

EVALUATION OF A MASTER THESIS BY THE OPPONENT

Title: Combinatorial Gap Label Cover

Author: Filip Bialas

SUMMARY OF THE CONTENTS

The PCP theorem is a fundamental result in complexity theory, as it implies the computational hardness of many optimization problems. In a recent publication [3], Barto and Kozik stated a “combinatorial” version of it, which, despite being weaker than the original theorem, still suffices for several applications, while having a comparably easy proof. The goal of the underlying thesis by Filip Bialas was to investigate possible improvements of the combinatorial PCP theorem, following some of the directions proposed in [3].

Chapter 1 of the thesis gives some background on the classical PCP theorem, and discusses its equivalence to the hardness of the Gap Label Cover Problem. Chapter 2 presents Barto and Kozik’s result (corresponding to the hardness of the Combinatorial Gap Label Cover Problem), while Chapter 3 introduces another *probabilistic* combinatorial variation of the PCP theorem. Finally, in Chapter 4 it is shown that no result analogous to Raz’ Parallel Repetition Theorem [11] holds for the Combinatorial PCP theorem (nor for the Probabilistic Combinatorial PCP theorem). This negatively answers one of the open questions in [3].

EVALUATION OF THE THESIS

The topic of this thesis can be considered to be well above the average level of difficulty: Not only did working on it require a good understanding of recent developments in theoretical computer science, but also the ability to conduct independent research (given that the proposed goals were open ended).

The student clearly stood up to this level of difficulty: His original results in Chapter 3 and 4 are interesting new contributions to the field and arguably strong enough to be published in a relevant scientific journal. Furthermore, also Chapter 1 and 2 can be considered to be the original work of the student, as he summarized material from several different sources (in particular [3] and [9]) in his own words, filling in gaps and adding examples.

The mathematical quality of the thesis is very high, and I could not find significant mistakes in any of the proofs. Thus, my only critique concerns formal aspects of the thesis. The writing style of the thesis is quite informal. I do not consider this to be a problem by itself. However, the occasional lack of rigour in definitions and proofs can create some ambiguity, which makes them harder to understand. Here some notable examples:

- In proof of Claim 3: “Combinations of values which the PCP theorem accepts are in the relation.” The PCP theorem is not an algorithm or decision problem, thus it is not clear what is meant by “accepts” here.
- Definition 2: “[The prover’s] goal is to answer the questions with the highest possible probability. The highest probability is called the value $\text{val}(G)$ ”. In what sense is $\text{val}(G)$ the highest probability? Does it depend on the question of the verifier (as suggested by the previous sentence)?
- Proof of Lemma 10: The combinatorial solution s should have codomain $\mathcal{P}(D)$, not $\mathcal{P}(Y)$.
- Definition 5: “An assignment $f: X \rightarrow D$ is called an m -solution of the partial assignment system if [...] $f \in (s_{K_M})|_K$.” Something seems wrong here. What is K ? (It was universally quantified in the previous sentence).

- Lemma 12: it is never explicitly mentioned what the "set size" of a partial assignment system is.
- Definition 8: "[...] $\text{pcval}(I)$ is equal to [...], where the expected value is took using the uniform distribution over all arrows." No expected value appears in the formula! (also "taken" vs. "took")
- p21: "The Parallel Repetition theorem can be stated in the combinatorial setting as follows:" Is the following statement supposed to be true and equivalent to Theorem 19? Or is it equivalent to the statement that is proven to be wrong in Theorem 21?
- p 21: $\beta = \sqrt{\frac{1}{\alpha(\text{val}(L))}}$ - what is L ?
- Several of the intratextual references are not very precise (e.g. Lemma 12 is often just referred to as "the lemma").

These are however only minor critiques points, given the otherwise high quality of the thesis and clear presentation of the author's own proof in Chapter 4.

CONCLUSION

In conclusion I highly recommend to recognize this thesis for a defense. The suggested grading will be communicated directly to the committee.

Michael Kompatscher, Ph.D.
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