

Opponent report on the Ph.D. thesis "*Classical operators of harmonic analysis and Sobolev embeddings in rearrangement-invariant function spaces*" by **Zdeněk Mihula**

The Ph.D. thesis of Z. Mihula is focused on the behavior of various operators on rearrangement-invariant function spaces with particular emphasis on the sharpness (optimality) of function spaces appearing either as domains or as targets.

The thesis is divided into five chapters, the first one being preliminary. The second chapter contains all the necessary one-dimensional inequalities which are later used to prove the main results via reduction principles. The main results are contained in Chapters 3-5. They are centered on classical operators of harmonic analysis, optimal Sobolev embeddings on the entire Euclidean space, and compactness of Sobolev embeddings with upper Ahlfors measures (called sometimes also Frostman measures), respectively.

The thesis is composed in the form of a comprehensive mathematical text (it is not just a collection of published or accepted papers). Therefore, it is easy to read, it makes use of unified notations and the sequence of results is natural.

The information about sharpness of a function space (considered within a prescribed pool of spaces that are allowed to compete) is important both for applications and for the further development of the theory. In some cases even the very existence of an optimal structure is highly nontrivial and of interest. This part of functional analysis has been studied intensively during the last two or so decades, and, while plenty of results have been obtained, it still contains plenty of open questions. Some of those are tackled and successfully solved in the thesis.

The thesis deals with interesting problems and demonstrates considerable technical skills, good theoretical background, and a high level of scientific insight. The results are deep, original and interesting, the proofs are given in good details, and each part is richly illustrated with examples.

The results contained in the thesis are the subject of six papers. In two cases Z. Mihula is a single author, the remaining ones were written jointly with co-authors, some of whom have positions at institutions abroad. All papers have been either published, or accepted for publication, in international journals with high reputation and good impact.

In my opinion, Z. Mihula has proved to be a very talented young mathematician who can carry out sensible mathematical research, both on his own and in collaboration in international research teams. He certainly has all the skills required to become a good academic. During his visit to Florence and during my visits to Prague I had a chance to discuss some mathematical problems with Z. Mihula and also to attend some lectures delivered by him. Therefore, I can confirm that he is not only an excellent researcher but also a good speaker. I believe he would be an asset of any research team.

In conclusion, the submitted work by Z. Mihula certainly fulfils (and in some ways overtakes) the requirements for the Ph.D. thesis, and I strongly recommend that the Ph.D. title is awarded to him.