REPORT ON FILIP STRAKOŠ MASTERS THESIS: *LEGENDRIAN SUBMANIFOLDS IN HIGH-DIMENSIONAL CONTACT TOPOLOGY*

Summary

The work under review deals with Legendrian submanifolds of contact manifolds. The thesis is well-written, showing a mature understand of the material in question. Since the material in question is an important and challenging area of contemporary mathematics, this is an impressive achievement for a masters student. Moreover, the student manages to establish two meaningful novel results. Hence I strongly recommend that the thesis be approved and that the student be awarded the highest mark of 1.

Mathematical Overview

The roots of contact geometry can be traced back to Sophus Lie who introduced the notion of contact transformation as a geometric tool to study systems of differential equations. The subject has numerous connections with other fields of pure mathematics, and as well as more applied areas such as mechanics, optics, thermodynamics, or control theory. Nonetheless, contact geometry has for a long time been receiving less attention than its even dimensional analogue, symplectic geometry. Contact topology is of more recent origin. However, since the mid 1980s, 3-dimensional contact geometry and topology have seen a number of breakthroughs.

The thesis under review treats Legendrian submanifolds of contact manifolds, a far reaching generalisation of Legendrian knots in $\mathbb{R}^3$. More specifically, it treats the classification of Legendrian submanifolds up to Legendrian isotopy. The study of the Legendrian submanifolds of a contact manifold is interesting for a number reasons, most notably because of the implications for the contact structure of the contact manifold itself.

Two of the most important classes of Legendrian submanifolds are known as *flexible* and *rigid*. The flexible family is studied using homotopy invariants, the so-called *classical invariants*. One of the main results of this thesis is to prove a limitation of the Legendrian product construction using the classical invariants.

The rigid class can be studied using the homological invariants, whose origins lie in symplectic field theory. The other original contribution of the student lies on the rigid side and describes the dga-homotopy criterion for augmentations of Chekanov-Eliashberg algebras of disconnected Legendrian sub-manifolds.

Material Presented in the thesis

§1: Basic Notions. As the title suggests, Chapter 1 introduces the fundamentals of contact and symplectic manifolds. A large number of helpful examples are provided, elucidating concepts such as contactomorphism, symplectomorphism, Darboux's theorem, Reeb vectors, symplectic bundles, and Legendrian isotropy.

This chapter does a very good job of presenting the necessary background material, giving the impression that the student has mastered the fundamentals of contact and symplectic geometry.

§2: Constructions of Legendrians. In Chapter 2, the constructions of Legendrian submanifolds are introduced, treating constructions such as products of Legendrian submanifolds. Moreover, a general overview of recent developments in this direction.

The content of this chapter suggests that the student has had no trouble in engaging with contemporary work in the field.

§3: Classical Invariants. In Chapter 3 introduces a number of classical invariants such as the Maslov index, the Conley–Zehnder index, the rotation class, and the Thurston–Bennequin invariant. Moreover, a nice survey of the applications of these invariants is given.

§4: Exceptional families of Tori. The first of the original contributions of the student can be found in this chapter, dealing with the subclass of flexible Legendrian submanifolds. A particular family of maps from $\mathbb{T}^3$ to the three sphere S^3, of non-zero degree, is constructed. It is then shown that this induces a map from $\mathbb{T}^3$ to U_3 whose homotopy class furnishes a non-splittable class. From this, it is possible to conclude that there exist toral embeddings into S^3 that are not isotropic to a Lengendrian embeddings of lower dimensional tori. This is a valuable contribution to resolving a question by Dimitroglou, Rizell, and Golovko, posed in *Algebraic and Geometric Topology.*

§5: Legendrian contact homology. In Chapter 5, the student treats the rigid class of Legendrian submanifolds. Here the approach is to associate a differenital graded algebra, the Chekanov–Eliashberg dga, to a Legendrian submanifold, and hence to provide invariants for the submanifold. Here the student establishes a number of new results, confirming his solid grasp of this contemporary approach to contact geometry.

Conclusion

From reading the thesis, it is clear that the student has absorbed a considerable amount of mathematics, not only from geometry, but also from modern algebra. His ability to navigate the subtle intersection between geometry and algebra is impressive. That he has control of these new techniques is demonstrated by the fact that he has been able to make novel contributions. I am convinced that the material is mathematically sound.

Thus I propose that the thesis be accepted with the highest mark of 1. As far as I understand, the student intends to continue in mathematics and pursue a PhD degree.

The thesis gives every reason to suspect that he will be successful in this endeavour, and I wish him the very best of luck.

General Suggestions to Improve the Thesis

In general the thesis is very well-written, and I commend the student for the care he took in his writing. However, there are a number of linguistic issues (which are only to be expected from a non-native speaker at masters level) that should be addressed.

Articles. There are consistent problems with articles. Sometimes articles appear when they should not, and sometimes they are missing when they should be present. When the article is identifying a unique object this can be quite confusing. Some examples are as follows:

1. **Sect.** 1.1: In Defn 1.1.5 *the contactization of P is a contact manifold* should read *the contactization of P is the contact manifold*
2. **Section 1.2**: In Definition 1.2.2 *The smooth map* should read *A smooth map.*
3. **Definition 1.3.1** should read *a symplectic* and *a smooth section*
4. **Top of page 20** should read *imitating the product* and perform surgery for Legendrian submanifolds
5. The title of §3.4 should read *The Might of classical invariants*
6. page 37 **Question 1** *When $n = 1$ the question . . .*

Non-Sentences. There are a number of instances where sentences are simply not sentences. These should be rewritten so that they make grammatical sense. Some examples are as follows:

- **Section** 1.2: Before Definition 1.2.1, the line *In is also . . .* does not make sense, I guess it should read *The complement of ξ in TM also plays a major role in contact topology.* Note the changes in the articles.
- **Section** 1.2: Before Definition 1.2.1, the line *In is also . . .* does not make sense, I guess it should read *The complement of ξ in TM also plays a major role in contact topology.* Note the changes in the articles.
- page 32 before Proposition 3.2.3,
- line 10 from the end of page 16,
- page 37 **Question 1** should read *Can any Lengendrian n-torus in* $(\mathbb{R}^n, \xi_{\mathrm{st}})$ *be attained . . .*
- 2nd and 5th lines on par 4, page 53 before Corollary 5.2.18,
- Proposition 5.5.2 on page 60

0.1. **Sentences beginning with a conjugation.** This complaint might be overly pedantic, but it is something that the student should avoid in his future writing. It is not correct to begin sentences with conjugations such as ”and” or ”because”.

- after figure 5.7, 5 lines from the end of page 21
- 9th line in section 2.1.4
- 3rd line from the of page 24

0.2. **Various Typos.** There are a number of (minor) typos throughout the thesis which should be addressed by a careful rereading. Some examples are as follows:

- Definition 1.4.3: in the first bullet point, it should read *except for finitely many*
- **Definition 2.1.3.2** should read *Reeb chord lengths*
- **Definition 3.3.1** there is a problem with the comma after $z(b) < z(a)$