FACULTY
OF MATHEMATICS
AND PHYSICS
Charles University

Opponent Report on Doctoral Thesis

Asset-Liability Management: Application of Stochastic Programming with Endogenous Randomness and Contamination

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The thesis is devoted to studying stochastic programming problems with decision dependent (endogenous) uncertainty, which are motivated by asset-liability management in finance. In my opinion, this is an important topic in the fields of mathematical optimization and operations research.

The introduction provides an extensive overview of the literature related to the elaborated topics and states the contribution of the work to the current knowledge.

Chapter 1 deals with the short-rate interest rate models which are used to generate scenarios in other parts of the work. In particular, the chapter is focused on the well-known Hull-White model and proposes new calibration procedure based on the maximum likelihood approach, cf. Section 1.2. The likelihood functions are derived under the risk-neutral as well as the real-world measures, where especially the second one brings significant improvement to fitting real data. The new method is applied in the extensive numerical study. In addition to parameters estimation on real data from financial markets, the section contains stability analysis, forecast construction and out-of-sample analysis,

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swaption price calibration, and scenario generation approach. Last but not least, the section lists other possible research directions in this area.

In Chapter 3, large-scale multistage stochastic programming problem with decision dependent uncertainty is elaborated. This problem is motivated by pricing decisions of a loan company providing loans to small investors. The endogenous uncertainty includes probabilities of the acceptance, prepayment and default which depend on the optimized loan interest rate. A scenario approach is applied where the scenario tree is constructed by combining the exogenous and endogenous uncertainty, i.e. by including the scenarios of the interest rate and all decision-dependent states mentioned above. A large number of constraints are formulated, which are always properly motivated and explained. The resulting large-scale problem is solved using GAMS-CONOPT tool-solver and a detailed sensitivity study with respect to various parameters (e.g. offered rate or market interest rate) is provided.

Chapter 4 is devoted to stress testing of stochastic programming problems using the contamination technique where the original distribution is contaminated by another usually "more sceptic" distribution. This chapter is divided into two significant parts: theoretical and numerical. The first part elaborates the contamination technique in details and provides new theoretical results focused on the construction of the contamination bounds. One of the new theoretical results can be found in Theorem 3 which provides a way for construction of a tight lower bound for problems with exogenous uncertainty. In Section 3.2, the case of decision dependent randomness is elaborated. Theorems 5 and 6 propose new lower and upper contamination bounds for the problems with fixed set of feasible solutions, whereas Theorems 7 and 8 generalize these bounds for the case with the feasibility set dependent on the probability distribution. The second large part focuses on the stress testing and construction of the contamination bounds for the asset-liability management problem introduced in the previous chapter. The contamination bounds are constructed under several choices of the contaminating distribution and the results are discussed in details. Moreover, the model is extended by including the Conditional Value at Risk constraint which brings another complexity into the construction of the contamination bounds.

I have several questions and comments:

1. Page 14, formulas (1.20), (1.21): I can see some simplifications/approximations of the likelihood functions. However, I am not able to quickly identify them. Would you mind explaining them?
2. Section 2.2: Is the resulting large-scale nonlinear programming problem convex or nonconvex? How influences it the obtained results? How large problems are you able to solve to optimality in a reasonable time?
3. Section 3.3: In some cases, you are able to compute the optimal value for the contaminated problem, so the contamination bounds are in principle not needed. Can it happen that the complexity of the contaminated problem is so high (in comparison with the problem under original distribution P or contaminating one Q) that it cannot be solved to optimality? So the bounds are really needed. I remember such problem from stochastic integer programming, but I am not sure about your case, i.e. the multistage problems with decision dependent randomness.
4. Section 3.3.1: In a mathematical text, I would avoid phrases like “pretty good approximation” or “completely different story”.
5. The reference Hull and White (1990) occurs twice in the bibliography.

The thesis is written very precisely. I found very limited number of typos or ambiguities. The candidate works correctly with the sources, they are properly cited in the text and his contribution is clearly specified.

The candidate has published 2 papers in conference proceedings and 2 papers in journals with positive WOS Impact Factor, where one of them appeared in *Annals of Operations Research* which is a leading journal in mathematical optimization/operations research area. Two other papers are submitted to high quality IF journals (*European Journal of Operational Research, Journal of Empirical Finance*) and I believe that they will be soon accepted for publication.

To summarize, the thesis brings original results which contribute significantly to the fields of stochastic optimization and operations research. The mathematical methods used to derive new results are original and are supplemented by extensive numerical studies. I consider the goals of the work fulfilled. I would like also to appreciate critical evaluation of the obtained results and outlined directions for further research, which conclude many parts of the thesis. On the basis of the thesis, I conclude that Tomáš Rusý is capable of performing research independently. I have no doubt that the submitted thesis fulfils the requirements for a doctoral dissertation thesis and **I recommend it for defense.**

In Prague, November 16, 2021

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