This thesis is devoted to a formal presentation of an alternative proof of Gödel's first incompleteness theorem, based on the Berry paradox ("the smallest number not definable in under 57 characters", with this definition having less characters and defining this number). The approach used was suggested by an article by G. Chaitin. We define the Kolmogorov complexity of a natural number m as the binary length of the smallest program for the universal Turing machine that on input 0 outputs the number m. Using a formal argument based on the Berry paradox, we show that the property of a (large enough) number n being a lower bound for the Kolmogorov complexity of a number m is not provable in any consistent recursively axiomatizable extension of Robinson arithmetic. But by a counting argument, for all n, it is true for all but finitely many m. This is used to prove the first incompleteness theorem. Another way (by G. S. Boolos) of formalizing the Berry paradox to prove the same theorem is put in the context of the presented approach.