Evaluation of a Bachelor Thesis by Supervisor

Title: Applications of algebraic geometry in mathematical contests
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Summary of the contents

The thesis focuses on explaining how to use selected facts from algebraic geometry for efficiently solving geometric problems in mathematical contests. The methods are illustrated on chosen examples of actual recent problems from the International Mathematical Olympiad. This can be also viewed a bridge between high school mathematics, which is expected to be used in the contests and involves very classical geometric arguments based on similarity of triangles, counting angles etc., and university-level algebraic geometry.

The author introduced basics of algebraic geometry and found elementary proofs for key results which he wanted to employ in solving problems. This is in fact interesting on its own right because the elementary nature is somewhat deceptive. For example the so-called Method of Animation is based on deeper phenomena (such as the projective elimination theory) in algebraic geometry and finding a direct and elementary proof required non-trivial effort. Even worse, we were not able to find any written proof for the results which this method is based on as it comes from an online forum of fans of the Mathematical Olympiad and they only provided statements. As already mentioned, the author complemented the theory with nice illustrating examples which he chose completely on his own as he has substantial experience with mathematical contests himself.

Evaluation of the thesis

Topic of the thesis. The topic was in fact chosen by the author himself based on his interests. On one hand, it more or less precisely satisfies the recommendations for bachelor theses in mathematics in that the required mathematical background is covered by standard lectures, but the thesis itself definitely goes beyond. On the other hand, the work itself was creative in that it involved finding direct proofs of special cases of deeper facts in algebraic geometry. In that respect, the difficulty was above the average.

Contribution of the student. He wrote an original text on the not completely usual (from my point of view) topic, provided missing proofs and illustrated the described techniques on detailed examples.

Mathematical quality. The text is rigorous and presented in a logical way.

Usage of the literature. The literature is used and cited appropriately.

Formal aspects. The thesis meets usual standards from the formal point of view. The final phase of polishing the text was already under time pressure, which resulted in some misprints not being caught (such as a forgotten 'TODO' phrase on page 22 or the misprint in the name of one of the authors of reference [6]). Despite that, the number of such minor omissions still stayed reasonable, not marring the quality of the text.

Conclusion

I recommend to recognize this nice piece work as a Bachelor thesis.

The suggested grading will be communicated directly to the head of the examination (sub)committee.

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May 27, 2022