PhD defense – Charles University (Prague, CZR), Department of Probability and Mathematical Statistics

Tomas Rusy PhD thesis and defense

Title: Asset-Liability Management: Application of Stochastic Programming with Endogenous Randomness and Contamination

Supervisor: Professor Milos Kopa

- **Overall assessment**

My assessment on Tomas Rusy (the *candidate*) work under Professor Kopa supervision is overall very positive when considering the contributions to the state-of-the-art in several areas, including interest rate models estimation and the extension of stochastic optimization methods to account for endogenous uncertainty. I believe that the thesis is first class in terms of clarity, rigorous mathematical development and relevance to the field. The candidate, this is very rare, almost unique, shows to master fully the topics, rather diversified, tackled in each chapter of the thesis.

The writing is very good and I would surely rate the work as belonging in my experience to the best 5-10% in terms of doctoral works I had the chance to supervise or assess in my academic career. Thank you then for the opportunity.

This report is divided in two parts: first part devoted to comments and remarks on the thesis content and the second part with questions I would put forward in the PhD defense.

PART 1] The thesis content and development.

One initial comment is due: it is rare and difficult even in scientific works to read such a good and comprehensive introductory section. The English is extremely good and there is no one unneeded paragraph nor any missing information, the spanning of the literature accurate and satisfactory as well. It was a pleasure to go through it.

Ch. 1: Interest rate modelling

This first section is devoted to a thorough discussion on the key properties of one factor models and then it develops to present the Hull-White (H-W) interest rate model and its key mathematical properties before presenting a novel estimation-calibration method based on the concept of *post-calibration*.

The study of the stochastic differential equation of the short rate process under either a risk-neutral measure Q or the real-world measure P is clear and very well presented. The H-W model is an affine interest rate model and the candidate clarifies through the chapter the relationship between the interest rate dynamics, the associated discount bond prices and through the relationship between the price yields under the two probability measures, the role played by market premiums. An empirical study is presented with the estimation period from 2017 to 2019 and forecasts over the remaining 2 years with statistics on the out-of-sample errors. Finally, the interest rate model is related to the 1 year default probability through a logistic function, anticipating the subsequent development of the ALM model with endogenous uncertainty.

- *Personal assessment*

I found the chapter very solid and well developed. The candidate contributions include: 1) the estimation procedure based on a post-calibration matrix and density estimate, which is clever and effective in terms of market validation, 2) the yield curve and density forecasts under the two probability measures, 3) the Hull and White model extension to the empirical measure.

Relevant topics: the candidate may focus during the discussion on a subset of the following points:

(i) The derivation of the conditional density of market yields and following profile log-likelihood function in eq. (1.21), pg 14.

(ii) The evidence presented on post-calibration errors

(iii) Insights and relevance of figures 1.3 and 1.4 in the economy of the research and their main implications.

(iv) The implications of the adopted default model and the relevance of the β_0, β_1 coefficients.

Ch. 2: An Asset-Liability Stochastic program with a pricing decision

This section is devoted to the key motivation for a stochastic program with endogenous uncertainty, the model description and a complete set of results for model validation. This is the core of the candidate contribution to quantitative ALM and the extension proposed is relevant. I believe that the content of this section is relevant to any credit institutions offering consumer loans. The structure of the asset and liability constraints is new due to the endogeneity of the decision process.

My assessment is extremely positive for the problem formulation and the numerical results: I believe that the description of the optimization problem and the lending company asset-liability model are outstanding and very advanced. The computational evidences are also relevant and appropriate. An important financial assumption of the A-L model is related to the floating interest rate on the lending company liability side and the fixed rate to propose.

From a very general perspective, I have two remarks: first, while reading the motivation of the model development I was considering the cost of accepting/rejecting the loan and conditional on accepting the loan the risk associated with either default or prepayment as potentially and properly captured by option contracts and associated exercise probabilities. Have the candidate considered this perspective? Why if so it turned out not to be appropriate?

Second remark: the optimization model is very good and well described. However due to the nonlinear feasibility region, I was wondering whether there was a risk for the problem to turn into a nonconvex program? Is this a risk which has been considered?

- *Personal assessment*

The contributions in this chapter are relevant and primarily related to: (i) the formulation of the ALM problem with endogenous interest rate, default intensity and prepayment rates is very relevant, (ii) the loan portfolio value maximization is new to me and very appropriate, (iii) the solution outcome and the sensitivity analysis are maybe not new in general but new in the given context and indeed very interesting.

Relevant topics: the candidate may focus during the discussion on a subset of the following points:

(i) Are other approaches to the default probability estimation possible? Is the proposed interest rate on the loan also indirectly leaving the borrower default probability non affected? Same for the prepayment probability.

(ii) Describe in detail the scenario generation approach.

(iii) Discuss and comment the interesting graphical evidences, especially the heatmaps.

Ch. 3: Stress-testing via contamination

This section extends several results on lower and upper bounds for contaminated stochastic programs to the case in which the probability space becomes decision dependent. This is probably the most innovative and rich of new results of the thesis. As the previous two chapters is very well written and properly structures and I found the numerical section very interesting and the introductory results, primarily theorems 7 and 8 in section 3.2.2 very relevant. It is of immediate understanding that particularly under an assumption of probabilities contamination specifically in the case in which such probability measure depends on the

decision process the entire optimization procedure may be jeopardized. The presented results clarify under which, relatively large, conditions this is not the case. The numerical section is also very clear and adequate.

It is therefore fully justified the submission of the results in this section to a first-class Journal. During the defense a mention to the current state of the submission would be welcome.

- *Personal assessment*

This section is full of nice results and the contribution, broadly speaking, extends contamination results established previously to the case of decision dependent probability spaces. By itself this is obviously a path-breaking contribution, which is then applied and studied in the context of the ALM problem introduced in section 2.

Relevant topics: the candidate may focus during the discussion on a subset of the following points:

(i) Qualitative assessment of the resulting ALM solution evidences related to the results in section 2,

(ii) Upper and lower bounds analyses under a contaminated stochastic program with endogenous uncertainty.

PART 2] PhD defense, questions, 14.12.2021

Further to the remarks and questions raised above at the end of every section as part of my assessment I indicate here next more general questions and topics that may be considered by the candidate and for which I would like to hear about. I am assuming a 2 hour discussion roughly and those below would represent question points to be considered in the say first hour before going more in detail.

A] General questions (overall research framework):

1) A general framework for stochastic programs with endogenous uncertainty: provide more general and relevant examples of uncertainty endogeneity, before presenting the case problem actually addressed in the study.

2) Present upfront the set of key contributions of the study, those most important from the candidate perspective.

3) Provide an overview of the data actually used to develop the project. Are the results consistent with the bank's operational practice.

B] General interesting issues from chapters 1, 2 and 3

4) Post-calibration approach to more general interest rate models (2 or 3 factor models): is the approach viable in this case?

5) ALM problem formulation and solution approach: after presenting the model, discuss if any and in which case, the limitations to consider for which the presented approach is not viable.

6) Stress testing in case of a contaminated problem with endogenous uncertainty: can distributionally robust optimization be regarded as a relevant associated reference mathematical domain?

Best Giorgio

gc/30112021

Opponent: Prof. Giorgio Consigli, Khalifa University of Science and Technology and University of Bergamo.