was no prospect of its positive effect on the future health state Ω_{i+1}. Hence, together with the assumption that lockdown effectively mitigates the spread and can not be violated by any agent, both variables are treated as endogenous.

In the event where health state Ω_i and lockdown level implemented by the government L_i are the only endogenous variables, the government is the sole active agent, and its objective is, in every situation, to maximize the societal welfare W. The government can only affect utility in the same period negatively, as higher lockdown L_i leads to lower consumption c_i. This is proven by calculating the effect of the lockdown policy in the second period L_2 on the societal welfare W through $u(c_2 + t\ \pi_2, \Omega_2)$ in (3.7), as $t \in \langle 0, 1\rangle$, $w > 0$ and $\Omega_i > 0$ the total effect is negative. This effect can be entitled as marginal disutility of lockdown on the current period due to the consumption loss. However, the decision to implement lockdown has a positive impact on the utility in the future periods through improved health state Ω_{i+1}. This model contains only two periods; hence this effect can influence only the following period. It makes no sense to introduce any positive lockdown in the second period, as the model ends after two periods, making the effect on Ω_3 completely negligible. At the same time, the choice of no lockdown, $L_2^* = 0$, maximizes the consumption c_2.

$$
\begin{aligned}
\frac{du(c_2 + t\ \pi_2, \Omega_2)}{dL_2} &= \frac{dc_2}{dL_2} + \frac{d(t\ \pi_2)}{dL_2} + \frac{d\Omega_2}{dL_2} = \\
&= w\frac{dl_2}{dL_2} + t\left((1-w)\frac{dl_2}{dL_2} + (1-r)\frac{dx_2}{dL_2}\right) + \frac{d\Omega_2}{dL_2} = \\
&= (w + t - tw)\frac{\partial l_2}{\partial L_2} = (tw - w - t)\Omega_2 < 0 \qquad (3.7)
\end{aligned}
$$

In the first period, the particular solution depends on the tradeoff between disposable income $c_1 + t\ \pi_1$ and the health state Ω_2. Effect on the utility in the current period (3.8) is analogous to the (3.7), while the marginal utility of lockdown on the future period due to the improved health is derived in (3.9) with the first two steps analogous to the (3.7).

$$\frac{du(c_1 + t\ \pi_1, \Omega_1)}{dL_1} = (tw - w - t)\Omega_1 < 0 \tag{3.8}$$

$$\begin{aligned}\frac{du(c_2 + t\ \pi_2, \Omega_2)}{dL_1} &= (w + t - tw)\frac{dl_2}{dL_1} + \frac{d\Omega_2}{dL_1} = \\ &= ((w + t - tw)(1 - L_2) + 1)\,\frac{\partial\Omega_2}{\partial L_1} = \\ &= ((w + t - tw)(1 - L_2) + 1)\Omega_1 > 0\end{aligned} \tag{3.9}$$

All the variables in the two equations above are already known, even the L_2, as described through (3.7). This implies that the solution will be a corner solution. It is possible to compare the effects, and from the domains of definition of t and w it is evident that the effect in (3.9) is stronger. Thus it is optimal for the government to implement a maximal lockdown, $L_1^* = 1$. It should be remarked that the policy choice L_i does not interact with the exogenous investments x_i. This observation will be essential and examined further in the event with endogenous investments x_i.

The optimal actions of government can be defined at the beginning. The existence of a commitment would have no effect, as the government is the only active agent, and therefore the commitment would not have any impact.

Model 2 summary: The investments x_i are exogenous, government sets the lockdown policy $L_1^* = 1$ and $L_2^* = 0$.

3.3 Models with strategic interaction without tax

This section, unlike the previous one, allows both firms and the government to optimize. The objective of firms is to maximize the profit, while the government concentrates on the maximal societal welfare possible. Nevertheless, both agents are rational and aware of the response they are going to elicit, creating an environment for a strategic interaction game. For this section, the tax rate t is set to zero, rearranging the utility function to $u(c_i, \Omega_i)$.

3.3.1 Model 3: Without policy commitment

To adequately describe the decision-making, the process will be solved backwards, i.e. beginning with stage four of the second period and ending with stage one of the first period. In other words, going through Figure 2.1 from right to left. It is worth reminding that no decisions are made in stages three and four of each period; hence they are not referred to further.

In the second stage of the second period, investments x_2 were already made, and the model ends after this period. The overall effect of the lockdown L_2 on the utility $u(c_2+t\,\pi_2, \Omega_2)$ is negative because the government faces an analogous optimization problem to the case where only the health state is endogenous, so $t = 0$ can be plugged into (3.7). The government thus decides to implement no lockdown, $L_2^* = 0$, under any circumstances.

Firms are choosing investment x_2 in order to maximize the profit π_2. They are able to foresee the lockdown policy L_2 and are aware that it is not dependent on the investment level x_2. The firms optimize as if only investments were endogenous, according to logic arising from (3.6).

The government has to take into account the fact that firm owners might use L_1 to predict L_2 while choosing x_2 in the following period. However, this will not happen as the policy choice for the second period follows from the fact that the model ends afterwards. Additionally, the government aims to maximize societal welfare W, independent of production y_i. The decision then follows the same pattern as if x_1 was exogenous because it is sunk. Plugging the values $t = 0$ and $L_2 = 0$ into (3.8) and (3.9) implies that the negative effect of the lockdown L_1 on the second period is greater than the positive effect on the first period, hence $L_1^* = 1$.

The optimal level of investments $x_1^* = \bar{x}$ using the same approach as in the second period. The decision the government will make regarding the policy in the first period L_1 can be again predicted.

Model 3 summary: The firms invest the maximal amount $x_1^* = x_2^* = \bar{x}$ and make a positive profit. Optimal lockdown policy is $L_1^* = 1$ and $L_2^* = 0$.

3.3.2 Model 4: With policy commitment

The procedure will be solved backwards, as in the previous case. The commitment, though, rearranges the order, as L_1 and L_2 are decided before period one, as seen in Figure 2.2. The government commits to the policy choice that will take place in stage two of periods one and two. The commitment assures the firms and they can play their best response accordingly.

The firms follow the same decision-making process as in the case with no commitment. Firms will use the logic succeeding from (3.6) to determine the investments $x_1^* = x_2^* = \bar{x}$. Moreover, they have perfect information about the optimal choice for the government and assurance that their decision can not change this choice. This is caused by the fact that the government's objective does not depend on the investments x_i and by the commitment itself.

In the pre-stage, the government chooses both L_1 and L_2. Choice of L_1 will be the same as in the case without commitment because the level of x_1 does not enter (3.8) and (3.9), whose comparison determines the policy, at all. $L_1^* = 1$ based on the strength of these effects. Moreover, the policy choice for the second period will be $L_2^* = 0$, as it makes no difference for the government whether they decide before or after the firms. This builds on the fact that the government maximizes the societal welfare W, which is not affected by investments x_2 and that the model ends after period two. Thus the implementation of any lockdown policy would decrease the societal welfare W with no future payoff.

Model 4 summary: The commitment does not change the outcome. Investment level is $x_1^* = x_2^* = \bar{x}$ and lockdown policy is $L_1^* = 1$ and $L_2^* = 0$. Zero tax revenues as a consequence of $t = 0$ are one of the reasons.

3.3.3 Role of taxation

It is evident that the existence of a government commitment in this setting makes no difference. The main cause is the form of the strategic interaction. The government had no incentive to optimize the profit π_i and therefore the production y_i and investments x_i, while the firms' profits depended on the lockdown policy L_i through labor supply l_i. This section has helped with the understanding of agents' motivation. However, the setting has to be changed in order to obtain a more complex strategic interaction problem.

3.4 Models with strategic interaction and tax

From this section further, only the tax rate values $t \in (0, 1\rangle$ will be considered for the abovementioned reasons. This section will follow the same steps as the previous one in order to demonstrate the role of taxation in this model and discuss the effect of a commitment.

3.4.1 Model 5: Without policy commitment

This solution shares similarities with the case where $t = 0$. The only exception in policy choice in the second period L_2 is the evaluation of (3.7). However, Section 3.2 can be referenced here, and the result is $L_2^* = 0$. Even though it maximizes the profit π_2, this choice is not driven by the utility increase from $t\ \pi_2$ but by the fact that there is no benefit from the implementation of a positive lockdown $L_2 > 0$ because of the model's termination. That is proven by the fact that this lockdown level is optimal even for $t = 0$. Firms maximize the profit through the logic of (3.6) and can not affect the policy L_i.

In the first period, the lockdown policy affects the utility in both periods. The effects are calculated in (3.8) and (3.9). The comparison of these two effects again shows that the positive effect on the future period is greater than the negative effect on the current period. Therefore the optimal lockdown policy is the implementation of a complete lockdown, $L_1^* = 1$. Firms set $x_1^* = \bar{x}$.

Model 5 summary: The solution does not change, even though the cause partially differs as the tax rate is now $t \in \langle 0, 1 \rangle$. Firms invest $x_1^* = x_2^* = \bar{x}$, government sets $L_1^* = 1$ and $L_2^* = 0$.

3.4.2 Model 6: With policy commitment

The decision for stage two of both periods is made in a pre-stage before the first period. Consequently, the firms will set the investments $x_1^* = x_2^* = \bar{x}$ based on implications of (3.6), assuring that the output is on their best response function. The commitment takes away the part where firms have to anticipate the lockdown level L_i to predict their profit π_i.

When committing to the lockdown policies L_1 and L_2, the government has to anticipate the impact on future investment levels, as no investments x_i are yet sunk. However, the government understands the firms' best response function, and by committing to a policy, it can determine the output that is preferable for its objective. In other words, firms always have one optimal investment for any policy L_i, and the government can predict that. Nevertheless, in this setting, the effects calculated in (3.8) and (3.9) are not impacted by the level of investments x_i at all. Therefore the policy for the first period $L_1^* = 1$ and the second period $L_2^* = 0$ follows the same logic as above.

Model 6 summary: Again, the commitment has not affected the choices of investments $x_1^* = x_2^* = \bar{x}$ and lockdown $L_1^* = 1$ and $L_2^* = 0$. Production function must be changed in omit corner policy solutions.

3.4.3 Effect of the tax

It is appropriate to evaluate the effects of the tax rate t on the marginal effects of the lockdown policies, particularly the L_1. This helps to understand the differences between the two cases provided above from the taxation perspective, even though the taxes t are set exogenously. The effect of the tax rate t on the marginal disutility of the lockdown L_1 is derived in (3.10). It shows that for wages $w < 1$, the disutility increases. This originates from the fact that a higher tax rate t ceteris paribus implies higher disposable income $c_1 + t\ \pi_1$. On the other hand, it increases the marginal utility of the lockdown L_1, as calculated in (3.11). This is due to the improved health state Ω_2 and its implications.

$$\frac{du(c_1 + t\ \pi_1, \Omega_1)}{dL_1\ dt} = \frac{\partial((tw - w - t)\Omega_1)}{\partial t} = (w-1)\Omega_1 < 0 \tag{3.10}$$

$$\frac{du(c_2 + t\ \pi_2, \Omega_2)}{dL_1\ dt} = \frac{\partial(((w + t - tw)(1 - L_2) + 1)\Omega_1)}{\partial t} = (1-w)(1-L_2)\Omega_1 > 0 \tag{3.11}$$

Notice that the sum of (3.10) and (3.11), under the optimal policy for the second period $L_2^* = 0$, equals zero. The intuition behind this result is that the level of tax rate t does not affect the optimal choice in the first period at all. This obviously holds even for the second period, as the optimal lockdown is always $L_2^* = 0$. A higher tax rate t only implies higher societal welfare W for any policy L_i but does not change the optimum.

3.5 Evaluation of the linear models

The optimal choices of the firms and the government are identical regardless of the commitment. This is caused, as explained above, by the fact that the effects from (3.8) and (3.9) do not contain the investments x_i. Furthermore, the optimal investment level x_i^* does not depend on the labor supply l_i as a consequence of the effect presented in (3.6). The reason for both problems is the shape of the production function $f(x_i, l_i)$. Thus, to achieve an advanced model in this thesis, the following condition must be satisfied by the production function $f(x_i, l_i)$.

$$\frac{df(x_i, l_i)}{dx_i \; dL_i} > 0 \tag{3.12}$$

This condition by itself should not and does not necessarily ensure that the commitment will change the outcome. Crucial is the overall strength of the effects, determined by the shapes of input functions, as forces driving the model to the corner solutions might occur. However, it is necessary for the investments x_i to have some effect on lockdown policy L_i and vice versa.

Chapter 4

Non-linear production models

It was observed and, in Section 3.5, explained that the linear relationships in the production function lead to corner solutions and the independence of particular effects on the desired parameters. Hence, a new production function is defined. This adjustment should allow a more profound commitment analysis. As a consequence of only the production function being non-linear, the models in this chapter will be referred to as non-linear production models.

4.1 Definition of functions

The production function $f(x_i, l_i)$ now has the additional condition from (3.12). It should be increasing in both arguments, investment x_i and labor supply l_i, which are rather complements. It is desirable to ensure constant returns to scale $\alpha f(x_i, l_i) = f(\alpha x_i, \alpha l_i)$. Cobb-Douglas production function will be used.

$$f(x_i, l_i) = \sqrt{x_i \; l_i} \tag{4.1}$$

The derivatives of function f in (4.2) show the complementarity of inputs. It may be observed that the function is considered well-behaving.

$$\begin{aligned}
\frac{\partial^2 f}{\partial x_i^2} &= -\frac{1}{4}\sqrt{\frac{l_i}{x_i^3}} \\
\frac{\partial^2 f}{\partial x_i \partial l_i} &= \frac{1}{4\sqrt{x_i \; l_i}} \\
\frac{\partial^2 f}{\partial l_i \partial x_i} &= \frac{1}{4\sqrt{x_i \; l_i}} \\
\frac{\partial^2 f}{\partial l_i^2} &= -\frac{1}{4}\sqrt{\frac{x_i}{l_i^3}}
\end{aligned} \tag{4.2}$$

4.2 Models without strategic interaction

The process from a linear model in Chapter 3 will be replicated with the alternated input functions defined in Section 4.1. The first step is the analysis of one agent's optimization in the event when the choices of the second agent are exogenous, as the applied logic and equations were fundamental for progress in the case where both agent's choices are endogenous.

4.2.1 Model 7: Firms' decision

The situation is analogous to the setting in Model 1, the only difference being the new production function defined in (4.1). In (4.3) is calculated the restriction for the price of an investment $r \in (0, \frac{1}{2}\sqrt{\frac{l_i}{x_i}}\rangle$ ensuring that the profit π_i is growing in investments. It is again necessary for labor supply l_i to have a positive effect on the profit π_i, which is fulfilled for wages $w \in (0, \frac{1}{2}\sqrt{\frac{x_i}{l_i}}\rangle$.

$$\begin{aligned} \frac{\partial y_i}{\partial x_i} &\geq \frac{\partial (r\ x_i + w\ l_i)}{\partial x_i} \\ \frac{1}{2}\sqrt{\frac{l_i}{x_i}} &\geq r \end{aligned} \tag{4.3}$$

$$\begin{aligned} \frac{\partial y_i}{\partial l_i} &\geq \frac{\partial (r\ x_i + w\ l_i)}{\partial l_i} \\ \frac{1}{2}\sqrt{\frac{x_i}{l_i}} &\geq w \end{aligned} \tag{4.4}$$

The profit type for the investment market has to be decided again. The (4.5) displays the general inequality. In this model, zero profits are generated for the price of an investment $r = \frac{1}{2}\sqrt{\frac{l_i}{x_i}}$, which can be written as $x_i = \frac{l_i}{4r^2}$. Unlike Model 1, here is in this case an optimal investment level, and it is the $x_i^* = \frac{l_i}{4r^2}$. Hence, the restriction expresses that the price of an investment r has to be set in a way that the $x_i^* \in (0, \bar{x}\rangle$. A positive profit generating setting has the price of an investment $r \in (0, \frac{1}{2}\sqrt{\frac{l_i}{x_i}})\rangle$ and leads to an analogous situation where the optimal investment level equals the maximal investment level $x_i^* = \bar{x}$. As it was one of the motivations for alternation of the production function to have an optimal investment level dependent on labor supply l_i, this chapter proceeds in a zero firms' profit mode. Note that also the possible wage levels are dependent on the investment level x_i and the labor supply l_i by the abovementioned interval $w \in (0, \frac{1}{2}\sqrt{\frac{x_i}{l_i}}\rangle$.

$$\frac{d\pi_i}{dx_i} = \frac{\partial(\sqrt{x_i\ l_i} - r\ x_i - w\ l_i)}{\partial x_i} \geq 0 \tag{4.5}$$

Model 7 summary: Investments are chosen by firms to make a non-negative profit $x_1^* = \frac{l_1}{4r^2}$ and $x_2^* = \frac{l_2}{4r^2}$. Lockdown policies L_1 and L_2 are exogenous.

4.2.2 Model 8: Policy choice

The policy choice in an advanced model again shares the basic logic with the linear model. Nevertheless, the equations have to be recalculated, and new conclusions must be deduced from the results. The effect of lockdown L_2 on the second period is calculated below in (4.6) and is negative. Thus the optimal lockdown policy for the second period is $L_2^* = 0$. For compactness of the solution, (4.6) and (4.8) do not break down the labor supply l_2 into $(1 - L_2)\Omega_2$ and the health state of period two Ω_2 into $\Omega_1 - l_1$ if not necessary.

$$\begin{aligned}\frac{du(c_2 + t\ \pi_2, \Omega_2)}{dL_2} &= w\frac{dl_2}{dL_2} + t\left(\frac{\partial\sqrt{x_2\ l_2}}{\partial L_2} - r\frac{dx_2}{dL_2} - w\frac{dl_2}{dL_2}\right) + \frac{d\Omega_2}{dL_2} = \\ &= \left(w + \frac{t}{2}\sqrt{\frac{x_2}{l_2}} - tw\right)\frac{\partial l_2}{\partial L_2} = \left(tw - w - \frac{t}{2}\sqrt{\frac{x_2}{l_2}}\right)\Omega_2 < 0\end{aligned} \tag{4.6}$$

Furthermore, for the optimal lockdown policy in the first period L_1, effects on both periods have to be calculated. The marginal disutility of the lockdown policy in the current period due to the consumption loss is in (4.7), and the marginal utility on the future period due to the improved health in (4.8).

$$\frac{du(c_1 + t\ \pi_1, \Omega_1)}{dL_1} = \left(tw - w - \frac{t}{2}\sqrt{\frac{x_1}{l_1}}\right)\Omega_1 < 0 \tag{4.7}$$

$$\begin{aligned}\frac{du(c_2 + t\ \pi_2, \Omega_2)}{dL_1} &= \left(w + \frac{t}{2}\sqrt{\frac{x_2}{l_2}} - tw\right)\frac{\partial l_2}{\partial L_1} + \frac{d\Omega_2}{dL_1} = \\ &= \left(\left(w + \frac{t}{2}\sqrt{\frac{x_2}{l_2}} - tw\right)(1 - L_2) + 1\right)\frac{\partial\Omega_2}{\partial L_1} = \\ &= \left(\left(w + \frac{t}{2}\sqrt{\frac{x_2}{l_2}} - tw\right)(1 - L_2) + 1\right)\Omega_1 > 0\end{aligned} \tag{4.8}$$

The result of (4.7) is negative, and (4.8) is positive. Therefore the particular solution is not apparent, and further comparison of the effects is needed. These effects can be thought about as the marginal cost, where it is important to mention that (4.7) has to be multiplied by minus one in order to represent it and the marginal benefit, respectively. Thus the optimal level of lockdown L_1^* can be expressed from an equation which sets these two effects equal.

$$-\frac{du(c_1 + t\ \pi_1, \Omega_1)}{dL_1} = \frac{du(c_2 + t\ \pi_2, \Omega_2)}{dL_1} \tag{4.9}$$

Model 8 summary: Investments x_1 and x_2 are exogenous. The government sets the lockdown policy L_1^* according to the result of (4.9) and $L_2^* = 0$.

4.3 Models with strategic interaction

This section continues to follow the same process as Chapter 3 while implementing the acquired knowledge regarding the taxation t from Section 3.3; hence considering only $t \in (0, 1\rangle$ for the abovementioned reasons.

4.3.1 Model 9: Without policy commitment

The policy choice in the second period L_2 shares with the previous parts the reasons for optimality of no lockdown in the second period $L_2^* = 0$. Firms will then set their investments x_2 through the result of (4.5) while foreseeing the government's decision.

The government in the first period knows the input values of the (4.7) and (4.8), including the investments x_1 and agents' choices for the second period. Therefore it uses the (4.9) to calculate the optimal lockdown policy L_1^*. As perfectly informed agents, firms are aware of this best response function of the government. They plug the result of this function into their profit-maximizing strategy (4.5) to express and evaluate the optimal investment level x_1^*.

Model 9 summary: Firms in both periods invest according to the (4.5). The optimal lockdown L_1^* is expressed from (4.9) an $L_2^* = 0$.

4.3.2 Model 10: With policy commitment

The lockdown policies L_1 and L_2 implemented in stages two of each period were decided in a pre-stage. Thus the firms know all the input variables and can use the (4.5) to set the levels of investments x_1 and x_2, as they are not the first agent to act.

Unlike in the linear model, in the model with non-linear production, the marginal effects in the (4.7) and (4.8) depend on the investment levels x_1 and x_2. Firms' best response function is the result of (4.5); therefore, the government can substitute it into the (4.9) for both x_1 and x_2. To calculate the optimal policy for the first period, the optimal policy for the second period L_2^* has to be substituted. Its value is once again zero from the model's termination. After substituting all these values, the lockdown L_1^* can be expressed.

Model 10 summary: The government, while setting the lockdown policy L_1^* according to the (4.9), observes (4.5) through which it also determines the investment level x_1^* towards its objective. In the second period, the optimal lockdown is $L_2^* = 0$ and firms respond again through (4.5).

4.3.3 Consequences of the policy commitment

Under no commitment, the government implements an optimal lockdown level L_i under given circumstances. These are the health state Ω_i, given by the labor supply in the previous period, and investments x_i, set by profit-maximizing firms. Nevertheless, committing to future lockdown policies enables the government to influence the setting of investments x_i with the intention of maximizing societal welfare. The commitment expands the government's pool of attainable outcomes. Hence new societal welfare-maximizing optimum might be reached while the societal welfare-maximizing optimum of the case with no commitment remains achievable. Thus, with a commitment to a future lockdown policy, the government can not end up worse off than without commitment, but it can end up better off.

4.3.4 Effect of the tax

In order to complete the comparison of the models, the effect of the tax rate will be calculated analogously for the model with non-linear production. The equations (4.10) and (4.11) show that the effects have the same direction as in the linear models. Like the production function, the marginal effect of l_i is different and consequently the wage restriction changes; the considered wage level is $w < \frac{1}{2}\sqrt{\frac{x_i}{l_i}}$. This is consistent with the restriction set at the beginning of this chapter. Also, the sum of (4.10) and (4.11) equals zero, which proves that the tax rate level t does not affect agent's optimal choices.

$$\frac{du(c_1 + t\ \pi_1, \Omega_1)}{dL_1\ dt} = \frac{\partial\left(\left(tw - w - \frac{t}{2}\sqrt{\frac{x_1}{l_1}}\right)\Omega_1\right)}{\partial t} = \left(w - \frac{1}{2}\sqrt{\frac{x_1}{l_1}}\right)\Omega_1 < 0 \tag{4.10}$$

$$\frac{du(c_2 + t\ \pi_2, \Omega_2)}{dL_1\ dt} = \frac{\partial\left(\left(\left(w + \frac{t}{2}\sqrt{\frac{x_2}{l_2}} - tw\right)(1 - L_2) + 1\right)\Omega_1\right)}{\partial t} =$$
$$= \left(\frac{1}{2}\sqrt{\frac{x_2}{l_2}} - w\right)(1 - L_2)\Omega_1 > 0 \tag{4.11}$$

4.4 Evaluation of the non-linear production models

Under no commitment, the government implements an optimal lockdown level L_i under given circumstances. These are the health state Ω_i, which is given in the first period or determined by the previous period's labor supply and health state, and investments x_i, set by the profit-maximizing firms. Nevertheless, committing to future lockdown policies enables the government to influence the setting of investments with the incentive to maximize societal welfare. The commitment expands the government's pool of attainable outcomes. Hence new societal welfare-maximizing optimum might be reached while the societal welfare-maximizing optimum of the case with no commitment remains achievable. Thus, with a commitment to a future lockdown policy, the government can not end up worse off than without commitment, but it can end up better off. It is worth reminding that such commitment has to be credible.

Chapter 5

Extensions

This chapter provides extensions which might address some anticipated readers' questions and concerns. At first, it elaborates on the possibilities of the input function alternations in a particular example simulating the approach from previous chapters. In the second section, changes in the definition of variables are discussed, particularly the maximal investment level $\bar{x}$ and the tax rate t. This emerges into an indication of possible future research directions.

5.1 Further non-linear alternations

The previous chapter displayed how the alternation of production function $f(x_i, l_i)$ affects the strategic interaction. It is only logical to investigate the effect that substitution of other input functions has on the solution and compare these two effects. This section hence provides a look at the changes in the shape of potential labor supply $\bar{l}(\Omega_i)$ and the evolution of the health state $g(l_i, \Omega_i)$ functions, whose definition is not part of the environment description. Then the effect of the non-linear shape on the decision-making process and solutions is briefly evaluated. The goal is to justify the choice of particular input function shapes in models throughout this thesis and emphasize its importance.

5.1.1 Definition of functions

The potential labor supply $\bar{l}(\Omega_i)$ has to remain increasing. Moreover, the concave shape is desired and will be achieved through the square root.

$$\bar{l}(\Omega_i) = \sqrt{\Omega_i} \tag{5.1}$$

Health state evolution function $g(l_i, \Omega_i)$ has to be decreasing and additionally concave, still respecting the intervals of the variables.

$$\Omega_{i+1} = g(l_i, \Omega_i) = (-l_i^2 + 1)\Omega_i \tag{5.2}$$

The concavity in both functions is necessary in order to achieve the correct shape of the marginal disutility of lockdown on the current period and the marginal utility of lockdown on the future period functions.

5.1.2 Model 11 to Model 14

The analysis process of the previous chapter is preserved. Therefore, at first, the cases without strategic interaction are evaluated, and the section proceeds with strategic interaction without and then with the commitment. Models in this section are analogous with the exception of shapes of potential labor supply $\bar{l}(\Omega_i)$ and the evolution of the health state $g(l_i, \Omega_i)$ functions.

The solution provided in this section will be brief as the reasoning for agents' optimal choices remains the same. There have been no major differences in the logic, such as between Chapter 3 and Chapter 4. However, the particular equations and evaluations have alternated, which affects the values of optimal investment levels x_1^* and x_2^* and lockdown policy L_1^*, as the model should more precisely reflect the variable behaviour.

5.1.3 Evaluation of the non-linear models

This section serves rather as an explanation of the function's shape choice. Moreover, and possibly more importantly, it answers the question regarding the impact of other alternations in the function definition, which might naturally emerge after Chapter 4. It also sheds even more light on an important takeaway for this thesis' model: the shape of the production function $f(x_i, l_i)$ is cardinal for achieving the desired effects for the reasons already mentioned in Section 3.5 and displayed by the outcome of Chapter 4.

5.2 Endogeneity of variables

This section briefly elaborates on the possible endogeneity of some variables, which were taken as exogenous in the thesis. It aims to provide thoughts for possible extension of the presented models and further helps the reader to understand the chosen setting.

5.2.1 Maximal investment level $\bar{x}$

The maximal investment level in this thesis was exogenously set on some inalterable level $\bar{x}$. The firms were able to use this amount independently of previous periods' choices. However, as the models operate in a short pandemic wave, a setting in which $\bar{x}$ denotes a maximal investment level for both periods is reasonable. The firms would then have to concentrate on the distribution of the investments x_i between periods. In that case, the government commitment would be even more important, as the firms would have to plan more.

Another possibility, interesting especially in a model with a longer horizon, would be a maximal investment level based on the sum of previous periods' profits π. This would lead the firms to an improved economization of their actions and an effort to utilize periods with a lenient lockdown to accumulate profit. Consequently, they might strive to save the investments x_i in periods with strict restrictions by not maximizing the current period's profit π_i. It is evident that credible commitment would again improve their possibilities. In both scenarios, it is also in the government's interest to commit to a future lockdown policy for the reasons already mentioned in Chapter 4. Moreover, the limited investments provide even stronger motivation as no agent wants them to be utilized during a heavy lockdown period, thus providing a lower payoff as a result of the labor supply restriction.

5.2.2 Tax rate t

Time inconsistency, the main concern of this thesis, has been studied predominantly in capital taxation. The introduced setting also contains a taxation variable, but it is exogenous and constant throughout the model. Nevertheless, in the event that the tax rate t would be set endogenously by the government, the time inconsistency would arise also from the taxation. This would be in addition to the inconsistency emerging from the lockdown, which has been the object of this thesis. The setting with endogenous tax rate t would require more modelling emphasis on the distribution of the profit, which might partially shift the agents' motivation. Therefore, the analysis of a commitment's effect in this situation would be beneficial.

Furthermore, the relationship between lockdown policies L_i and tax rates t and the corresponding choices of the policymaker would generate interesting dynamics. The government might be moderately able to interchange between the variables and would achieve even greater flexibility if the tax rates t would be set for every period separately. However, this would depend on the particular features of the model. Again, the effects would become strengthen especially in a longer horizon model.

Chapter 6

Conclusion

Multiple two-period models simulating the economy hit by a pandemic wave have been built. The short-term approach enriches the existing infinite-horizon literature on this topic, as the reality of the COVID-19 pandemics shares some resemblances with the timeline introduced in this thesis.

At first, a linear model with simple-form input functions is introduced. The solution of this model presents the interaction process and emphasizes the role of taxation for this and the following models. The process, however, exposes certain limitations of the linear model, which concern especially the production function and provide essential takeaways for its definition. Hence, a non-linear production model is constructed. This model is then used to demonstrate the time inconsistency problem and the effect of government commitment on it. The results indicate a fundamental role of a credible commitment. These findings are consistent with those in the existing literature on the time inconsistency in the context of predominantly capital taxation.

The contribution of this thesis lies not only in this recommendation for policymakers but also in the provided theoretical framework. The models can be used as a simplified reality to emphasize the role of included relationships. The approach of a model's gradual improvement has led to a thesis that can present the time inconsistency in the context of COVID-19 pandemics and its modelling to undergraduate students interested in intermediate-level macroeconomics and intermediate-level game theory. The stress on models' composition further adds to the recent literature. The implications regarding the government commitment and credibility should lead to discussions and improvements in the decision-making processes not only in the lockdown policy and capital taxation area but to a broader range of time inconsistency problems.

Further research on the time inconsistency in pandemics is necessary. This thesis is one of the stepping stones, as it mainly elaborates on the foundations of modelling and its findings are rather theoretical indications. The inclusion of some finite-horizon thoughts should be considered in order to achieve a closer reality resemblance. Calibration with the data from COVID-19 pandemics and benchmark of the countries with different approaches to lockdown policies would reveal the most efficient methods. The ultimate goal of this research should be to provide a particular recommendation that leads to the higher efficiency of lockdown measures without a severe impact on productivity.

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