

Review of the Master's thesis "Prime geodesic theorem for the Picard manifold"

Department of Algebra

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Reviewer: Matteo Bordignon

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Study program: Mathematics

Study Branch: Mathematical structures

One of the greatest achievements of Number Theory is the prime number theorem which, defining $\pi(x)$ the counting function for all the primes smaller than x , states

$$\pi(x) \sim \frac{x}{\log x}.$$

Many generalizations of this result were produced, and these are an highly active research topic. The goal of this thesis is, drawing inspiration from the work in two dimensions of Phillips in [1], to obtain a weighted first moment of the error term of the approximation of the counting function of prime geodesics on the three dimensional Picard variety $SL(2, \mathbb{Z}[i]) \backslash \mathbb{H}^3$. The error is explicitly written using the Selberg trace formula which relates geometrical information with the spectrum of the Laplace operator on the Picard manifold. This result is new and, in the opinion of the review, worth of being published in a peer reviewed journal.

The thesis appears well structured and relatively easy to follow. The reviewer was able to find only two points, of not high importance, that could be improved upon. The first is that the introduction appears to be not as accessible, to a wide audience, as it would have been expected from a Master's degree thesis. The second is that in the thesis the author didn't motivate enough why the main result is of interest and more comparison could have been drawn to other similar results. The reviewer was able to find only one, and at that minor, mathematical mistake in the thesis. Namely that comparing equation (3.23) with (3.81) and (3.84), gives that in equation (1.12) we should have -2κ instead of 2κ . This doesn't affect the strength of the result as it only

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implies that e^ρ is on average larger than $\psi_g(e^\rho)$ and not vice-versa. Also some, while not important, typos are present. I will now list them:

- In page 2 appears a $\text{li}(N)$ that should have been a $\text{li}(x)$,
- In some of the matrices appears an unusual spacing in the left hand side,
- Some of the parentheses don't seem to correctly match the size of the formulas they are associated with,
- In equation (3.33) one parenthesis is misplaced,
- In page 27 'chose' should be 'chosen'.

I recommend the thesis to be defended as Master's thesis and awarded the highest grade of 1.

REFERENCES

- [1] R. Phillips, *Conjugacy classes of $\Gamma(2)$ and spectral rigidity*, Math. Comp. 64 (1995), no. 211, 1287–1306, S35–S37.