Generalising the notion of an ultrafilter to structured sets, we construct the ultrafilter monad in the categories of partially ordered sets and finitely colourable graphs. This is done similarly to codensity monads, knowing that the codensity monad of the inclusion of finite sets into sets is the ultrafilter monad. We derive an equivalent definition of an ultrafilter on an object applicable for general graphs, also giving rise to a monad. We show that ultrafilters on a poset can be completely characterised in terms of suprema or infima of directed subsets when the poset has only finite antichains. We attempt to classify algebras over the poset ultrafilter monad; our results completely classify the algebras with all antichains finite as posets with a particular compact Hausdorff topology.