

February 5 2022

Evaluation of the PhD thesis “Classical operators of harmonic analysis and Sobolev embeddings in rearrangement-invariant function spaces” in Mathematics
by **Zdenek Mihula**

In his PhD thesis some new results concerning the boundedness properties of some classical operators of harmonic analysis (e.g. the Hilbert and Riesz transforms, the Riesz potentials and (fractional and nonfractional) maximal operators) are derived and applied. Moreover, some new Sobolev-type embeddings on the entire space are proved and discussed. The compactness of Sobolev trace embeddings is also investigated. A great focus is put on the optimality of these new results within the class of rearrangement-invariant function spaces.

The obtained results are also of great interest in the area of Hardy-type inequalities since some of the crucial questions can be reduced to equivalent problems concerning appropriate Hardy-type operators acting on functions of a single variable. More exactly, first the behaviour of some Hardy-type operators on rearrangement-invariant function spaces is investigated. These new, and also some well-known, results concerning some Hardy-type operators are used as the building blocks when the main results in this PhD thesis are proved, discussed and applied. All these new results are put into a more general frame by presenting the PhD thesis as a monography.

This PhD thesis contains totally 159+iii pages. It is divided into five different Chapters and List of References containing 97 items. The new results in this PhD thesis by the author are based on six papers with the candidate as author or co-author. Four of these papers are already published in very good international Journals, one is accepted for publication and the sixth one is submitted for publication. The candidate is sole author of two of these papers. In this connection it is also worth to be mentioned that the candidate is co-author for further two papers submitted for publications.

My only really minor critical remark is that even if the list of references is extensive and clearly acceptable I mean that it could be updated and complemented a little concerning Hardy-type inequalities.

Remark: Before I present my judgement I want to mention that I have myself been a supervisor of 66 students with PhD exams and also been the opponent or corresponding to around 40 PhD:s so I have much to compare with.

My overall judgement: I judge that the submitted PhD thesis is clearly acceptable as a doctoral dissertation of very high level.



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