

The work presents results concerning the self-smallness and relative smallness properties of products in the category of abelian groups and in $Ab5$ -categories. Criteria of relative smallness of abelian groups and a characterization of self-small products of finitely generated abelian groups are also given. A decomposition theory of UD -categories and in consequence a unified theory of decomposition in categories of S -acts is developed with applications on (auto)compactness properties of S -acts. A structural description of compact objects in two categories of S -acts is provided. The existence of projective covers in categories $S - \overline{Act}$ and $S - Act_0$ and the issue of perfectness of a monoid with zero are discussed.