A module over a ring R is R-projective if it is projective relative to R. This module-theoretic notion is dual to the notion of an R-injective module that plays a key role in the classic Baer's Criterion for Injectivity. This Thesis is concerned with the validity of dual version of Baer's Criterion. It also introduces a concept of projectivity in a general category-theoretic setting.

DBC is known to hold for all perfect rings. However, DBC either fails or it is undecidable in ZFC for non-perfect rings. In this Thesis we deal with the subclass of non-perfect rings, which are small, regular, semiartinian and have primitive factors artinian. Trlifaj showed that there is an extension of ZFC in which DBC holds for such rings. Especially, it is enough to consider extension of ZFC in which the weak version of Jensen's Diamond Principle holds. This combinatorial principle is known as the Weak Diamond Principle.

Apart from an overview of the properties of rings mentioned above and introduction of the necessary set-theoretic notions, the Thesis also contains a proof of this new result by Trlifaj published in the paper "Weak diamond, weak projectivity, and transfinite extensions of simple artinian rings" in the J. Algebra in 2022.