## **Opponent's Report on Dissertation Thesis**

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Title of the Thesis:	On Market Efficiency, Optimal Distributional Trading Gain, and
	Utility Maximization
Type of Defense:	DEFENSE
Date of Defense	June 7, 2022
Opponent:	Prof. Jiří Witzany (VŠE)

Address the following questions in your report, please:

- a) Can you recognize an original contribution of the author?
- b) Is the thesis based on relevant references?
- c) Is the thesis defendable at your home institution or another respected institution where you gave lectures?
- d) Do the results of the thesis allow their publication in a respected economic journal?
- e) Are there any additional major comments on what should be improved?
- f) What is your overall assessment of the thesis? (a) I recommend the thesis for defense without substantial changes, (b) the thesis can be defended after revision indicated in my comments, (c) not-defendable in this form.

The thesis brings both new theoretical results as well as practical numerical applications in in the area of portfolio optimization, trading strategy development, or a specific passport option analysis and valuation. The main chapters contain clear original contributions of the author and are based on three articles published in respected journals (AIS Q3 and 2x AIS Q2). However, it should be noted that all the three articles are coauthored by the adviser and another researcher. The thesis is based on an extensive list of relevant references and would be definitely defendable at my home institution (Faculty of Finance and Accounting, Prague University of Economics and Business). My more detailed comments and questions to individual parts of the thesis are given below.

The first three chapters cover the definitions and key results from stochastic calculus and dynamic programming that are needed in the main three chapters. My reservation to this part is that it is sometimes a little bit too sketchy or unbalanced, defining some really basic notions, but missing definitions of some more advanced concepts. For example, defining filtration, but not the conditional expectation, The Wiener process properties, etc. In Section 3.2 the butterfly arbitrage is defined, but not the calendar spread arbitrage. The main part of the Theorem 3.2.4 is not proved in the "Proof", but only its second part. A minor factual remark: Definition 3.2.1 defines the total variance as  $w = T\sigma$  where  $\sigma$  is the implied volatility, as if  $\sigma$  was in fact variance. The notion of volatility is sometimes confused with variance, but in the previous formula is stands for standard deviation (of log returns), and so the total variance should be rather  $w = T\sigma^2$ ?

Chapter 4 is based on a published article named as the chapter, i.e. "Equity Market Inefficiency During the COVID-19 Pandemic". It can be also interpreted as a classical attempt to develop an efficient trading strategy maximizing the power utility function. The approach is using the general optimal portfolio results combining a risk-free asset and a risky asset (different ETFs) based on estimated drift and volatility. The empirical approach to estimate the drift and volatility is relatively simplistic combining several univariate regressions based on a list of suggested explanatory variables. It is surprising that all the univariate regressions are combined with equal weights. In a classical econometric analysis, importance of the variables would be analyzed and only the significant variables would be left. I miss a detailed table reporting statistical properties and importance of the considered explanatory variables. Since the approach does not work outside of the Covid period, one would expect that the Covid related explanatory variables give the relatively high explanatory power. Is it the case? The model also requires to select two key hyperparameters. The article firstly shows detailed results with an ad hoc proposed couple of hyperparameters and then deals with hyperparameter selection with optimal hyperparameters being almost the same as the initial ad hoc proposal. This looks suspicious and the approach might lead to the phenomenon of backtest overfitting or "data snooping". Can the author exclude this issue? It is also surprising that according Figure 4.6 the final portfolio values almost do not depend on the coefficient a although the risky asset allocation is proportional to 1/a (equals to  $(\mu$  $r)/a\sigma^2$  with  $\pm 1$  limit on long short positions) and so one would expect a significant relationship between a and the expected profitability. Can the author explain better this surprising empirical outcome?

The next Chapter named "Utility Maximization of the Discrepancy between a Perceived and Market Implied Risk Neutral Distribution" assumes that the underlying asset follows the Geometric Brownian Motion but there is a discrepancy between the market taker and maker view on its parameters. It is shown how to express analytically the optimal payoff of the market maker given a power or logarithmic utility function. According to a general result the optimal payoff function can be replicated by o portfolio bonds, forwards, and options. Since the number of instruments needed for precise replication is generally infinite, the remainder of the chapter deals with the numerical mathematics problem how to approximate the optimal payoff with a finite set of instruments showing that a relatively low number of options is needed to obtain a good precision. My practical comment to this analysis is a classical one: implementation of theoretically designed optimal strategies often fails when transaction costs are considered. Since the replication portfolio probably usually consists of long and short option positions, the transaction costs (in the form of bid-mid-ask spreads) will play larger and larger role when the precision is increased, possibly going to infinity. Was the effect of transaction costs taken into account? Another minor remark: in the maturity gain-loss function is defined  $g(X, K, m) = (X - K)^+ - p(K, m)$ , the cost p(K, m) should be the call option premium plus the accrued interest, not just the premium?

The last chapter "Options on a traded account: symmetric treatment of the underlying assets" analyses a specific passport option, i.e. an option where the option holder trades two assets with certain restrictions over a time horizon and keeps the profit if any, but the losses are covered by the option seller. The key problem is the optimal strategy determination and the option valuation. The proposed passport option imposes symmetrical restrictions on both assets and the optimal strategy is measured with respect to the reference (numeraire) index mixing the two assets in 1:1 proportion. A relatively technically difficult derivation leads to a surprisingly simple optimal strategy investing always 100% of funds into the weaker asset in terms of the relative performance. The optimal strategy maximizes  $E^{I}[(X_{I}(T) - K)^{+}]$  where I is the chosen reference index. Would the result (optimal strategy) change if the reference

index was defined in a different way? If the answer was yes, would not the dependence on the chosen index be an issue in terms of practical valuation of the option? A minor remark is that in Theorem 6.4.2 and elsewhere the expectation is incorrectly stated as  $E^{I}[X_{I}(T) - K]^{+}$ .

Overall, in spite of some questions and minor remarks, the thesis contains valuable and original contributions of the author. **To conclude, I recommend the thesis for defense without substantial changes.** 

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