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A referee's report on the Ph.D. thesis

Robert Navrátil: On Market Efficiency, Optimal Distributional Trading Gain, and Utility Maximization

In the doctoral thesis author studies several problems from contemporary mathematical finance such as optimal distributional training gain problem, utility maximization, implied volatility surface parametrization, market efficiency during the COVID-19 pandemics, passport options (options on traded account). All these areas fit well into the specialization *Probability and mathematical statistics* and the topic as a whole **is suitable** to be defended at the Department of Probability and Mathematical Statistics, Faculty of Mathematics and Physics, Charles University.

Presented thesis serves as a well-arranged summary of results obtained in three independent research papers that the author wrote together with his supervisor and two other co-authors, in particular in Navratil, Taylor, and Vecer (2021, 2022) and Vecer, Kampen, and Navratil (2020) respectively. Summary of these papers is presented in Chapters 4, 5 and 6 respectively and since it is expected that they were thoroughly peer-reviewed in the respectful impacted journals, no additional review of these parts is necessary and hence the major comments are focused on the first three chapters.

In Chapter 1, author tries to present necessary preliminaries from stochastic calculus and dynamic programming. This part unfortunately contains several formal flaws and imperfections that are often seen in works of early stage researchers before review. What follows is an unsorted list of some of these issues:

– inconsistency in denoting probability measure sometimes as P and sometimes as $\mathbb{P}$ (p. 6 and on),

– not detailed referencing to books (i.e. missing section and/or page numbers), such as Karatzas and Shreve (1991) (p. 6 and on),

– using terms that were not defined earlier, such as $\mathcal{F}_t$-adapted (p. 6, l. -6), $\mathcal{F}_t$-stopping times (p. 6, l. -1), $\mathcal{F}_t$-Wiener process (p. 7, l. -6), $\mathcal{F}_t$-progressively measurable (p. 9, l. 10), strong solution (Def. 1.2.3) including the missing conditions for existence and uniqueness, etc.,

– in Thm. 1.1.6 there should be probably $\langle M\rangle = \langle M, M\rangle$,

– Thm. 1.1.8 is multi-dimensional Itô's formula, but the proof refers to a one-dimensional version only,

– in Thms. 1.2.4 and 1.2.5 b and σ satisfy *at most* linear growth in x variable, similarly *at most* polynomial growth for L and Ψ in the paragraph below this theorem,

– p. 13 and on, it is not very wise to use W to denote more different objects (Wiener process, value function, solution of the dynamic programming equation, etc.),

– what is $\mathcal{A}^v$ in (1.6)?

Chapter 2 introduces both the efficient market hypothesis (Section 2.1) and the optimal distributional trading gain problem (Section 2.2). At the beginning of both sections there is a nice literature review making the text self-contained. Tiny comments:

– not detailed referencing to the book by Vecer (2011) (p. 17 and on),

– Arrow-Debreu securities are not explained before the proof of Thm. 2.1.4,

– what is F on p. 21, l. 4?

In Chapter 3, the so called stochastic volatility inspired (SVI) parametrization of the implied volatility surface is introduced. Unfortunately the quality of the text here follows some of the *bad* habits of the original texts by Gatheral (2006) (the reference in the thesis uses a later printing – online publishing) and Gatheral and Jacquier (2014), not mentioning the really poor quality of the unpublished preprints written by Ferhati (2020a,b) (the latter having almost 70% of the content identical to the former). Some comments:

– density q mentioned for the first time on p. 26, l. -1 should have been introduced earlier,

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– Def. 3.2.2 and Thms. 3.2.3 and 3.2.4 work with calendar spread arbitrage and butterfly arbitrage, but these terms are not properly defined before, only mentioned in the text on p. 28,

– The first paragraph below Def. 3.2.6 is little misleading. Cited Theorem 4.1 from Gatheral and Jacquier (2014) does not refer neither to raw SVI parametrization nor to butterfly arbitrage. Necessary and sufficient condition for a general parametrization to be free of butterfly arbitrage is $g(k) \geq 0$ as mentioned in Thm. 3.2.4 that is equivalent to Lemma 2.2 in Gatheral and Jacquier (2014). However, it is not clear what are the equivalent conditions for the parameters $(a, b, m, \varrho, \sigma)$ of the raw SVI parametrization to guarantee $g(k)$ to be non-negative and there exist many examples for which the raw SVI parametrization is not arbitrage-free. First attempt to find these conditions is probably mentioned in the Theorem 2.1 in the unpublished preprint by Ferhati (2020b), however, these conditions are derived only from the linear asymptotes for $k \to \pm\infty$ and I am afraid that they cannot be considered neither as sufficient nor as necessary conditions.

It is worth to mention that the last comment should not influence the results obtained in Navratil, Taylor, and Vecer (2022), provided the implementation of the constrained optimization checks the positivity of $g(k)$ rather than the conditions mentioned by Ferhati (2020b). One suggestion for the further research therefore could be to use a different implied volatility surface parametrization such as the SSVI (Gatheral and Jacquier 2014) or a more general version introduced by Guo, Jacquier, Martini, and Neufcourt (2016).

Unfortunately, there are no supplementary codes and especially data to check and analyse the correctness of the numerical and empirical results neither as a supplement to the thesis nor to the published papers. Although it is not explicitly required by the dissertation guidelines, in my personal opinion, any result in mathematical finance should be presented together with numerical confirmation as well as open source codes and data. In my opinion, Charles University and Faculty of Mathematics and Physics should adapt the FAIR principles.

Apart from the several above mentioned issues, the overall thesis is well written in a coherent form and together with the published papers is of very high quality. Ph.D. candidate was studying and solving state-of-the-art problems of modern mathematical finance and showed a potential and preconditions for independent research. For this reason **I recommend the thesis to be defended**. During the defence I suggest the candidate to clarify the own contribution in each of the published papers together with the further issues (possible further research directions) and comment on the implementation of all numerical codes.

Ing. Jan Pospíšil. Ph.D.

References

Ferhati, T. (2020a), *Robust calibration for SVI model arbitrage free*, available at SSRN: https://ssrn.com/abstract=3543766.

Ferhati, T. (2020b), *SVI model free wings*, available at SSRN: https://ssrn.com/abstract=3557646.

Gatheral, J. (2006), The volatility surface: A practitioner's guide. Wiley Finance, Hoboken, New Jersey: John Wiley & Sons, ISBN 978-0-471-79251-2/hbk, 978-1-118-04645-6/e-book.

Gatheral, J. and Jacquier, A. (2014), *Arbitrage-free SVI volatility surfaces.* Quant. Finance 14(1), 59–71, ISSN 1469-7688, DOI 10.1080/14697688.2013.819986, Zbl 1308.91187, MR3175969.

Guo, G., Jacquier, A., Martini, C., and Neufcourt, L. (2016), *Generalized arbitrage-free SVI volatility surfaces.* SIAM J. Finan. Math. 7(1), 619–641, ISSN 1945-497X, DOI 10.1137/120900320, Zbl 1410.91451, MR3546976.

Karatzas, I. A. and Shreve, S. E. (1991), Brownian motion and stochastic calculus, vol. 113 of *Grad. Texts Math.* New York: Springer-Verlag, second edn., ISBN 978-0-387-97655-6/pbk, Zbl 0734.60060, MR1121940.

Navratil, R., Taylor, S., and Vecer, J. (2021), *On equity market inefficiency during the COVID-19 pandemic.* Int. Rev. Financ. Anal. 77, 101820, ISSN 1057-5219, DOI 10.1016/j.irfa.2021.101820.

Navratil, R., Taylor, S., and Vecer, J. (2022), *On the utility maximization of the discrepancy between a perceived and market implied risk neutral distribution.* European J. Oper. Res. 302(3), 1215–1229, ISSN 0377-2217, DOI 10.1016/j.ejor.2022.01.048.

Vecer, J. (2011), Stochastic finance. A numeraire approach. Chapman & Hall/CRC Financial Mathematics Series, Boca Raton, FL: CRC Press, ISBN 978-1-4398-1250-1/hbk; 978-1-138-11641-2/pbk, 978-0-429-09240-4/e-book, DOI 10.1201/9781439812525, Zbl 1208.91005, MR2681007.

Vecer, J., Kampen, J., and Navratil, R. (2020), *Options on a traded account: symmetric treatment of the underlying assets.* Quant. Finance 20(1), 37–47, ISSN 1469-7688, DOI 10.1080/14697688.2019.1634278, Zbl 1431.91406, MR4040261.

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