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Faculty of Social Sciences
Institute of Economic Studies



BACHELOR'S THESIS

**Assessing Sustainability of the PAYG Pension
System in the Czech Republic and the Effect of
Automatic Balance Mechanism**

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Academic Year: **2021/2022**

Declaration of Authorship

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Prague, January 04, 2022

Signature

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Abstract

The financial sustainability of pension systems facing the ageing population is subject to extensive discussion by researchers. This thesis aims to assess the financial sustainability of the Czech PAYG pension system and then to design the Automatic Balance Mechanism (ABM), which is a recently developed strategy to restore the sustainability of the PAYG pension systems, which has not been introduced to the Czech pension system yet, according to the author's knowledge. The ABM reside in the calculation of the future course of three key variables, the contribution rate, the age of retirement and the indexation of pensions while ensuring financial sustainability and the liquidity of the pension system for all years of the study. As the results show, without applying the ABM rules, the Czech pension system does not seem to be sustainable in the period of 25 years.

Keywords

pension system, demographic change,
automatic balance mechanism, optimisation

Abstrakt

Finanční udržitelnost důchodových systémů, které čelí stárnutí populace, je předmětem rozsáhlé diskuse výzkumníků. Tato práce si klade za cíl zhodnotit finanční udržitelnost českého průběžně financovaného penzijního systému a následně navrhnout automatický vyvažovací mechanismus (ABM), což je nedávno vyvinutá strategie obnovy udržitelnosti průběžně financovaných penzijních systémů, která dosud nebyla aplikována na český penzijní systém. ABM spočívá ve výpočtu budoucího vývoje tří klíčových proměnných, sazby důchodového pojištění, věku odchodu do důchodu a valorizace důchodů při zajištění finanční udržitelnosti a likvidity důchodového systému po všechny roky studie. Jak ukazují výsledky, bez aplikace ABM se český důchodový systém v období 25 let nejeví jako udržitelný.

Klíčová slova

penzijní systém, populační vývoj, ABM,
optimalizace

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Acronyms

PAYG	Pay-as-you-go pension system
DC	Defined contribution
DB	Defined benefit
NDC	Notional defined contribution
OECD	Organisation for Economic Co-operation and Development
OLG	Overlapping generations model
AAM	Automatic adjustment mechanism
ABM	Automatic balance mechanism
NLO	Nonlinear optimisation problem

Thesis Proposal

Author:	Lukáš Pokorný
Supervisor:	doc. Petr Janský, M.Sc., Ph.D.
Proposed Topic:	Assessing Sustainability of the PAYG Pension System in the Czech Republic and the Effect of Automatic Balance Mechanism

Research question and motivation:

The issue of pension systems and their sustainability is considered to be one of the most substantial social challenges nowadays. Numerous countries around the world, including the Czech Republic, have to deal with the changes in their population structure caused mainly by the decrease of birth rates and the increase of life expectancy. The major focus of my thesis will be brought on the very situation in the Czech Republic. The pension system in the Czech Republic still largely depends on a pay-as-you-go (PAYG) system, whose long-term financial sustainability has been repeatedly called into question. Recently, new strategies to restore the sustainability of the PAYG pension systems have been designed, called the Automatic Balance Mechanism (ABM), which I would explore deeper in my thesis and apply it to the Czech pension system. My thesis will be written in the English language.

Contribution:

This thesis should provide new insights into the issue of the financial sustainability of the PAYG system in the Czech Republic. To best of my knowledge, the ABM has not been applied to the Czech pension scheme yet. This mechanism may offer an interesting perspective on that matter.

Methodology:

The goal of my thesis is to calculate the optimal paths of the key variables (contribution rate, retirement age and indexation of pensions) of the PAYG pension system in the Czech Republic, which would guarantee the long-term sustainability of the pension system. Following the method of ABM developed by Godínez-Olivares et al. (2016), I will begin with specifying the general optimisation model depending on parameters such as the age-dependency ratio, the population size, the salary growth, the expenditure on pensions and the key variables, whose optimal paths I am interested in. Afterwards, using the most recent data regarding population projections obtained from Eurostat Database, I will apply the Generalised Reduced Gradient algorithm (following Godínez-Olivares et al. (2016)) to calculate the optimal paths of the key variables that would guarantee the long-term financial sustainability of the system.

Preliminary hypothesis:

1. The PAYG pension system in the Czech Republic is not financially sustainable in the long run (even with the application of ABM).
2. To keep the indexation of pensions positive and the contribution rate below 20%, the age of retirement needs to be above 70.
3. To keep the age of retirement below 70, we need to have the contribution rate above 20% or negative indexation of pensions.

Outline:

1. Introduction
2. Literature Review
 - a. Pension Systems around the World
 - b. Pension System in the Czech Republic
 - c. Financial Sustainability of Pension Systems
3. Methodology
 - a. Methodological Background
 - b. Data Description
4. Discussion of Results
5. Conclusion

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Author

Supervisor

1 Introduction

Population ageing has an adverse effect on the changes in the population structure. According to Cipra (2012), population ageing is not a new phenomenon that would be characteristic of our present time. It has been observed for more than a hundred years. However, the sociological aspect regarding the retirement perspective has been changed. In the last century, retirement has been looked upon as a synonym for poverty in some developed industrial countries, and workers were generally retiring because of their loss of physical or mental ability.

While the proportion of pensioners in the total population rises, the number of working-age individuals decreases, which causes weaker productivity of the economy and growth of the government spendings. Next to its possible negative impact on the health care system, the public pension system is often cited as one of the most influenced spheres by the demographic shift.

The pension system in the Czech Republic still heavily depends on a pay-as-you-go (PAYG) system, whose long-term financial sustainability has been repeatedly called into question.¹ The life expectancy in the Czech Republic significantly increases,² which makes the prospect of maintaining the Czech pension system stable uncertain. There are several challenges for the PAYG pension systems around the world, emerging not solely from the adverse demographic change. Despite the increase of life expectancy and the fertility rate decline, emigration is often cited among threats for the financial stability of the PAYG system, as well as tax avoidance or the unemployment rate increase in some countries. The PAYG pension system heavily relies on the so-called intergenerational agreement, which stands on ensuring an adequate standard of living for retirees and simultaneously on not putting an unbearable financial burden on the working population. Since those two tasks are by nature contradictory, the whole PAYG system is somewhat prone to be a subject of political competition, and its

¹ E.g. by Alonso-García et al. (2018), Gawthorpe & Šafr (2019) or Cipriani (2014).

² The Czech Statistical Office estimates 75 years for newborn men and 81 years for newborn women in 2020, as opposed to 69 years for newborn men and 76 years for newborn women in 1993.

incautious alteration could result in labour market disruption and intergenerational conflict.

Various studies have assessed the possibility of introducing the so-called automatic balance mechanism (ABM) to PAYG pension systems in the past few years.³ These strategies have an ambitious goal of restoring the sustainability of PAYG pension systems and maintaining their financial stability in the long run.

The general goal of this thesis is to provide new insights into the issue of the financial sustainability of the PAYG system in the Czech Republic, to introduce the ABM method to the Czech pension system, and to simulate a future course of the system with the application of the ABM rules with the intention of ensuring of the system's long-run sustainability. The ABM model used for the purpose of this study is an optimisation problem of a determined function settled in a nonlinear framework, whose minimisation with a simultaneous forecast of the key variables results in the ABM rules.

The thesis is structured as follows: Chapter 2 briefly describes and clarifies the general classification of pension systems, characterises the Czech PAYG pension scheme, assesses the financial sustainability of pension systems and addresses its situation in the Czech Republic. Chapter 3 covers the theoretical principles of the ABM methodology for restoring the sustainability of pension systems and discusses a course of the key variables' development. Chapter 4 describes the data necessary for the ABM modelling and clarifies a calibration of the ABM model. Chapter 5 discusses the results of a simulation of the future course of the Czech pension system and an application of the ABM model to the Czech public pension system and discusses its effect. Chapter 6 summarises the study's findings.

³ E.g. Godínez-Olivares et al. (2016b), Ferrer Fernández & Boado-Penas (2017) or Gannon et al. (2014).

2 Pension Systems

Barr (2002) states that the general goal of a pension system is to provide income, consumption smoothing and poverty prevention for the pensioners. The following commonly used criteria can help to distinguish pension systems:

- **Public/private pension systems** differentiate whether the pension system is administered by the government (or authorised public institution) or by commercial institutions (such as insurance companies).
- **Mandatory/voluntary pension systems** might be either public or private systems. It is usually compulsory to participate in public pension systems as established by law.
- **Funded pension systems** do have available enough assets to cover their future pension liabilities, while the **unfunded pension systems** do not, and they rely on continuous financing through contributions. Furthermore, the participant's accumulated assets in funded systems are being continuously invested, and they are later used with the earnings from investments directly for the participant's future pension. In contrast, the contributions made to the unfunded systems are used for simultaneous pension payments to the present-day pensioners. Unfunded plans are referred to as the pay-as-you-go (PAYG) systems as well. Although unfunded schemes are usually less prone to be affected by inflation or financial market performance than funded schemes, they are highly affectable by demographic changes. It is worth mentioning the **notional defined contribution (NDC) pension systems**, which have similar features to the unfunded systems. However, by creating seemingly separate accounts for individuals and behaving partly as funded pension schemes, they adapt better to the adverse demographic shift and economic changes.
- **Defined contribution (DC) pension systems** are funded schemes having defined participant's contributions to the system, whereas the **defined benefit (DB) pension systems** can be both funded and unfunded schemes having determined the level of pensions paid.

The so-called World Bank terminology defines the pension systems classification as a three-pillar arrangement, where every pillar has a different existential purpose, financing structure, and they differ in their mandatory status.

- **The first pillar** is commonly known as the mandatory public PAYG pension system, which represents the traditional tool for reducing poverty among retirees and is usually conditioned by reaching certain retirement age and simultaneously satisfying a requirement of contributing to a pension system for an established number of years. It is usually unfunded and administrated by a public institution with a governmental guarantee. As a universal system depending on intergeneration solidarity and including all economically active citizens, it is still a mainstay of pension systems in many countries. It is based on the foundation of cumulating contributions from the working population, from which pensions are simultaneously paid out to the pensioners.
- **The second pillar** does, on the other hand, generally rely on the individual's occupation since it is usually managed by employers or work associations. This kind of pension plan is generally understood as supplementary to the first pillar pension system. It is mostly mandatory as well, but it is funded, and it does reflect the individual's preceding income more accurately. Hence, it is more earnings-related.
- **The third pillar** is a funded scheme based solely on private savings with a possible governmental contribution or tax exemption. It is supposed to be an additional pension plan to both previous pillars. Thus, it is voluntary, and it relies on citizens' initiative and commercial supply (usually provided by insurance companies).

The three-pillar classification has been later extended and redefined to a five-pillar system to delineate current pension systems more sensitively and precisely. The so-called null and fourth pillars were introduced, and the previous three pillars were slightly redefined while preserving their primary distinction. The definition of a particular pillar might slightly differ in various countries. The newly introduced pillars are delineated as follows:

- **The null pillar** is widely understood as elementary protection against poverty. It is clarified as a non-contributory pension system. A specified level of pension is provided even to citizens that had not been contributing to a pension system

at all. The requirements for receiving pensions vary, customarily a reaching of the retirement age and the individual's social situation are taken into account.

- **The fourth pillar** is a summarising term for various social programs and services from medical and nursing services to accommodation or charitable programs.

For a more detailed description of pension systems, refer to Cipra (2012) or Slaný et al. (2004).

2.1 Pension System in the Czech Republic

With the purpose of providing for citizens facing old age, invalidity or loss of a breadwinner, as described in Act No. 155/1995 Coll. (the Pension Insurance Act), the pension insurance in the Czech Republic is under the competence of the Czech Social Security Administration. There are four types of pension insurance benefits provided: old-age, invalidity, widow's/widower's and orphan's pensions. As 82% of all expenditures on pension insurance comprises expenses on old-age pensions,⁴ the financial sustainability and stability of the whole pension insurance system heavily rely on the condition of the old-age pensions, on which, for the purpose of this research, the focus is brought.⁵

After the collapse of the planned economy in 1990 and resulting market liberalisation, the pension system in the Czech Republic, at that time solely dependent on the PAYG system, underwent through several changes and redefinitions in the subsequent years. Besides several parametric adjustments, the separation from the government budget and the introduction of the supplementary third pillar rank among the most significant alterations made to the Czech pension system in the 1990s. In 2010, the age of retirement and the insurance period were redefined, as well as the pension assessment. For a brief period between 2013 and 2015, a second pillar type pension system existed in the Czech Republic before it was cancelled due to its unpopularity among the majority of citizens and legislators. Savings from the funds

⁴ An average value of 2013-2020, according to the Czech Social Security Administration.

⁵ If not specified otherwise, the term pension (and pensioners) refers to an old-age pension (and old-age pensioners) in this thesis.

were transformed to either individuals' bank accounts or third pillar type funds. Since 2018, the age of retirement is capped at 65 years.

2.1.1 PAYG System

Under the current legislation in effect of the Pension Insurance Act (with several amendments), an entitlement to receive an old-age pension is conditioned by reaching specific retirement age and fulfilling a required insurance period. For insured individuals born after 1971, the age of retirement has been unified to 65 years regardless of gender nor the number of children raised. For individuals born before 1971, see Table 2.1. The necessary insurance period has increased since 2010 from 25 years to the current 35 years. If the minimum required insurance period condition is not fulfilled, an individual is entitled to receive a so-called proportional old-age pension under certain conditions.

Table 2.1: Age of retirement (years + months) in the Czech Republic

Year of birth	Men	Women with no children	Women with one child	Women with two children	Women with 3-4 children	Women with 5+ children
1953	63	62	60+8	59+4	58	56+8
1954	63+2	62+4	61	59+8	58+4	57
1955	63+4	62+8	61+4	60	58+8	57+4
1956	63+6	63+2	61+8	60+4	59	57+8
1957	63+8	63+8	62+2	60+8	59+4	58
1958	63+10	63+10	62+8	61+2	59+8	58+4
1959	64	64	63+2	61+8	60+2	58+8
1960	64+2	64+2	63+8	62+2	60+8	59+2
1961	64+4	64+4	64+2	62+8	61+2	59+8
1962	64+6	64+6	64+6	63+2	61+8	60+2
1963	64+8	64+8	64+8	63+8	62+2	60+8
1964	64+10	64+10	64+10	64+2	62+8	61+2
1965	65	65	65	64+8	63+2	61+8
1966	65	65	65	65	63+8	62+2
1967	65	65	65	65	64+2	62+8
1968	65	65	65	65	64+8	63+2

1969	65	65	65	65	65	63+8
1970	65	65	65	65	65	64+2
1971	65	65	65	65	65	64+8

Source: Czech Social Security Administration (www.cssz.cz).

The amount of an old-age pension received is based on basic and percentage assessments. The basic assessment is determined by the appropriate Act of parliament on pension insurance, and its amount is altered by executive orders issued by the government and is equal for all individuals. The percentage assessment is, on the other hand, derived from the individual's insurance period and the salary size. In 2021, the basic assessment was CZK 3 550, and the minimal amount of the percentage assessment is CZK 770 (per month both).⁶

Refer to Czech Social Security Administration (2021) and Slavík (2006) for a comprehensive description of the Czech pension system.

2.2 Financial Sustainability of Pension Systems

In simple terms, a pension system can be financially sustainable by itself in the long run if the sum of pensions paid out to retirees does not tend to exceed the contributions paid by the working population. However, the adverse demographic shift causes an imbalance between the working population and pensioners, which results in a disproportion between paid contributions and received pensions and creates difficulty to secure sufficient resources in the future, as Šimková et al. (2016) observe.

The financial sustainability of pension systems facing the ageing population is subject to an extensive discussion by researchers utilising diverse approaches for its assessment. Alonso-García et al. (2018) name three main issues that the PAYG systems currently face. Besides ensuring the adequacy of provided pensions and their fairness in relation to the contributions paid during the productive age, they emphasise restoring the financial sustainability of the public pension systems. According to Jedynek (2018), a pension system could be financially sustainable in the long run, only if it is financially stable, it does not experience any abrupt changes, and it maintains adequacy of pensions and intergenerational fairness.

⁶ According to the Czech Social Security Administration.

As Káninský (2016) puts it, pension systems are heavily dependent on demographic, economic and political factors, with the latter one being the most influential. Magnani (2018) stresses that the non-sustainability of PAYG systems is not as caused by the population ageing as it is caused by inappropriate and ineffective policies and by the treatment of such pension systems in the past. The study names incorrect anticipation of the life expectancy increase, its subsequent insufficient recognition and failure to accumulate adequate pension reserves during baby boom periods as the main reasons. As Barr (2002) points out, a necessity for a sustainable pension system is an effective government.

While assessing the demographic shift from the political perspective, the increase of the proportion of pensioners in population creates a potentially powerful electoral force, as Zaidi (2010) points out. In the case of inadequate response to the ageing trend by policymakers and failing to prevent retirees from living on the poverty level due to the insufficient pension level, the study concludes that the stability of the political system might be severely endangered.

Various studies utilise the overlapping generations (OLG) modelling for examining the effect of population ageing on the sustainability of PAYG systems. Using a simple OLG model, Cipriani (2013) shows that PAYG pension benefits are negatively affected by an increase in longevity, which is, therefore, an issue for the sustainability of PAYG pension systems. Alonso-García et al. (2018), using a dynamic OLG model, design flexible and tractable risk-sharing mechanisms that restore the financial sustainability with changes in the contribution rate and indexation of pensions while examining the effect on adequacy and fairness of pensions of such mechanisms. Using demographic projection of the Belgian population, they conclude that without immediate implementation of some automatic mechanisms, the financial sustainability of the public mandatory pension system is compromised. Using an OLG model, Magnani (2018) shows that a PAYG pension system is sustainable in the face of the demographic shift only if the population shock is anticipated and adequately recognised.

Using a mathematical model for maintaining financial sustainability, Nepp & Okrah (2017) explore the impact of demographic uncertainty on the PAYG pension system and the relationship between the level of pensions, the age dependency ratio⁷ and the

⁷ The working-age population to the number of pensioners, also known as the dependency ratio.

indexation of wages, while proving that the financial sustainability of a PAYG system heavily depends on the growth rate of wages and contribution rates.

Barr (2002) and Káninský (2016) state that there are four ways of improving the financial stability of a pension system. The system can be restored by either lowering pensions, increasing contributions, postponing the retirement age or designing policies that would increase the national output. Since the problem regarding the national output and labour productivity, including the effect of migration, exceeds the scope of this thesis, the focus is brought merely on the three remaining means.

Although, as Slavík (2006) and Káninský (2016) point out, it would not be correct to assume that pension systems could be treated similarly across various countries since they usually differ in many specifics.

Even though Rytířová (2013) sees the excessive solidarity principle and the beforehand unknown individual's pension amount among the most significant drawbacks of the PAYG systems, a sustainable PAYG system proved itself as important social security, especially in times of financial crisis, which tend to have a significant negative impact on funded pension systems.

2.2.1 Assessment of the Czech Pension System

Although the total population in the Czech Republic has been relatively stable over the past few decades,⁸ the population structure has suffered from severe changes. The Czech Republic ranks among countries with an evident ageing population (as, e.g. Austria, Hungary and Finland), whose median age is above 43.⁹ The proportion of the 65 and more years old population to the total population permanently increases (from 12.5 % in 1990 to 19.9 % in 2020).¹⁰ Even though the fertility rate has experienced a subtle increase in the past two decades,¹¹ it still does not match its values prior to 1990, and its increase does not deflect the population ageing trend. Thanks to the increasing longevity, the average length of pension receiving in the Czech Republic continually increases as well, as Figure 2.1 exhibits, with a faster pace for women.

⁸ According to the Czech Statistical Office, the population of the Czech Republic has been fluctuating within the range of 10.2 to 10.7 millions since 1980.

⁹ According to Eurostat, as of 2020.

¹⁰ According to Eurostat.

¹¹ From 1.14 in 2000 to 1.71 in 2020, according to the Czech Statistical Office.

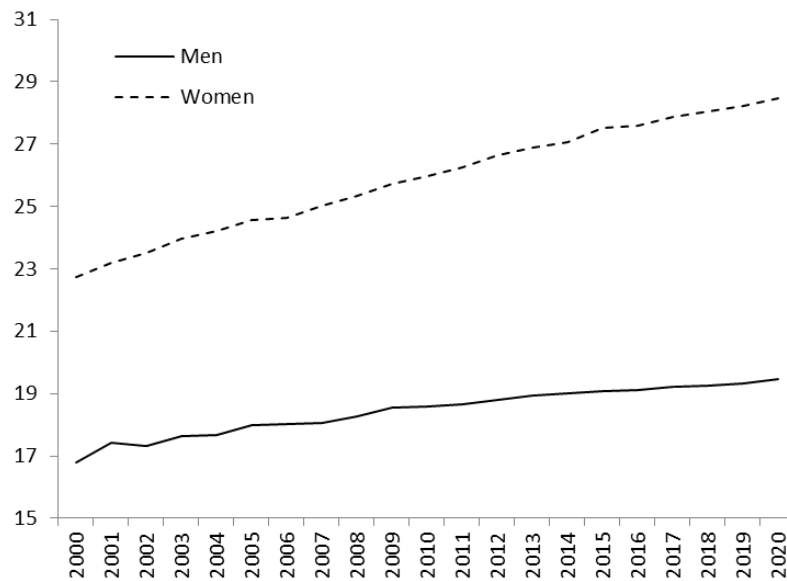
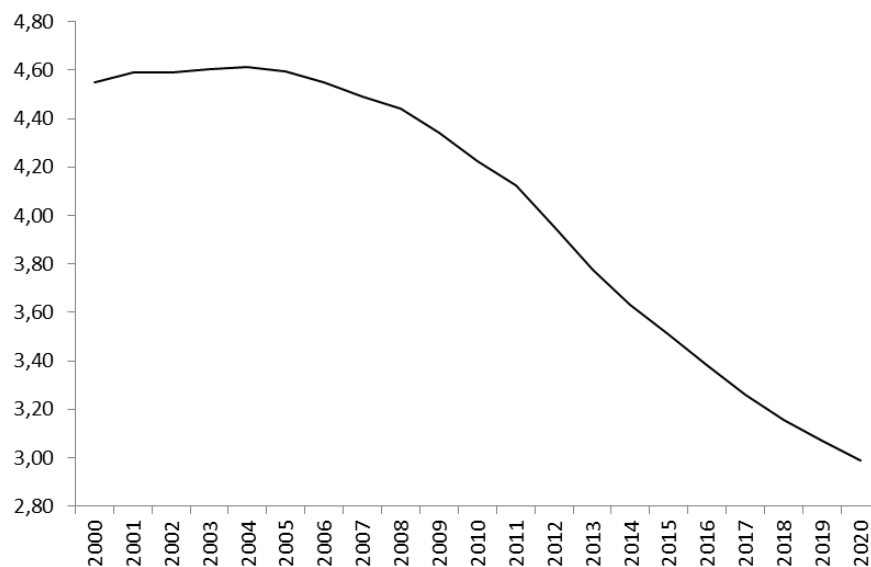


Figure 2.1: The average length of a pension receiving (in years)

Source: Czech Social Security Administration.

A helpful indicator of the potentially adverse demographic course is the age dependency ratio. As seen in Figure 2.2, the age dependency ratio experiences a clear downward trend,¹² which expresses that for every individual in the productive age, there are annually more and more pensioners.



¹² The age dependency ratio is computed by a generally accepted definition, the working-age population is considered to be at least 20 years old, and 65 years old or older individuals are regarded to be pensioners.

Figure 2.2: The age dependency ratio

Source: Eurostat (ec.europa.eu/eurostat) and author's computation.

The expenditure on pensions stands essentially on the number of pensioners and the pension amount disbursed. As seen in Figure 2.3, the number of pensioners in the Czech Republic increases in the long term.¹³ The abrupt increase of pensioners between 2009 and 2011 might be due to the legislative change of status of invalidity pensioners aged 65 or more to old-age pensioners.

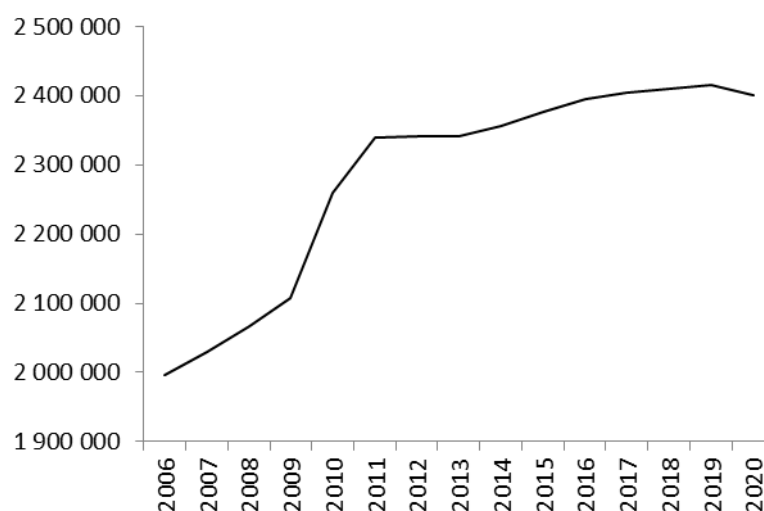


Figure 2.3: The number of pensioners (in December of a given year)

Source: Czech Social Security Administration.

Figure 2.4 depicts that the average net wage grows at a faster pace than the average pension does,¹⁴ accelerating since 2013. There is an evident long-term trend of a decline of the pension to wage ratio,¹⁵ accelerating in the past few years. In December of 2018, the average amount of pension paid out was CZK 12,435. It is worth noting that the average pension growth does not depend solely on the indexation of pensions,

¹³ Formally speaking, for the purpose of this study (old-age) pensioners include retirees receiving proportional pensions and the combined old-age pensions (with widow's/widower's pensions) as well, if not specified otherwise.

¹⁴ In this case, it does not include proportional nor combined pensions.

¹⁵ Defined as a relationship between the level of average pension and the average net wage, sometimes referenced to as the benefit ratio in literature.

but it might be profoundly affected by the so-called generational renewal as well.¹⁶ It is evident that with the ongoing trend of the decreasing pension to wage ratio, the well-being of pensioners solely dependent on the PAYG system is called into question. For a more detailed summary of various data regarding the Czech pension system, refer to Czech Social Security Administration (2021).

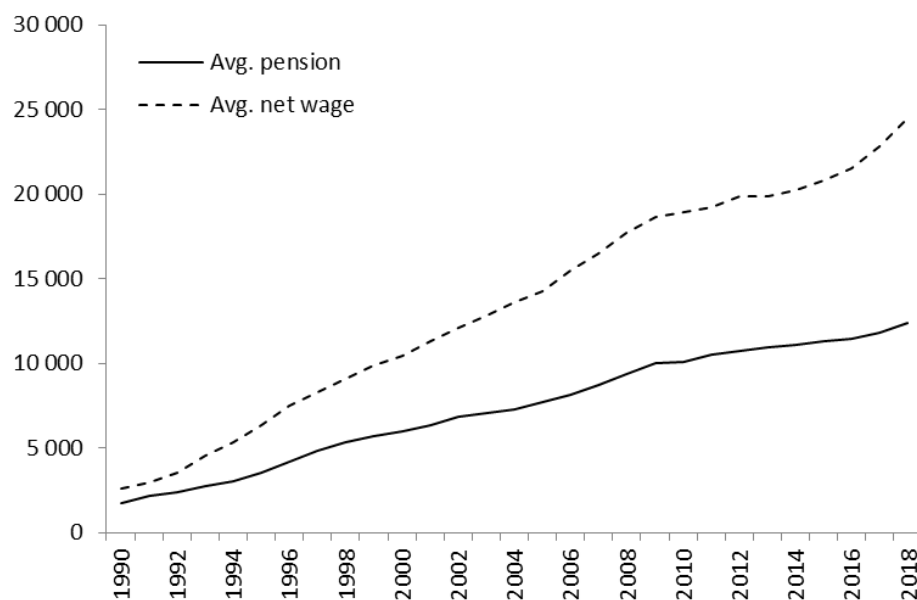


Figure 2.4: Average net wage and pension (monthly in CZK)

Source: Czech Statistical Office (www.czso.cz) and Ministry of Labour and Social Affairs (www.mpsv.cz).

Ambriško et al. (2017) show that at the beginning of the 2030s, the adverse demographic development would cause the primary government balance in the Czech Republic to turn negative and a debt to GDP ratio to grow. The study shows that this would necessarily lead to the unsustainability of Czech public finance, including the pension system.

Since the public PAYG pension system is the main form of financial protection of the elderly citizens in the Czech Republic, Balteş & Jimon (2018) claim that restoring and maintaining the financial sustainability of the public pension system should be a priority. While comparing a situation regarding public pension systems in various countries of Central and Eastern Europe, they conclude that the increase of both birth

¹⁶ As newly awarded pension benefits are on average generally based on higher wages thanks to the natural growth of nominal wages, the pensions' growth reflects that as well.

and employment rates, decrease of emigration and efficient integration of immigrants should be the cardinal directions to ensure the sustainability of the PAYG system. The study points out that even though the Czech Republic is among countries with the highest positive increase of population since 1990 in Central and Eastern Europe, it is also a country with the highest age dependency ratio and with the highest proportional amount of population aged 65 years and more.

To determine the effect of the ageing population of the Czech Republic on the economy and standard of living of the elderly, Gawthorpe & Šafr (2019) utilise a New Keynesian general equilibrium model. While using an additive utility function to measure individuals' well-being, the study's goal is to detect the necessary technological progress and government expenditures to ensure an unaltered and stable standard of living while facing a demographic shift. The study proves that without increasing technological progress or higher social security payments, the ageing phenomenon would have a significant impact on the economic welfare in the Czech Republic.

Since participation in the third pillar scheme is not mandatory in the Czech Republic, many citizens still rely solely on the PAYG system. Therefore, maintaining the financial stability of the PAYG system is an essential task for securing a functional social system in the Czech Republic.

2.2.2 Pension Reform Proposals

Besides preventing social exclusion and preserving an adequate standard of living of pensioners, Slaný et al. (2004) accent the permanent financial stability and long-term sustainability in the face of unpredictable challenges to be among the most crucial goals of potential pension reform. Vostatek (2017) emphasises the necessity of structural reform of the Czech pension system and Štěpánek (2019) stresses that postponing of finding a suitable solution makes the potential adjustments of the pension system more difficult in the future. However, OECD (2019) mentions the lack of long-term conceptuality of the policies regarding the Czech pension system, as partly documented by the recent reversals of previously adopted reforms.¹⁷ Zaidi (2010) assesses the importance of making widely unpopular but crucial suitable public policy choices and raising awareness of the need for such unnecessary reforms. While examining

¹⁷ Regarding the second pillar cancellation and the retirement age recalculation.

prospects of the development of pension systems in the EU countries, the study lists the Czech Republic among countries with a relatively high risk of a significant decline of pension values between 2007 and 2060.

According to Barr (2002), beliefs that a transformation of unfunded systems to funded systems would resolve the demographic issue have vague arguments. Moreover, the transformation from a PAYG system to a funded system might be undesirably costly, and there is no guarantee that the resulting pensions would not be even lower, as Štěpánek (2019) points out. Šimková et al. (2016) add that this transition would theoretically not raise the overall wealth of society but merely redistribute the income. Thus, although it might motivate individuals for more extensive savings and resulting investments, it would not resolve the sustainability issue. Even though that an NDC system promises a more transparent scheme with a more precise relationship between contributions paid and consequent benefits earned, Slaný et al. (2004) and Magnani (2018) claim that additional adjustments for the restoration of financial sustainability would be required as well.

Slavík (2006) argues that debates about the pension reforms focus excessively on the long term stability of the system and should instead consider their potential impact on the economic output and employment, among other economic indicators. The study recommends that government should be focused on short term goals as pension systems automatically balance themselves in the long run thanks to the pressure (e.g. to increase the contribution rate) made on political representatives by pensioners facing deteriorated financial situation due to the poor condition of the pension system account.

Mertl et al. (2019) introduced a theoretical concept of a transition to a merit-based PAYG scheme based on prolonging the productive age and labour market participation of elders with a gradual decrease of working hours while not increasing the statutory age of retirement at the same time. Similarly, Kubíčková et al. (2018) notice the willingness of a significant portion of the elderly to remain work active in the Czech Republic. Although the study does not confirm a financially insufficient standard of living as one of the reasons for active labour market participation of the elderly, the positive effect of endorsement and support of the active work participation on public finances is indisputable (especially on the income, consumption taxes or the financial stability). According to Káninský (2016), it is worth discussing an ambitious suggestion of implementing a universal basic pension, which is based on a guaranteed equal pension provided to every entitled citizen. Even though this possibility appears

to be quite radical, it might benefit from transparency, low administrative costs and no need for future excessive interventions. Nevertheless, as Barr (2002) stresses, and a recent development concerning the introduction and then cancellation of the second pillar scheme in the Czech Republic supports, the political stability of potential pension reform should not be underestimated. For an extensive discussion about possibilities of a reform of the Czech pension system, refer to Vostatek (2017).

Even though a discussion about the necessity of structural reform of the pension system has been taking place vigorously in the past two decades, the willingness for such reform is not emphatic by the policymakers and political representatives. Various suggestions of structural reforms were proposed over the years by policymakers, such as the transformation of the first pillar to an NDC scheme or to enable to opt-out from the PAYG system and rely solely on the third pillar. However, except for the introduction and then cancellation of the second pillar, no significant structural reform has been implemented recently.

Since the PAYG system depends heavily on the so-called intergeneration solidarity, its inconsiderate and nonconceptual modification might even lead to intergenerational tension and animosity. Political proposals might often lack rational bases and incline to more publically appealing, and populist decisions and unpopular but crucial structural changes might be postponed. As Mertl et al. (2019) claim, a sustainable pension system free of unsystematic political interventions is highly desirable.

3 Methodology

A concept of an automatic alteration of one or more parameters of a pension system is the fundamental idea of automatic adjustment mechanism (AAM) and more complex and strict automatic balance mechanism (ABM) based on a set of predefined rules that do not require any subsequent political interventions. As OECD (2019) summarises, AAMs, which are rules that usually link various parameters of a pension system to particular measures, are present in mandatory pension schemes in several OECD countries to a certain extent. Among the most frequent adjustments, there are various links to life expectancy, affecting either benefits (e.g. in Spain, Finland or Japan) or the age of retirement (e.g. in Portugal, Netherlands or Estonia).

A positive contribution of implementation of specific automatic mechanisms to a pension system is discussed by OECD (2019) with an accent on the reduction of an undesirable political influence and a necessity of repeated interventions. Arbatli et al. (2016) assess potential benefits arising from implementing AAM rules in Asian countries facing adverse population change, demonstrated by Japan's positive experience with AAM rules, and emphasise a possibility of linking the age of retirement to life expectancy. Even though the automatic mechanisms are occasionally criticised for their alleged anti-democratic nature, according to OECD (2019), the mechanisms do not prevent the government with sufficient political capital from any possible future intervention. As OECD (2021) mentions, the Czech Republic, alongside countries like Austria, Belgium or Hungary, has not implemented any automatic mechanisms into its pension system yet.

Despite their positive effect on the financial stability of pension systems, as Gannon et al. (2016) state, the AAM rules are not sufficient for ensuring long-term financial sustainability, which is a goal of the ABM. However, the distinction between AAM and ABM rules is not clear in some instances.

3.1 Automatic Balance Mechanism

Vidal-Meliá et al. (2010) define the ABM as a set of rules ensuring the financial stability of a pension system that is established by law and triggered when required by

a specific sustainability indicator. Efficient AMB rules promise to make a pension system, *inter alia*, more transparent, credible and financially sustainable in the long run and the concept has an ambitious aspiration to avoid or at least to minimise the political influence on the pension system. Since the implementation of ABMs to a pension system consists of the incorporation of specific predefined rules and not of direct implementation of any unpopular reductions, as Jedynak (2018) states, its political cost and risk are relatively low.

To a certain extent, specific ABM rules have already been introduced to various pension systems. Jedynak (2018) assesses the suitability of ABM rules and discusses their possible effect on the restoration of the financial sustainability of pension systems. The study evaluates the implementation of specific ABM rules in DB pension schemes in Canada, Germany and Japan and in the Swedish NDC scheme. As the case study summarises, Canada employs a semi-automatic balance mechanism, which does not fully guarantee financial sustainability, but it serves as a last resort concerning the contribution rate if a political agreement is not reached. The ABM rules in Germany are based on pension reduction in case of a decrease in the age dependency ratio. Thus, the goal of the German ABM is to guarantee the long-term sustainability of the German pension system. Although, as Jedynak (2018) emphasises, there is a possibility of an excessive reduction of pensions, which might result in a loss of their adequacy. The ABM rules in Japan aim to restore financial sustainability as well. Similarly to Germany, the ABM rules in Japan reside in altering the pensions paid, but they rely on a projected financial imbalance in a long-term period, the inflation rate and the salary growth rate. The ABM rules for maintaining the financial sustainability used in Sweden are built on a different foundation utilising a solvency ratio and could not be applied to a DB pension system due to the significant distinction between DB schemes and the Swedish NDC scheme. Since the ABM implementation is relatively recent, the assessment of its actual impact on the sustainability of pension systems is not entirely feasible yet.

Specific methods for designing an ABM model have been subject to various studies. Gannon et al. (2016) build a smooth AMB model and apply it to the US Social Security program for an evaluation of its financial sustainability. Gannon et al. (2014) then study a possible effect of the developed ABM model on the restoration of financial stability of the French first pillar pension system for private-sector employees, showing that a

reduction in pensions and an increase of the contribution rate is necessary for restoring the sustainability of the pension system.

Godínez-Olivares et al. (2016a) proposed an ABM model for restoring the financial sustainability and liquidity of the system for a period of 20 years. Among the most significant advantages of the Godínez-Olivares et al. (2016a) method, when compared to previously carried research, is the use of a logarithm function. As Godínez-Olivares et al. (2016a) emphasised, instead of stabilisation of the key variables' paths around their initial values,¹⁸ which is the basic idea of models relying on a quadratic function,¹⁹ the goal is to minimise the percentage changes of the variables over time. The Godínez-Olivares et al. (2016a) method is adjusted and extended to a period of 75 years by Godínez-Olivares et al. (2016b), who present two methods for designing the ABMs. The first one is based on measuring the difference of the net present value of the future income from contributions and the expenditure on pensions at year n and ensuring its stability in the long run. The second method takes into account a so-called buffer fund,²⁰ which either accumulates (or disperses) financial assets and potentially generates returns emerging from an annual surplus (or deficit) of the pension system. Godínez-Olivares et al. (2016b) then evaluate a theoretical application of both ABMs to a generic pension system using European population structure while ensuring long-term sustainability.

Following the Godínez-Olivares et al. (2016b) approach, Ferrer Fernández & Boado-Penas (2017) examine designing the AMB rules for pension systems in Japan, Germany and India, which are countries with very different population structures. The study shows an unfavourable development of key parameters of the pension system in all three countries, even with the application of the ABM rules. As the results show, the age of retirement increases to 65.4, 67 and 68 years in India, Germany and Japan, respectively, the contribution rate grows to 16%, 17.5% and 20%, while the indexation of pensions decreases to 1.2%, 0.6% and even -2% by 2030. Despite their significant unfavourable change even in a country with a relatively young population as India, the parameters stabilise after a few years in all three countries and more importantly, the financial sustainability of the pension systems is restored for a long-term horizon with no additional interventions needed. Run & Kilicman (2018) evaluate a theoretical

¹⁸ An extensive discussion regarding the key variables is in the following section 3.1.1.

¹⁹ E.g. presented by Gannon et al. (2016).

²⁰ Referred to as a contingency fund as well in literature.

application of ABMs on a funded DC pension system in Malaysia following the Godínez-Olivares et al. (2016b) ideas. Boado-Penas et al. (2020) then extend and evaluate the Godínez-Olivares et al. (2016b) method for a theoretical application to a mixed pension scheme consisting of a PAYG system with a complementary funded DC system.

The framework for calculating the automatic balancing mechanism strategy for a PAYG pension system in the Czech Republic in this thesis is inspired by the Godínez-Olivares et al. (2016b) method. The goal is to compute the optimal path of the three key variables, which would guarantee the financial sustainability of the public pension system without any necessary interventions while facing the uncertain demographic shift and fluctuation of crucial parameters such as longevity or fertility rates. The successful implementation of ABM rules into a pension system would then aim for annual liquidity of the system and ensure the self-reliant solvency of the system for a long-term period.

3.1.1 Key Variables

As presented in Chapter 2, the age of retirement, the contribution rate and indexation of pensions are identified as the crucial parameters affecting the financial stability of PAYG pension systems, and they play an essential role in the design of the ABM rules. This section aims to cover the possible future development of the key variables with regard to their potential social impact. Even though the age of retirement is currently capped at 65 years in the Czech Republic, while creating a model of Czech public finance to assess the ageing population effect, Ambriško et al. (2017) prove that this policy goes strictly against the sustainability of the Czech pension system. This is supported by Fiala (2016) as well, who argues that the retirement age should be linked to life expectancy, and pensioners should, on average, ideally receive pensions roughly for the last quarter of their lives. The study concludes that it might be necessary for the age of retirement to exceed 65 years in the future. Štěpánek (2019) examines the effect of the demographic shift on the long-term financial stability of the Czech PAYG pension system. Within the standard general equilibrium modelling framework, the study shows that the pension account may be balanced in the long run while facing the demographic shift if the age of retirement increases to 67 by 2042 and continues to increase in the subsequent years. An increase in the retirement age is also supported by Barr (2002) as one of the most suitable solutions for ensuring the financial

sustainability of a pension system facing a population ageing. An option that is no longer effective once the pension system is fiscally unstable in the long run, according to Barr (2002). Šimková et al. (2016) even state that according to their estimation, the age of retirement needs to increase to 73 years by 2050 for maintaining a stable age dependency ratio.

Nevertheless, it may not be sufficient only to increase the age of retirement without additional arrangements. As Slavík (2006) points out, the increase of the retirement age is only effective for the financial soundness of the pension system under the assumption that there is no substantial age discrimination or unemployment of older people. Otherwise, in the event of significant employment issues of the elderly, the increase of the retirement age would have a considerably lesser positive effect on the financial sustainability of the pension system than expected. On the other hand, Šetek (2018) argues that increasing the age of retirement above 65 is unacceptable for certain professions (namely metallurgists, coal miners or marines). The study emphasises that such professions, for which the so-called amortisation of human capital proceeds at a faster pace due to the tremendous physical strain and other biological factors, have even significantly lower age of retirement at a level of 50-55 years old in many European countries. With regard to the aforementioned arguments, even though an increase in the retirement age might be inevitable for maintaining the sustainability of the PAYG system, it should be done carefully with complete comprehension of its possible drawbacks.

According to OECD (2019), the Czech Republic belongs among OECD countries with the highest contribution rate. An increase in the contribution rate lowers the current standard of living of the economically active population as a consequence. Hence by nature, an abrupt increase of the contribution rate is not received positively by the working population. However, as Ramaswamy (2012) states, the increase of the age dependency ratio due to the population ageing phenomenon puts pressure on an extensive increase in the contribution rate.

While the contribution rate increase reduces the living standard of the working population, inadequate or even negative indexation of pensions causes lowering the standard of living of pensioners. Thus, there seems to be an apparent trade-off between the current financial well-being of pensioners and the working population, which should be treated with vigilance. Furthermore, Káninský (2016) stresses that all three key variables are highly socially sensitive, and their alteration should be done

cautiously. With regard to the observations mentioned above, any abrupt negative change in the indexation of pensions may be politically undesirable, especially if the pensions are the only income for a considerable part of the population, which is the case of the Czech Republic.

3.1.2 The Model

This thesis builds on the ideas introduced by Godínez-Olivares et al. (2016a) and Godínez-Olivares et al. (2016b) for the designing of the ABM model. Formally, designing an ABM model is set as a general nonlinear optimisation problem (NLO), described as a calculation of decision variables d_n representing the three key variables, that is

$$d_n = (c_n, x_n^{(r)}, \lambda_n) \in D, \quad (3.1)$$

where $D \in R^n$ is a decision space representing a set of all feasible solutions, c_n stands for the contribution rate, $x_n^{(r)}$ for the age of retirement, λ_n for indexation of pensions, $n \in N$ represents a particular year, and which minimises an objective function of the NLO,

$$f_n(d_n, n). \quad (3.2)$$

The set of feasible solutions is limited by constraints, which Godínez-Olivares et al. (2016b) identify as lower and upper bounds of the key variables, their change rate and the liquidity restriction at a year n . Moreover, the paths of the key variables are forced to be monotone. In other words, the result of the ABM modelling would be an optimal $d^* \in D$ for which

$$f(d^*) \leq f(d), \text{ for } \forall d \in D. \quad (3.3)$$

It is worth emphasising the importance of the constraints for the path of the key variables, which ensure their smoothness and value boundaries. The smoothness of the

²¹ There is a chance of having no feasible solution (possibly due to too strictly defined constraints). In that case there would no d^* minimising the objective function.

optimal paths has to be secured since it is not desirable for the paths of the three key variables to experience abrupt and sudden changes (i.e. a sudden unfavourable change of the contribution rate or the indexation of pensions should be avoided to, *inter alia*, ensure political stability and support of the ABM rules). Furthermore, the constraints ensuring monotony of the variables avoids an undesirable change of direction of the key variables. With a lack of setting adequate restraints for values of the key variables, undesirable or unrealistic extreme values might occur (i.e. excessive retirement age or contribution rate).

The ABM model is based on the minimisation of the difference between the net present value of the future income from contributions and the expenditure on pensions. Thus, the objective function to use is formally defined as

$$\min_{c_n, r_n, i_n} \left(\sum_{n=0}^N \frac{c_n W_n(g_n, x_n^{(r)})}{(1+\delta)^n} - \sum_{n=0}^N \frac{B_n(g_n, x_n^{(r)}, \lambda_n)}{(1+\delta)^n} \right), \quad (3.4)$$

where $c_n W_n(g_n, x_n^{(r)})$ represents the total contribution depending on the contribution rate, the growth of wages g_n and the age of retirement, $B_n(g_n, x_n^{(r)}, \lambda_n)$ stands for the total spending on pensions also depending on the growth of wages, the age of retirement and the indexation of pensions at year n and δ is the discount rate. The objective function is then subject to the constraints mentioned above, specified as

$$\begin{aligned} c_{min} &\leq c_n \leq c_{max}; c_{n+1} - c_n \leq c_{\Delta}; c_n \leq c_{n+1}; \\ \lambda_{min} &\leq \lambda_n \leq \lambda_{max}; \lambda_n - \lambda_{n+1} \leq i_{\Delta}; \lambda_{n+1} \leq \lambda_n; \\ x_{min}^{(r)} &\leq x_n^{(r)} \leq x_{max}^{(r)}; x_{n+1}^{(r)} - x_n^{(r)} \leq r_{\Delta}; x_n^{(r)} \leq x_{n+1}^{(r)}; \end{aligned} \quad (3.5)$$

$$\frac{c_n W_n(g_n, x_n^{(r)})}{B_n(g_n, x_n^{(r)}, \lambda_n)} \geq 1.$$

It is worth emphasising the liquidity restriction specification. The idea is to guarantee the liquidity of the pension system, and the design ensures that the total contribution is equal to or higher than the total spendings for all n , and the system is thus balanced. Since the paths of the retirement age and the contribution rate are designed to be nondecreasing and the path of the indexation of pensions to be

nonincreasing, the ABM model designed in this thesis is a case of a so-called asymmetric design.

An essential task in the designing of the ABM model is the estimation of the future development of all variables affecting the total contribution base and the total expenditure on pensions. Besides the age of retirement and the contribution rate, whose future courses are the outcome of the ABM model, the total contribution base $c_n W_n(g_n, x_n^{(r)})$ depends on an average wage and its growth in a particular age group. Moreover, the total expenditure on pensions $B_n(g_n, x_n^{(r)}, \lambda_n)$ is based on the age of retirement, indexation of pensions, growth of wages and the average level of pension benefit in a particular age group.

Generally speaking, the total contribution base $c_n W_n(g_n, x_n^{(r)})$ is calculated as the total earnings by the economically active population multiplied by the contribution rate. Formally, the total earnings from the working-age population at time $n = 0$ (the initial year of the model) are calculated as

$$W_0 = \left(\sum_{x=x_e}^{\lfloor x_1^{(r)} \rfloor - 1} w_{x,0} * wage(x) \right) + (x_0^{(r)} \bmod \lfloor x_0^{(r)} \rfloor) * w_{\lfloor x_0^{(r)} \rfloor, 0} * wage(\lfloor x_0^{(r)} \rfloor), \quad (3.6)$$

where $w_{x,0}$ represents the number of economically active people at age x , $wage(x)$ is their average gross monthly wage, $x_0^{(r)}$ is the age of retirement at time $n = 0$, and x_e is the entry age to the labour market,²² $\lfloor x_0^{(r)} \rfloor$ is the floor function of the retirement age and $x_0^{(r)} \bmod \lfloor x_0^{(r)} \rfloor$ is the modulus operation.²³

Out of the working-age population, only the employed, self-employed, and voluntarily participating part of that population contribute to the system.²⁴ The number of contributors is modelled as the number of economically active citizens older than

²² The number of working population (and pensioners as well) is assumed to be distributed uniformly over the whole year.

²³ As Godínez-Olivares et al. (2016a) describes, the floor function maps a real number to the largest previous integer and the modulus operation finds the remainder of the division of the real number by the largest previous integer.

²⁴ A voluntarily participating person is a contributor that is not obliged to pay pension insurance contributions, but decides to do so anyway. According to the Czech Social Security Administration, the voluntarily participating people comprised 0.2% out of all contributors in 2019.

the entry age and younger than the age of retirement. The number of economically active people at age x is computed as the number of working-age people at age x , which is derived from the demographic projection, multiplied by the expected rate of economic activity of people at age x . Unemployment is disregarded in this model.

For the subsequent years (for $n > 0$), the modelling of the total earnings introduces the growth of wages. Formally, the calculation is described as

$$W_n = \left(\sum_{x=x_e}^{\lfloor x_n^{(r)} \rfloor - 1} w_{x,n} * wage(x) * (1 + g)^n \right) + (x_n^{(r)} \bmod \lfloor x_n^{(r)} \rfloor) * w_{\lfloor x_n^{(r)} \rfloor, n} * wage(\lfloor x_n^{(r)} \rfloor) * (1 + g)^n, \quad (3.7)$$

where the average gross monthly wage at age x increases annually by the growth of wages g , which is assumed to be constant over time.

The total expenditure on pensions $B_n(g_n, x_n^{(r)}, \lambda_n)$ is then modelled as the number of pensioners multiplied by the average amount of pension. Thus, it holds that

$$B_n = \left(\left(1 - (x_n^{(r)} \bmod \lfloor x_n^{(r)} \rfloor) \right) * p_{\lfloor x_n^{(r)} \rfloor, n} \right) * P_{\lfloor x_n^{(r)} \rfloor, n} + \sum_{x=\lfloor x_n^{(r)} \rfloor}^{\omega} P_{x,n} p_{x,n}, \quad (3.8)$$

where $P_{x,n}$ represents the average pension and $p_{x,n}$ the number of pensioners at age x , ω is then the maximum age occurred in the population,²⁵ and $\lfloor x_n^{(r)} \rfloor$ is the ceiling function of the age of retirement.²⁶

The number of pensioners in the ABM model is estimated as the number of people living above the age of retirement. In practice, those two numbers might, however, not equal each other for various reasons.²⁷

²⁵ For simplification, the maximum age occurred in the population is set to 100, people that are older than 100 are considered to be 100 as well.

²⁶ Similarly to the floor function, the ceiling function maps a real number to the smallest previous integer.

²⁷ E.g. because of premature or belated retirement, significantly lower retirement age for women with children, etc.

The average pension at age x and year n , where $x > x^{(r)}$ at year n , is then computed as the average pension at the previous year, and age $x-l$ increased by the indexation of pensions, formally described as

$$P_{x,n} = P_{x-1,n-1} * (1 + \lambda_{n-1}). \quad (3.9)$$

The newly awarded level of pension at the age of retirement for $n = 0$ is set in advance based on the data used,²⁸ and the newly awarded level of pension at the subsequent years is then increased by the constant growth of wages. Formally,

$$P_{x_n^{(r)},n} = P_{x_n^{(r)},n} \mathbf{1}_{\{x_n^{(r)},n=0\}} + P_{x_n^{(r)},n} * (1 + g)^t \mathbf{1}_{\{x_n^{(r)},n>0\}}, \quad (3.10)$$

where $\mathbf{1}_{\{x_n^{(r)},n=0\}}$ takes the value of one if the condition in the braces is satisfied and zero otherwise.

For detailed mathematical preliminaries and definitions of the ABM modelling, refer to Godínez-Olivares et al. (2016a).

Similarly to Godínez-Olivares et al. (2016a),²⁹ this study uses nonlinear optimisation techniques for solving a minimisation problem with inequality constraints to find the optimal path of the key variables. The optimisation is carried out in R utilising the NLOpt R interface for nonlinear optimisation, as specified in detail in Appendix A.

²⁸ Specified in Chapter 4.

²⁹ E.g. utilised by Godínez-Olivares et al. (2016b), Ferrer Fernández & Boado-Penas (2017) or Run & Kilicman (2018) as well.

4 Data Description

This chapter aims to present the data necessary for the modelling of ABM, calibrate the ABM parameters for its use in the PAYG pension system in the Czech Republic, and set the key variables' initial values.

4.1 Demographic Projection

Because of the uncertain future demographic development, the ABM modelling heavily relies on demographic projections. Whereas some studies assessing the sustainability of pension systems utilise the Eurostat baseline population projections,³⁰ others use either the United Nations population projections³¹ or population projections obtained from respective national statistical offices.³²

Figure 4.1 offers a comparison of several demographic projections of the Czech population for a period between 2020 and 2100. The CZSO Medium variant, the UN Medium variant and the Eurostat demographic projections all expect a similar course of the demographic shift in the Czech Republic. Both the CZSO and the UN Low variants seem not to differ significantly, whereas the UN High variant projects visibly more favourable development of the Czech population.

For the purpose of this study, the CZSO Medium variant population projection is used for the ABM modelling.

³⁰ E.g. Magnani (2018), Fiala (2016) or Godínez-Olivares et al. (2016b).

³¹ E.g. Ferrer Fernández & Boado-Penas (2017).

³² E.g. Godínez-Olivares et al. (2016a).

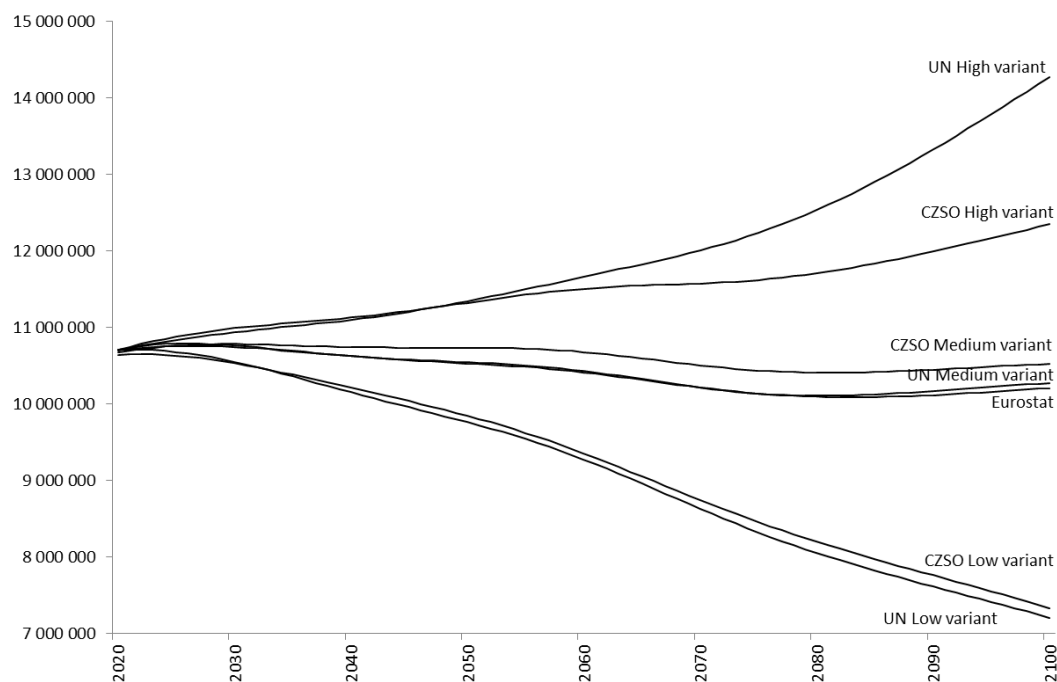


Figure 4.1: Comparison of Czech Republic's population projections

Source: Czech Statistical Office, Eurostat and The Population Division of the United Nations Department of Economic and Social Affairs (un.org/development/desa/pd).

When comparing the three variants of the CZSO demographic projection, the age dependency ratio is projected to gradually decrease in all three of them until 2060, after which it experiences a subtle increase. As can be seen in Figure 4.2, the age dependency ratio is not expected to significantly differ for all three CZSO demographic projection variants until 2070. Thus, all three variants assume the same proportion of the working-age population to the number of pensioners until 2070.

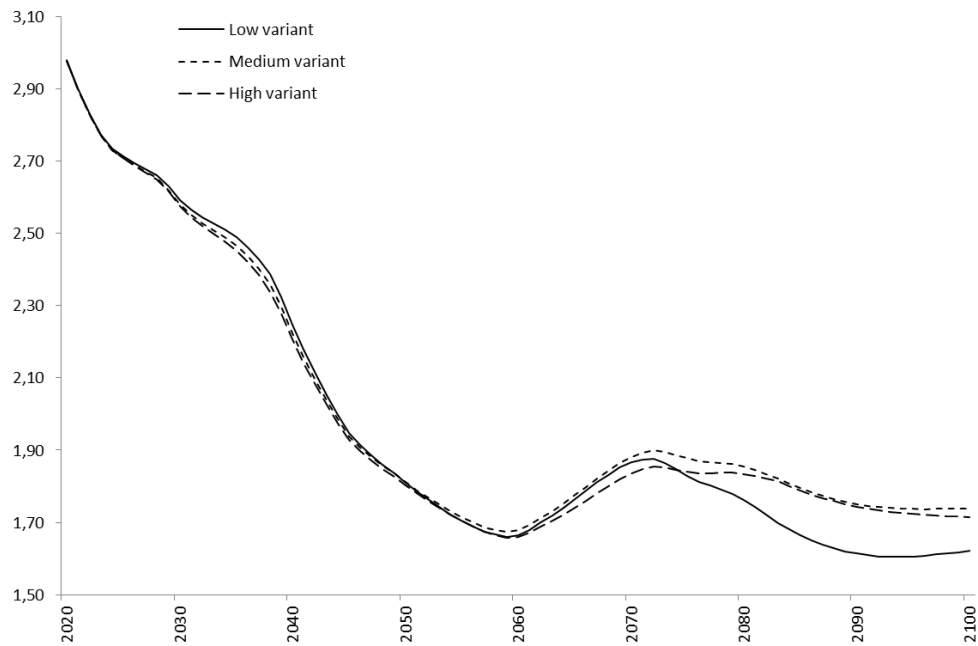


Figure 4.2: Projection of the age dependency ratio

Source: Czech Statistical Office and author's computation.

Figure 4.3 depicts the CZSO Medium variant population projection structure in the Czech Republic in two time periods, 2020 and 2070. There is a clear peak in the age group of 40 to 49-year-olds in 2020, which corresponds with the baby boom generation in the 1970s. Moreover, the increasing life expectancy and trend of the ageing population is evident.

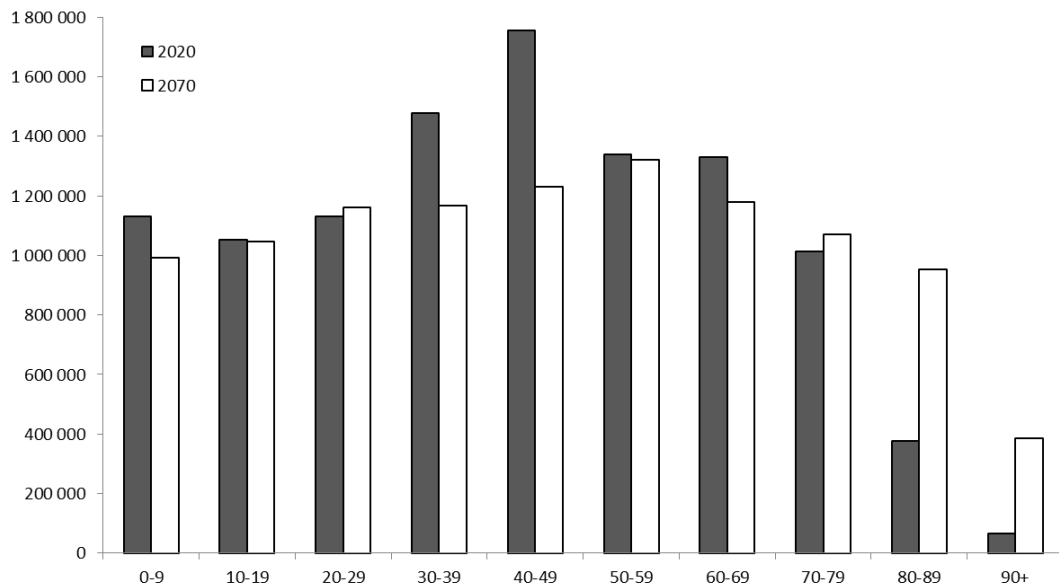


Figure 4.3: Czech population structure in 2020 and 2070

Source: Czech Statistical Office and author's computation.

4.2 Calibration

This section provides a necessary calibration of parameters of the ABM model presented in Chapter 3, which determines the ABM design.

Under the current legislation, a person can enter the labour market in the Czech Republic at the age of 15.³³ Thus, the entry age x_e in the ABM model is set to be 15.

Table 4.1 depicts the economic activity rate of the Czech population by age in 2020, calculated as an average of the quarterly data, which is used for the ABM model in this study. Following a simplified idea by Ambriško et al. (2017), the economic activity rate is assumed to be constant for younger age groups over time, whereas the increasing age of retirement is meant to increase the economic activity rate of the oldest age group before the retirement age. Therefore, the economic activity rate of the age group 65 and more is disregarded since it exceeds the average age of retirement in 2020,³⁴ after which the population is considered to be entirely economically inactive. The economic

³³ Under the condition of completion of the compulsory education, and with several restrictions concerning the working time.

³⁴ The average age of retirement in 2020 was 62.15 according to the Czech Social Security Administration.

activity rate of the age group 60 to 64 is thus extended over the age of 64 if retirement age exceeds 64 in the future development.

Table 4.1: Economic activity rate by age in the Czech Republic in 2020

Age groups	Economic activity rate
15 to 19	5.4%
20 to 24	49.3%
25 to 29	80.1%
30 to 34	80.7%
35 to 39	86.3%
40 to 44	93.5%
45 to 49	94.8%
50 to 54	93.6%
55 to 59	89.5%
60 to 64	49.1%
65 or more	6.9%

Source: Czech Statistical Office and author's computation.

Besides the structure of the economic activity rate of the population by age, the modelling of the ABM requires the earnings structure by age as well. Figure 4.7 depicts the average gross monthly earnings structure by age in the Czech Republic in the past four years. The values have been normalised to represent a percentage of the average gross monthly earnings in a given year. There seems not to be an apparent change in the earnings structure in time. Thus, the normalised earnings structure from 2020 is applied to the ABM model used in this thesis.

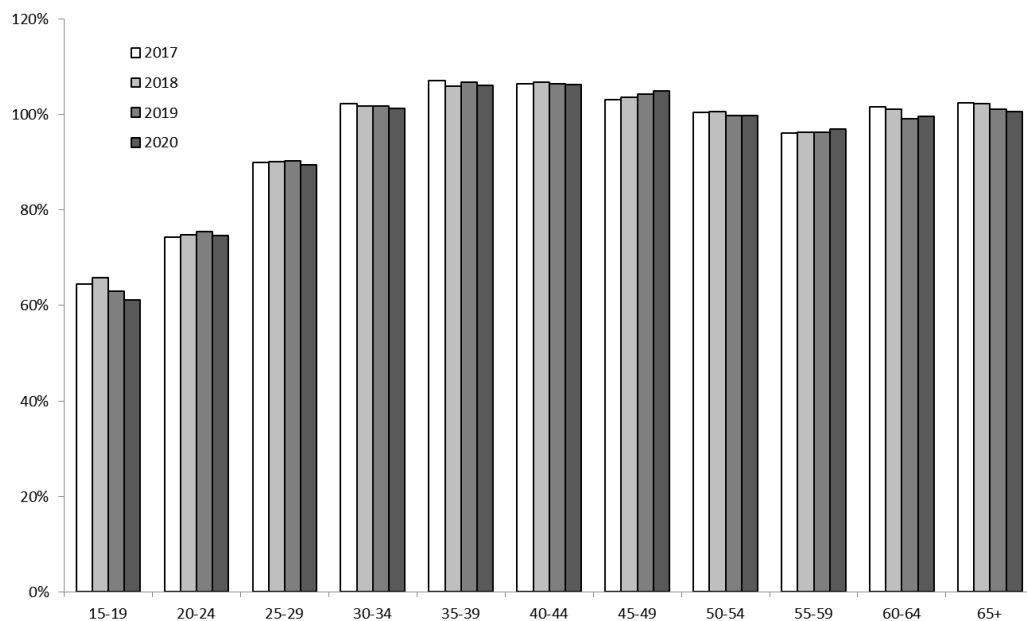


Figure 4.4: The earnings structure by age in the past four years

Source: Czech Statistical Office and author's computation.

Since the pension level of pensioners at age x is assumed to be the same for all pensioners at age x , the difference between the pension levels of pensioners at a different age is solely dependent on the difference between the newly awarded level of pension at the age of retirement in different years. Because the newly awarded level of is on average increasing with years thanks to the generational renewal, the average pension of pensioners at age x in a particular year is assumed to be decreasing with the age of the pensioners. Based on the average pension, the newly awarded level of pension at the age of retirement and the population structure by age in 2019, the pension level is estimated to be gradually decreasing from the initial pension level by 1.8% with the one-year increase of the pensioner's age in the initial year of the ABM model.

Wages are expected to grow constantly by 4.0% a year. This number is based on the average gross nominal wage growth in the past ten years, which is presented in table 4.2.

Table 4.2: Growth of wages in the Czech Republic

Year	Nominal growth of wages

2011	2.3%
2012	2.6%
2013	-0.1%
2014	2.8%
2015	3.2%
2016	4.4%
2017	7.0%
2018	8.0%
2019	7.2%
2020	2.7%

Source: Czech Statistical Office and author's computation.

Moreover, based on the ABM modelling specification described in Chapter 3, the newly awarded level of pension at the retirement age is assumed to increase in accordance with the growth of wages, i.e. annually by 4%.³⁵ The replacement rate is therefore assumed to be constant for all studied years.³⁶

For the purpose of this study, the inflation rate is expected to be constant over time at an annual level of 2.0%, which corresponds with the long-term inflation targeting in the Czech Republic.³⁷

The initial year of the ABM model ($n = 0$) is set to be 2020, and the time horizon for the model is set for 25 years. Thus, the future development of the pension system is modelled until 2045, and the AMB rules are designed to ensure financial sustainability for a 25-year period.

4.3 Initial Values

A crucial part of the ABM design is to set the initial values of the three key variables for $n = 0$, which would enter the model.

³⁵ According to the Czech Social Security Administration, the initial level of pension at the retirement age has been increasing annually on average by 4.6% from 2013 to 2020.

³⁶ The replacement rate is understood as the ratio of the initial pension to the gross final salary.

³⁷ Since 2010, according to the Czech National Bank. Utilised by Ambriško et al. (2017) as well. From 2010 to 2020, the average actual annual rate of inflation has been 1.8%, according to the Czech Statistical Office.

Under the current legislation, the contribution rate to the entire pension insurance (including all four types of pension insurance benefits specified in Chapter 2) is 28% of the individual's gross wage, out of which 21.5% is imposed on an employer and 6.5% on employees.³⁸ Since the expenditures on old-age pensions comprise 81.9% of all expenditures on pension benefits,³⁹ the initial value of the contribution rate on old-age pensions used in the ABM model is 22.9%.⁴⁰

As noted in Chapter 2, the age of retirement does differ for men and women with children for people retiring in the present day, even though it is intended to unify the age of retirement for people born after 1972. Moreover, it is possible to retire earlier (or later) than the official retirement age.⁴¹ For the purpose of this study, the age of retirement is therefore defined as the average age at which both men and women retire. Since the average retirement age was 62.15 in 2020,⁴² the initial value of the age of retirement is set to be 62.15.

Under the current legislation, the average indexation of pensions in the Czech Republic is annually approximately equal to the inflation rate plus one half of the growth of wages.⁴³ Thus, since the average wage growth is assumed to be 4% and the expected inflation rate is equal to 2%, the initial indexation of pensions is set to be 4% for all pensioners.

It is worth stressing out that according to the ABM model specifications described in Chapter 3, while the values of the retirement age and the contribution rate at year n affect the same year's balance, the value of the indexation of pension at year n affects, on the other hand, the following year's balance. Moreover, the initial values of the age of retirement and contribution rate (at $n = 0$) are predefined, and the minimisation does

³⁸ The contribution rate is the same for self-employed and voluntarily participating people as well.

³⁹ According to the Czech Social Security Administration, based on the average value in years 2013-2020.

⁴⁰ Similarly, Mertl et al. (2019) assumes the contribution rate to be 22% in their analysis of the Czech PAYG pension system.

⁴¹ Detailed requirements are described in the Act No. 155/1995 Coll. (the Pension Insurance Act).

⁴² According to the Czech Social Security Administration.

⁴³ The indexation of pensions in the Czech Republic is more complex, since the basic assessment of the pension can be increased by fixed amount and the percentage assessment by percentage points. Moreover, the pensioners can receive a one-time pension benefits issued by the government as well.

not change them, whereas the last value of the indexation of pensions (at $n = N$) is disregarded since it does not affect the outcome of the objective function.

The initial average gross wage in the initial year is set to be CZK 34,119 per month,⁴⁴ and the average pension is CZK 14,479 per month.⁴⁵

With the purpose of ensuring a smooth path of the key variables, the maximum possible annual increase in contribution rate is set to be 0.5%, the indexation of pensions is allowed to decrease annually by 0.5% at the highest, and the age of retirement can annually increase by two months, which is in line with the current change rate of the retirement age in the Czech Republic. The maximum contribution rate is then assumed to be 25%, the age of retirement at 72 years and the minimum indexation of pensions is set to be 0%. Since the ABM model under the asymmetric design is used in this study, the minimal (maximal) possible values of the retirement age, contribution rate (and the indexation of pensions) is equal to their value at $n = 0$.

Table 4.3 summarises all initial values and parameters specified and used in the ABM modelling.

Table 4.3: Summary of the initial values and parameters

Parameter	Value
Entry age	15
Growth of wages	4%
Expected inflation rate	2%
Initial year of the model	2020
Last year of the model	2045
Initial average wage per month (CZK)	34,119
Initial average pension per month (CZK)	14,479
Initial contribution rate	22.9%
Initial indexation of pensions	4%
Initial age of retirement	62.15
Maximum contribution rate	25%
Minimal indexation of pensions	0%

⁴⁴ According to the Czech Statistical Office.

⁴⁵ According to the Czech Social Security Administration.

Maximum age of retirement	72
Highest change in the contribution rate	0.5%
Highest change in the indexation of pensions	0.5%
Highest change in the age of retirement	2 m

5 Discussion of Results

This chapter provides a comprehensive overview of the results of the ABM model. First, the financial sustainability of the Czech pension system under its current setting is assessed. After that, a future course of the key variables is presented if just one variable is allowed to change to ensure liquidity. Furthermore, the results of the ABM model, allowing a simultaneous change in all three key variables, are assessed.

5.1 Financial Sustainability

To assess the financial sustainability of the Czech pension system without any intervention, the current expectation by the legislation of the future course of the key variables is used, and the future development of the pension system is modelled as described in Chapters 3 and 4 without the optimisation of the objective function. Therefore the contribution rate is expected to be constant and equal to 22.9%. The indexation of pensions is assumed to be constant as well, and equal to 4%. The age of retirement is then expected to increase by two months from its initial value of 62.15 every year until it reaches 65, which it does in accordance with the mechanism described in Table 2.1.

A valuable indicator of the liquidity of a pension system at time n is the liquidity ratio L_n ,⁴⁶ which is defined as a share of the system's total contribution to its total spending, formally as

$$L_n = \frac{c_n W_n(g_n, x_n^{(r)})}{B_n(g_n, x_n^{(r)}, \lambda_n)}. \quad (5.1)$$

As Figure 5.1 depicts, without any intervention, the current state of the Czech pension system does not appear to be financially sustainable in the long run, not even

⁴⁶ E.g. utilized Alonso-García et al. (2018) or Godínez-Olivares et al. (2016b) as well.

liquid after a few years, since the liquidity ratio of the systems decreases below one without any indication of changing its course.

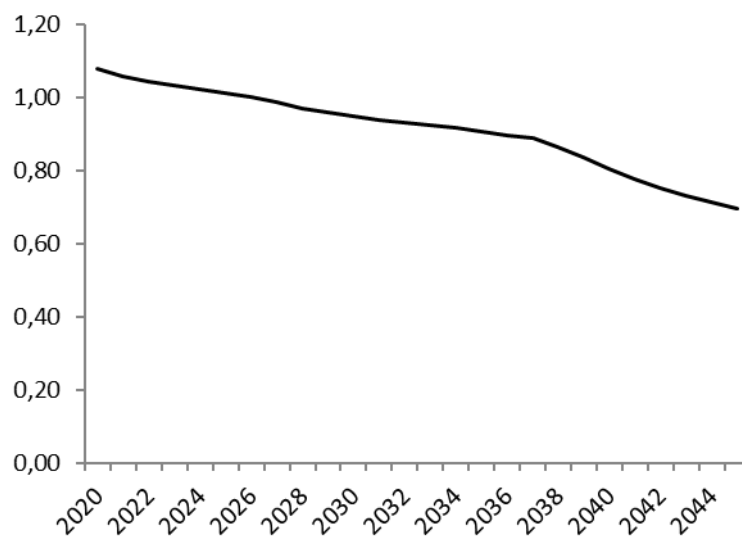


Figure 5.1: Liquidity ratio – pension system without ABM

Source: Author's computation.

5.2 Modifying One Variable

While allowing only one key variable to change in the ABM model and relaxing the constraints regarding the minimal and maximal possible values of the key variable and its change rate, as well as allowing the path of the key variable to be increasing as well as decreasing, there is exactly one solution to the minimisation problem for all n ,⁴⁷ and hence overall as well, since the optimisation forces the system to be precisely balanced for all n . The only active constraint is the liquidity restriction. The other two key variables are then predefined according to their statutory expectation.

Figure 5.2 depicts a path of the contribution rate that would ensure the liquidity of the Czech pension system in the horizon of 25 years if the retirement age and indexation of pensions remain without any additional intervention.

⁴⁷ The relaxed constraints would have no purpose if only one variable is changing, moreover, their inclusion might make the solution unfeasible.

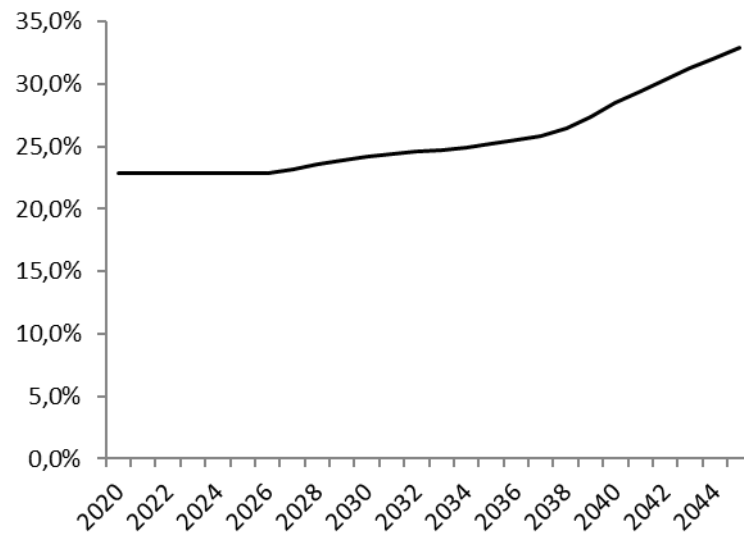


Figure 5.2: Modification of the contribution rate

Source: Author's computation.

Whereas the contribution rate would have to increase from its current 22.9% to 32.9%, the retirement age would need to grow from 62.15 years to 69.39 to make the system liquid every year, as Figure 5.3 illustrates.

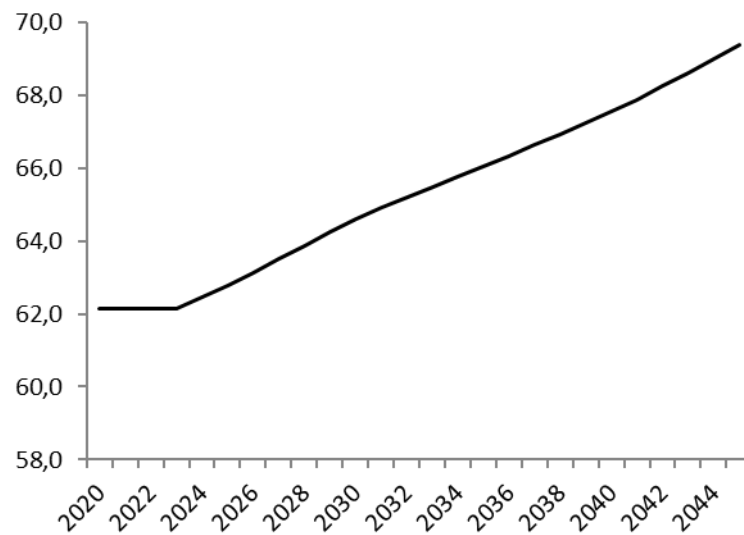


Figure 5.3: Modification of the retirement age

Source: Author's computation.

If both contribution rates and the age of retirement, on the other hand, remain on their current statutory paths, the indexation of pensions would need to decrease to

negative values, which would not only mean enlarging the difference between wages and pensions but heavily worsening the financial situation of pensioners as well.

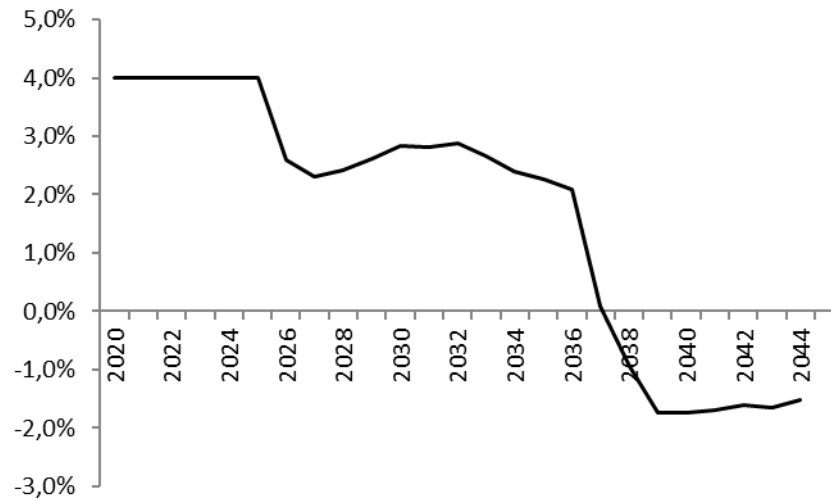


Figure 5.4: Modification of the indexation of pensions

Source: Author's computation.

5.3 Simultaneous Modification of Key Variables

This section provides the results of the ABM modelling with a simultaneous change of all three key variables' paths with all constraints, as defined in Chapter 3. To make the system financially sustainable in the horizon of 25 years and liquid every year, the results of the designed ABM rules show that the contribution rate would have to reach its maximal allowed value by the end of the period, the age of retirement would need to increase to 64.6 years, and the indexation of pensions would need to drop to 0 %, as depicted by Figures 5.5, 5.6 and 5.7 respectively.

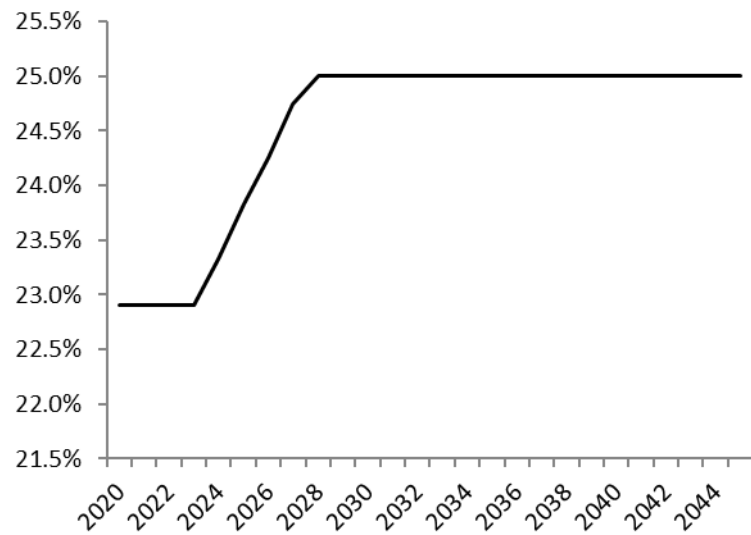


Figure 5.5: Contribution rate with the application of the ABM

Source: Author's computation.

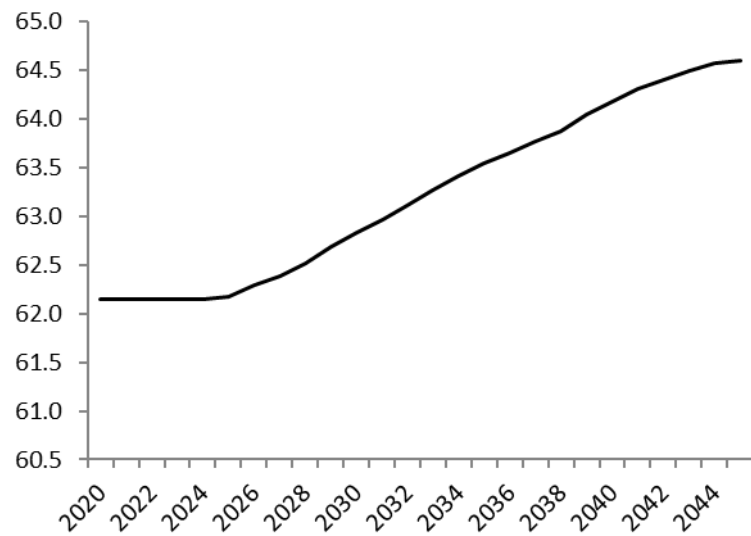


Figure 5.6: Age of retirement with the application of the ABM

Source: Author's computation.

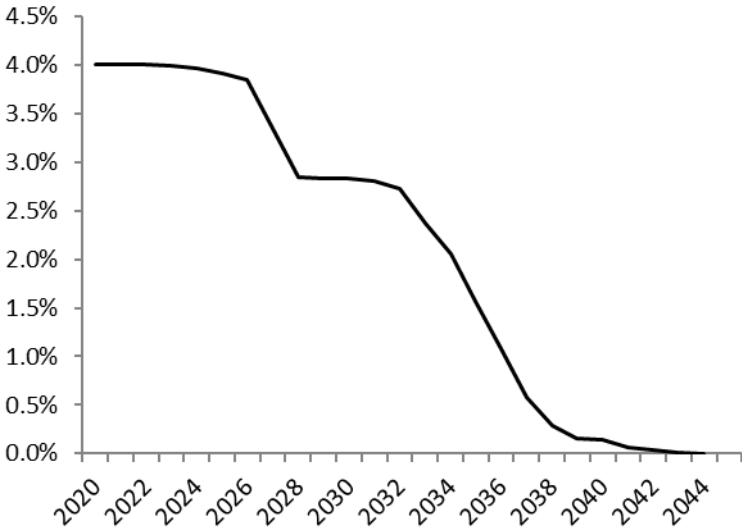


Figure 5.7: Indexation of pensions with the application of the ABM

Source: Author's computation.

See Appendix B for tables of detailed selected results.

6 Conclusion

The aim of this thesis is to provide new insights into the issue of the financial sustainability of pension systems. The primary focus of this thesis is brought on the situation of the Czech pension system, which still largely depends on a PAYG system.

As the results have shown, the PAYG pension system in the Czech Republic does not seem to be financially sustainable in the horizon of 25 years without any additional interventions. To make the system sustainable and liquid, the ABM rules are introduced, whose assessment shows that either the contribution rate or the age of retirement (or the indexation of pensions) would need to significantly increase (decrease) to ensure the financial sustainability of the system.

The use of the ABM rules in practice might be desirable to prevent reliance on political decisions and to rely strictly on the projected values.

There is a vast opportunity for further research on this topic, residing in the subsequent development of the objective function and constraints, as well as in the possibility of introducing the buffer fund, including other macroeconomic parameters in the model or extending the time horizon.

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Appendix A: R script

The R script written for this thesis aims to calculate resulting numbers for the assessment of the financial sustainability of the statutory expected future development of the Czech pension scheme at first, then to compute the results of the ABM modelling, where only one key variable is allowed to change, and finally to calculate the ABM model with simultaneous change of all key variables.

The optimisation for the cases where just one key variable is modified is carried out independently for every year n of the study since there is precisely one optimal solution of the model. On the other hand, the optimisation where all key variables are modified simultaneously is carried out gradually from setting the model's final year to be $n = 2$ to set the final year as $n = N$ (the last year of the studied period). The optimal values of the optimisation are then used as the initial values of the key variables entering the optimisation problem extended by one additional year until the final year of the optimisation reaches the last year of the study (the initial value entering the model for its final year n of the optimisation is set to be equal to the initial value at $n - 1$).

The key variables are scaled when performing the optimisation with their simultaneous change so that they represent comparable ratios of the difference between their resulted values and their initial values entering the optimisation to their highest possible change.

For a full R script used in this thesis, see the attached file *Appendix A.r*.

Appendix B: Selected Results

Table B.1: Czech pension system without any intervention

Year	N of contributors	N of pensioners	Contribution base (nominal in billions)	Total spendings (nominal in billions)	Liquidity ratio
2020	5 042 126	2 520 393	472.50	437.71	1.08
2021	5 023 376	2 531 324	489.76	462.93	1.06
2022	5 021 589	2 532 184	509.31	487.16	1.05
2023	5 021 051	2 529 738	529.63	511.61	1.04
2024	5 014 930	2 527 493	550.07	537.08	1.02
2025	5 006 850	2 525 766	570.80	563.68	1.01
2026	5 002 246	2 525 574	592.57	591.73	1.00
2027	4 996 863	2 533 359	614.95	623.08	0.99
2028	4 994 057	2 544 312	638.35	656.66	0.97
2029	4 993 400	2 553 293	662.82	691.11	0.96
2030	4 991 351	2 557 109	687.98	725.45	0.95
2031	4 989 353	2 556 425	714.25	759.74	0.94
2032	4 982 924	2 553 368	741.11	794.58	0.93
2033	4 975 804	2 548 904	769.00	830.23	0.93
2034	4 961 763	2 546 342	796.97	867.88	0.92
2035	4 941 139	2 546 309	824.90	907.85	0.91
2036	4 919 240	2 548 619	853.68	950.24	0.90
2037	4 895 645	2 551 009	883.19	994.26	0.89
2038	4 857 561	2 587 169	911.12	1 054.12	0.86
2039	4 812 181	2 637 748	938.56	1 123.10	0.84
2040	4 763 403	2 698 767	966.16	1 200.25	0.80
2041	4 721 072	2 757 635	995.84	1 280.34	0.78
2042	4 683 270	2 813 313	1 027.27	1 362.91	0.75
2043	4 648 125	2 864 669	1 060.49	1 447.37	0.73
2044	4 614 136	2 913 756	1 095.05	1 534.74	0.71
2045	4 581 589	2 958 200	1 131.32	1 623.75	0.70

Table B.2: Modification of the contribution rate

Year	Contribution rate	N of contributors	Contribution base (nominal in billions)	Total spendings (nominal in billions)	Liquidity ratio
2020	22.9%	5 042 126	472.50	437.71	1.08
2021	22.9%	5 023 376	489.76	462.93	1.06
2022	22.9%	5 021 589	509.31	487.16	1.05
2023	22.9%	5 021 051	529.63	511.61	1.04
2024	22.9%	5 014 930	550.07	537.08	1.02
2025	22.9%	5 006 850	570.80	563.68	1.01
2026	22.9%	5 002 246	592.57	591.73	1.00
2027	23.2%	4 996 863	623.08	623.08	1.00
2028	23.6%	4 994 057	656.66	656.66	1.00
2029	23.9%	4 993 400	691.11	691.11	1.00
2030	24.1%	4 991 351	725.45	725.45	1.00
2031	24.4%	4 989 353	759.74	759.74	1.00
2032	24.6%	4 982 924	794.58	794.58	1.00
2033	24.7%	4 975 804	830.23	830.23	1.00
2034	24.9%	4 961 763	867.88	867.88	1.00
2035	25.2%	4 941 139	907.85	907.85	1.00
2036	25.5%	4 919 240	950.24	950.24	1.00
2037	25.8%	4 895 645	994.26	994.26	1.00
2038	26.5%	4 857 561	1 054.12	1 054.12	1.00
2039	27.4%	4 812 181	1 123.10	1 123.10	1.00
2040	28.4%	4 763 403	1 200.25	1 200.25	1.00
2041	29.4%	4 721 072	1 280.34	1 280.34	1.00
2042	30.4%	4 683 270	1 362.91	1 362.91	1.00
2043	31.3%	4 648 125	1 447.37	1 447.37	1.00
2044	32.1%	4 614 136	1 534.74	1 534.74	1.00
2045	32.9%	4 581 589	1 623.75	1 623.75	1.00

Table B.3: Modification of the retirement age

Year	Age of retirement	N of pensioners	Contribution base (nominal in billions)	Total spendings (nominal in billions)	Liquidity ratio
2020	62.15	2 520 393	472.50	437.71	1.08
2021	62.15	2 552 059	488.78	467.40	1.05
2022	62.15	2 570 476	507.42	495.75	1.02
2023	62.15	2 587 792	526.63	525.16	1.00
2024	62.45	2 571 318	547.72	547.72	1.00
2025	62.79	2 548 862	569.51	569.51	1.00
2026	63.13	2 528 178	592.42	592.42	1.00
2027	63.50	2 508 979	616.43	616.43	1.00
2028	63.87	2 491 484	641.66	641.66	1.00
2029	64.24	2 474 809	667.94	667.94	1.00
2030	64.59	2 457 198	694.76	694.76	1.00
2031	64.91	2 439 782	722.49	722.49	1.00
2032	65.21	2 421 545	750.79	750.79	1.00
2033	65.49	2 403 736	780.08	780.08	1.00
2034	65.76	2 384 700	809.80	809.80	1.00
2035	66.04	2 364 480	839.91	839.91	1.00
2036	66.33	2 345 121	871.16	871.16	1.00
2037	66.63	2 325 915	903.29	903.29	1.00
2038	66.92	2 308 535	937.01	937.01	1.00
2039	67.23	2 291 985	971.96	971.96	1.00
2040	67.56	2 277 031	1 008.53	1 008.53	1.00
2041	67.88	2 264 800	1 047.34	1 047.34	1.00
2042	68.24	2 254 253	1 088.02	1 088.02	1.00
2043	68.62	2 245 047	1 130.52	1 130.52	1.00
2044	69.01	2 236 643	1 174.64	1 174.64	1.00
2045	69.39	2 229 033	1 220.45	1 220.45	1.00

Table B.4: Modification of the indexation of pensions

Year	Indexation of pensions	N of pensioners	Contribution base (nominal in billions)	Total spendings (nominal in billions)	Liquidity ratio
2020	4.0%	2 520 393	472.50	437.71	1.08
2021	4.0%	2 531 324	489.76	462.93	1.06
2022	4.0%	2 532 184	509.31	487.16	1.05
2023	4.0%	2 529 738	529.63	511.61	1.04
2024	4.0%	2 527 493	550.07	537.08	1.02
2025	4.0%	2 525 766	570.80	563.68	1.01
2026	2.6%	2 525 574	592.57	591.73	1.00
2027	2.3%	2 533 359	614.95	614.95	1.00
2028	2.4%	2 544 312	638.35	638.35	1.00
2029	2.6%	2 553 293	662.82	662.82	1.00
2030	2.8%	2 557 109	687.98	687.98	1.00
2031	2.8%	2 556 425	714.25	714.25	1.00
2032	2.9%	2 553 368	741.11	741.11	1.00
2033	2.7%	2 548 904	769.00	769.00	1.00
2034	2.4%	2 546 342	796.97	796.97	1.00
2035	2.3%	2 546 309	824.90	824.90	1.00
2036	2.1%	2 548 619	853.68	853.68	1.00
2037	0.1%	2 551 009	883.19	883.19	1.00
2038	-1.0%	2 587 169	911.12	911.12	1.00
2039	-1.7%	2 637 748	938.56	938.56	1.00
2040	-1.7%	2 698 767	966.16	966.16	1.00
2041	-1.7%	2 757 635	995.84	995.84	1.00
2042	-1.6%	2 813 313	1 027.27	1 027.27	1.00
2043	-1.6%	2 864 669	1 060.49	1 060.49	1.00
2044	-1.5%	2 913 756	1 095.05	1 095.05	1.00
2045		2 958 200	1 131.32	1 131.32	1.00

Table B.5: Simultaneous modification of all key variables

Year	Contribution rate	Indexation of pensions	Age of retirement	Total spendings (nominal in billions)	Liquidity ratio
2020	22.9%	4.0%	62.15	450.67	1.09
2021	22.9%	4.0%	62.15	481.73	1.06
2022	22.9%	4.0%	62.15	511.43	1.03
2023	22.9%	4.0%	62.15	541.80	1.01
2024	23.3%	4.0%	62.15	569.53	1.00
2025	23.8%	3.9%	62.17	594.60	1.00
2026	24.2%	3.8%	62.30	618.53	1.00
2027	24.7%	3.3%	62.39	645.39	1.00
2028	25.0%	2.8%	62.52	677.03	1.00
2029	25.0%	2.8%	62.69	706.81	1.00
2030	25.0%	2.8%	62.83	736.98	1.00
2031	25.0%	2.8%	62.96	766.31	1.00
2032	25.0%	2.7%	63.10	798.75	1.00
2033	25.0%	2.4%	63.26	832.47	1.00
2034	25.0%	2.1%	63.41	867.89	1.00
2035	25.0%	1.6%	63.55	906.56	1.00
2036	25.0%	1.1%	63.65	940.16	1.00
2037	25.0%	0.6%	63.76	977.25	1.00
2038	25.0%	0.3%	63.88	1 012.16	1.00
2039	25.0%	0.2%	64.04	1 044.32	1.00
2040	25.0%	0.1%	64.18	1 077.86	1.00
2041	25.0%	0.1%	64.31	1 113.66	1.00
2042	25.0%	0.0%	64.41	1 151.26	1.00
2043	25.0%	0.0%	64.49	1 190.64	1.00
2044	25.0%	0.0%	64.57	1 231.87	1.00
2045	25.0%		64.60	1 275.06	1.00