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The Predictive Power of the Yield Curve: Some Empirical Evidence

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Predikčná schopnosť výnosovej krivky: empirický dôkaz

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Abstrakt

Ekonómovia často využívajú zložité matematické modely na predkciu budúceho vývoja ekonomiky a pravdepodobnosti recesie. Ale aj jednoduchšie ukazovatele ako úrokové miery, akciové indexy a peňažné agregáty taktiež obsahujú významné informácie o budúcej ekonomickej aktivite. V tejto práci prehodnocujeme jeden z indikátorov, výnosovú krivku, špecificky, rozdiel medzi výnosom z 10–ročného a 3–mesačného vládneho dlhopisu. Za pomoci štyroch rozdielnych modelov vo vybraných európskych krajinách skúmame či rozdiel vo výnosoch – spread – má stále predikčnú silu pre budúci vývoj reálnej ekonomiky. Navyše porovnávame predikčnú silu spreadu s viacerými premennými. Rozložíme výnosový spread na efekt očakávaní a "term premium" efekt aby sme zistili, ktorý faktor viac prispieva k predpovedi rastu reálneho HDP. S použitím vlastnej definície recesie dospejeme k záveru, že výnosový spread stále obsahuje informácie na predpovedanie budúcej ekonomickej aktivity, hoci jeho predikčná sila sa zhoršuje.

Klúčové slová: výnosová krivka, spread, recesia

Abstract

Economists often use complex mathematical models to forecast the future path of the economy and the likelihood of recession. But more simple indicators such as interest rates, stock price indices, and monetary aggregates also contain some relevant information about future economic activity. In this thesis we revisit the usefulness of one such indicator, the yield curve or, more specifically, the spread between the interest rates on the ten-year Treasury note and the three-month Treasury bill. By using four different models we examine whether the yield spread has still some predicitve power for future real GDP growth in selected european countries. What is more, we are comparing the predictive power of the yield spread with different variables, both in–sample and out–of–sample. We decompose the yield spread into expectations effect and term premium effect in order to investigate which factor contributes more to predicting real GDP growth. Using modified definition of recession we conclude that that yield spread still contains some useful information for predicting future economic activity, although its predictive power deteriorates.

Index words: yield curve, yield spread, recession

Declaration

Prehlasujem, že som predkladanú diplomovú prácu vypracoval samostatne a použil iba uvedené pramene a literatúru.

Súhlasím s tým, aby moja diplomová práca bola sprístupnená verejnosti pre účely štúdia a výskumu.

Hereby I declare that I compiled this thesis independently, using only the listed literature and resources.

I agree with my diploma thesis to be at disposal to public for research and study purposes.

V Prahe dňa Jozef Jamriška

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Dedication

I would like to dedicate this diploma thesis to my parents.

Table of Contents

List of Figures

List of Tables

Introduction

Economists often use complex mathematical models to forecast the future path of the economy and the likelihood of recession. But more simple indicators such as interest rates, stock price indices, and monetary aggregates also contain some relevant information about future economic activity. The primary focus of this thesis is to test simple financial variables whether they may be useful predictors of future recession and real GDP growth. The variables examined are the yield spread, inflation[1], stock price indices of corrensponding countries, 3–month Treasury bond rate and 10–year treasury bond rate.

The slope of the yield curve should be an excellent indicator of a possible future recession for various reasons. First of all, the current monetary policy has a serious impact on the yield spread and thus on real economic activity over the several quarters ahead. An increase in the short rate has tendency to flatten the yield curve and to slow real growth of the economy in the near term as well. [23] But the yield spread does not reflect only current state of monetary policy but also a private expectations concerning future inflation and interest rates.

By using binary choice models we examine whether the yield spread can predict economic recesion in United Kingdom, Germany, France, Slovak Republic, Czech

[1]By inflation we mean q–o–q growth in Harmonized Consumer Price Index (HCPI).

Republic, Hungary and Poland between one and eight quarters ahead. We also use linear models to test whether the yield spread has still some predicitve power for future real GDP growth. For United Kingdom, Germany and France we partially confirm previous studies that the spread between 10–year Treasury bond rate and 3–month Trasury bill rate contains information about future real GDP growth. However, we find that even growth in stock price indices for corresponding countries shows a significant predictive ability. For the three countries above we then decompose the yield spread into the expectations effect and term premium effect in order to investigate to which extent they contribute to yield spread's significance. We find that only expectations effect makes statistically significant contribution. Regarding out–of–sample results, in most cases we find the yield spread to be the best forecasting variable together with corresponding stock price indices.

The diploma thesis begins with a review of previous studies and usefullness of the yield spread in predicting future economic activity. In chapter 2 we present the rationale behind the term structure of interest rates in order to clarify and explain how and why are we using particular models. Chapter 3 describes models and data used. In chapter 4 we present our findings. The last chapter concludes that overall we find mixed results but still we can say that for some countries the yield spread contains some useful information which can be used to predict future economic activity.

Chapter 1

Literature overview

Interest rates and interest–rate spreads has recieved great attention from academics and market analysts in the past two decades. To be more precise, the relationship between short– and long–term interest rates with respect to maturity – the yield curve – rises many questions among investors and policymakers.

Analysis of the behavior of interest rates of different maturities over the business cycle goes back to Mitchel (1913). However, Kessel (1965) was the first one that made specific reference to the behavior of term spreads. He found out that different spreads between long– and short–term rates tend to be low at the beginnig of recessions – the business cycle peak – and high as expansions get under way after a cyclical trough.[1] Butler (1978) made a connection between yield curve as a predictor of short–term interest rates and implications of declining short–term rates for contemporaneous economic activity, foreshadowing some of the later logic. He correctly predicted that there would be no recession in 1979, though a year later the situation would have been quite different. [19]

[1] He used data which dates back to the 1858. He found some evidence of the leading indicator properties of the spread in time periods from 1914 to 1933 and from 1954 to 1961.[19]

Almost all of the research of the yield curve as a predictor of changes in real output has occured over the past two decades. The earlier work includes Evans (1987) and Laurent (1988, 1989) who offered the yield spread as a simple method for predicting growth of future output. Also Harvey (1988) noticed that there is predictive relationship between comsumtion and the slope of the yield curve.[35] Stock and Watson (1989), probably in their most influential study, developed a sophisticated model of leading indicators, with yield spread among them.[2][12]

Estrella and Hardouvelis (1991) documented that the yield spread between the 10–year Treasury bond and 3–month Treasury bill rate is a usefull predictor consumption, investment, the probability of recession, the cumulative growth up to four years in the future and the marginal economic growth rates up to seven quarters in the future. Further they found that the spread contains information for future economic growth not already embodied in current economic growth, in the current growth rate of the index of leading economic indicators, in the current level of interest rates or in the inflation rate.[21] Plosser and Rouwehorst (1994) examined the information contained in the term structure about future real economic growth in three industrialized countries. They found that the term structure has significant predictive power for long–term economic growth and showed that the term structure contains information about future real economic activity that is independent from information about

[2] Their approach was to examine combinations of 55 various economic variables and select the combination that best predict economic activity. To made their search managable they limited their index to seven variables. One of the variables that is an important component of their leading economic indicator is the spread between the ten–year and one–year U.S. Treasury bond. Because their research was fairly exhaustive, the finding that the yield spread is an important element of their indicator gave incentive to exploring the predictive content of this variable in isolation.[15] Stock and Waston (2003) still continue to keep the yield spread as a part of their index. [56]

current or future monetary policy. An unusual feature of their paper was the use of discount equivalent yields and the fact that they matched the maturity structure of the interest rate spread with the forecast horizon being studied.[15]

Haubrich and Dombrosky (1996) using a linear model found that the yield spread is a relatively accurate predictor of four–quarter economic growth but its predictable ability has changed over time. To be more precise they concluded that the yield spread's ability to predict economic growth has deteriorated since 1985.[40]

Other papers have concentrated on different feature, namely the ability of the yield spread to signal probability of a recession. Estrella and Mishkin (1998) showed that the yield spread is the best out–of–sample predictor of the probability of a recession occuring in the next four quarters.[35] Dotsey (1998) has thoroughly investigated the forecasting properties of the yield spread for economic activity. He concluded that the spread contains useful information beyond that contained in past economic activity or past monetary policy, although over more recent periods the spread has not been nearly as informative as it has been in the past.[15]

Cambell and Shiller (1987) showed that the yield spread can be decomposed into a weighted sum of future expected changes in the short rates and the term premium. Yet, the spread can be large due to expected future monetary policy to be tight or because the term premium is large as investors want to be compensated for taking on high risks in bad times.[26] Consequently, the yield spread decomposition into a term reflecting the term premium and the term reflecting future monetary policy might be helpful to understand why the yield curve fluctuations predict future economic activity and also why the spread's predicitve ability fluctuates over time. However, such a decomposition is difficult in practice as it requires expected future short–term interest rates.[3]

Recent studies used the decomposed yield spread to understand why it is a good predictor of real economic activity. The first paper that examines the importance of decomposition of yield spread for forecasting was Kim and Hamilton (2002), which forecast future GDP growth using a spread between the ten–year Trasury bond and three–month Treasury bill. The inovation of their paper is that it decomposes the yield spread into the expectations component and term premium component using ex–post short–term interest rates as a substitute for ex–ante expected rates. The authors achieve this separation by considering the ex–post realizations of short–term rates, using instruments ex–ante to isolate the expectations part. They concluded that both components contribute significantly to real GDP growth but the future expected change of short–term rates is significantly bigger than the that of term premium. They find that the coefficients β_2 and β_3 are indeed statistically significantly different from one onother, although both coefficients have positive signs. Note that positive value of β_3 implies that a decline in the term premium is associated with slower future growth.[35] Moreover, they state that volatility of short–term interest rates should also matter for the term premium. By using a two–factor affine pricing model of the

[3] The decomposition might differ fundamentally depending on how the expectations formed. Another possibility is to use survey data. See Fuhrer (1996)[33]

term structure they reached a conclusion that although interest rate volatility is a important determinant of the term structure of interest rates, the explanation why the yield spread helps to forecast economic growth must be sought elsewhere.[35]

Ang et al. (2004) used a VAR dynamic model for GDP growth and yields that completely characterizes expectations of GDP. The VAR is estimated on the full sample and the projections are made in–sample.[2] Favero et al. (2004) however argues that this procedure cannot simulate the investors' effort to use the model in real time to forecast short–term interest rates, as the information from the whole sample is used to estimate parameters while investors can use only historically available information to generate predictions. Favero et al. (2004) hence estimated a VAR at each point in time, using historically available information, and then project short-term rates out–of–sample. They concluded that the best forecasting model for output is obtained by considering the term premium, the short–term interest rate and inflation as predictors. [4][26]

Rudebush et al. (2007) concentrate on the explanation of recent experience of a declining term premium in long–term interest rates by using structural model.[51] This topic is very important and upcoming because of the practical inplications for conduct of monetary policy. The authors' results, contrary to Kim and Hamilton

[4]The authors also find that the coefficient β_3, representing the term premium in decomposed yield spread, has a positive value. Similar to Kim and Hamilton (2002) this indicates that the decline in term premium is associated with slower future GDP growth.

(2002) and Favero et al. (2004), suggest that a decline in the term premium has typicaly been associated with higher future GDP growth.[5]

The slope of the yield curve should be an excellent indicator of a possible future recession for various reasons. First of all the current monetary policy has a serious impact on the yield spread and thus on real economic activity over the several quarters ahead. An increase in the short rate has tendency to flatten the yield curve and to slow real growth of the economy in the near term as well. [23]

This relationship, however, is only one part of the explanation for the yield curves' usefulness as a forecasting tool. Expectations of future real interest rates and inflation which are part of the yield curve spread also seem to play an important role when predicting future economic activity. The yield curve spread variable examined corresponds to a forward interest rate applicable from three months to ten years into the future. As explained in Mishkin (1990a, 1990b), this rate can be decomposed into expected real interest rate and expected inflation components, each of which may be helpful in forecasting. The expected real rate may be associated with expectations of future monetary policy and hence of future real growth.[23] Moreover, because inflation is usually positively related to economic activity, the expected inflation component may also bear some information about future economic growth. Although the yield curve proved to have clear advantages as a predictor of future economic events,

[5] Which is consinstent with the speech of Federal Reserve Chairman Ben Barnanke before the Economic Club of New York (2006). He said: "To the extent that the decline in forward rates can be traced to a decline in term premium,. . . the effect is financially stimulative and argues for greater monetary restraint, all else being equal. Specifically, if spending depends on long-term interest rates, special factors that lower the spread between short–term and long-term rates will stimulate aggregate demand. Thus, when the term premium declines, a higher short-term rate is required to obtain the long–term rate and the overall mix of financial conditions consistent with maximum sustainable employment and stable prices."

several other variables have also been widely used to forecast the future development of the economy. Together with financial variables, stock prices have been extensively examined. Finance theory suggests that stock prices are determined by expectations about future dividend streams, which in turn are related to the future state of the economy. Among macroeconomic variables, the Conference Boards index of leading economic indicators appears to have an established performance record in predicting real economic activity.[23] Nevertheless, its record has not always been subjected to careful comparison tests. In addition, because this index has often been revised after the fact to improve its performance, its success could be overstated. An alternative index of leading indicators, developed in Stock and Watson (1989), appears to perform better than the Commerce Departments index of leading economic indicators.[56] In the discussion below, we compare the predictive power of various variables with that of the yield curve.

The literature found that the most significant results coincides with horizons 4 to 6 quarters ahead either in US and in European countries [34]. Many other papers have also demonstrated the predicitive power of the spread for future economic activity.

Economists often use complex mathematical models to forecast the path of the UK economy and the likelihood of recession. But simpler indicators such as interest rates, stock price indexes, and monetary aggregates also contain some information about future economic activity. In this thesis I examine the usefulness of one such indicatorthe yield curve or, more specifically, the spread between the interest rates on the ten-year Treasury note and the three-month Treasury bill.

Chapter 2

The term structure of interest rates

A abundant economic literature has examined various variables that help to predict the economic activity. Interest rates and interest–rate spreads has recieved great attention from academics and market analysts in the past two decades. To be more precise, the relationship between short– and long–term interest rates with respect to maturity – the yield curve – rises many questions among investors and policymakers.

The yield curve, or in other words the term structure of interest rates, started to be popular in mid–1990s and it continues to recieve great attention since then. In 1994 the Federal Reserve became eagerly aware of the yield curve as initially small increases in short–term interest rates induced unuexpectedly vigorous changes in long–term yields. Meanwhile, the U.S. Treasury began to shorten the maturity of the government debt in the hope of lowering federal interest costs.[6]

Various questions did emerge, for example: why should the yield spread help to predict future economic activity? Aside from ingenious macroeconomic models is there any place for a unsophisticated indicator like the yield spread? Before we find answers to these questions it is worth to give heed to the understanding of the term structure.

Bonds are one type of fixed income securities that are traditionally defined as instruments that make pre–specified payments to investors. Thus to calculate the price of the bond one need not to forecast random future payments but can simply discount known payments to the present. This makes the bond pricing relativelly clear and easy to understand but in practice many bonds differ from this theoretical approach.[1] [6]

2.1 The basic concept

Treasury securities are very suitable for our analyses while they are very close to the theoretical ideal due to negligible default risk. Treasury bills promise to make a single payment, principal, at a particular date in future. This day is called the maturity day and the time of the existence of the bill is called the maturity of the bill. Treasury bills have always maturity less than one year. On the other hand Treasury bonds are coupon–bonds that have maturity longer than one year. They promise a series of payments, known as coupons, every twelve months until the maturity day when a final coupon together with bond's face value is repaid. But we can think of these Treasury bonds as series of zero–coupon securities because the coupon payments can be traded separately of the Treasury bond. This process is called "stripping" of the

[1] It may easily happen that the issuer of the bond defaults and the payments on their bonds became virtually random. During the 1980s, this became particularly obvious when some of the third world countries failed to make scheduled interest payments on loans and with development of junk bonds, a class of securities with enough default risk to be intermediate between traditional bonds and equity. In order to focus only on the relation between yields and maturity I have to abstract from other factors that influence the bond's yield. For example we have to avoid those bonds that recieve special tax treatment–flower bonds, corporate and municipal bonds, which are not actively traded as Treasury securities. We have to abstract from the callable bonds, which means that they grant the borrower the right to repurchase the bond back at a predetermined price at some point in the future. [28]

bond. As a result of this process, zero–coupon securities with long maturities have been traded this way since mid–1980s. [28]

Since a zero–coupon bond makes a single payment at maturity date the investor has a guaranteed return over the life of the bond. Such a measure of return is the bond's yield to maturity. Following the notation of Campbell (1995) the yield to maturity $Y_{m,t}$ satisfies

$$P_{m,t} = \frac{V}{(1 + Y_{m,t})^m} \tag{2.1}$$

where the $P_{m,t}$ is the price of the m–period bond at time t with a face value of V.[2]

If the bond is sold before the maturity, the investor will recieve a holding period return. This holding period return is not known in advance because it depends on the price of the bond. From the equation 2.1 it is clear that if the yield to maturity remains unchanged the return will equal the initial yield. But if the bond's yield falls then the holding period return is greater than the initial yield and vice–versa.

To understand this, assume holding a m–period zero–coupon bond for one period. Then the holding–period return is just the change of the bond's price durig this period

$$r_{m,t+1} = p_{m-1,t+1} - p_{m,t} = y_{m,t} - (m-1)\ (y_{m-1,t+1} - y_{m,t}) \tag{2.2}$$

[2] Campbell uses the term "gross yield" by which he denotes the one plus yield to maturity. So if the yield to maturity $Y_{m,t}$ is equal to 5% then the gross yield is 1.05. He also states that it is often more convenient to work with such a continuously compounded, log yield, which is easily calculated as natural logarithm of the gross yield. For the small yields such a aproximation holds very well. By introducing the continuously compounded yield we can easily show that the bond's yield to maturity has a direct relation to the natural logarithm of the bond's price. By taking logarithms of the equation 2.1 we get $p_{m,t} = -m\, y_{m,t}$ where $p_{m,t} \equiv \log(P_{m,t})$ is log bond price and $y_{m,t} \equiv \log(1 + Y_{m,t})$ is continuously conpounded yield to maturity. Appearently, the maturity m measures the proportional change in the price of the zero–coupon bond when bond's continuously compounded yield changes by 1% point.[6]

where the $r_{m,t+1}$ is logarithm of the 1–period return on a bond purchased with maturity m in year t and sold with remaining maturity $m-1$ in year $t+1$. Thus the 1–period return is the initial yield, minus the the change in the yield durig the period the bond is held times the maturity of the bond when it is sold, $(m-1)$.[6]

By substracting short–term yield from both sides we get excess return on the left side and yield spread less $(m-1)$ times the change in long bond on the other side of the equation [3]

$$r_{m,t+1} - y_{1,t} = (y_{m,t} - y_{1,t}) - (m-1)\ (y_{m-1,t+1} - y_{m,t}) \tag{2.3}$$

The difference between the long–term yield and the short–term yield is called the yield spread. For any point in time the yields can be summarized in a plot of maturity against yields to maturity. This plot is known as a yield curve. The yield curve is normaly upward–sloping, but it can also be downward–sloping, inverted, or even hump–shaped.

From the equation 2.3 we can see that if long–term yields change over time, the big changes occur to the prices of long–term bonds and volatile returns on long bonds. Campbell (1995) investigated this relationship. He found out that the mean excess return was positive rising with maturity at first, but started to fall at a maturity of one year and was actually negative for 10–year zero–coupon bonds. This result was surprising because long–term bonds ordinary offer higher yields. But with reference

[3] [47]

to the second component of the equation 2.3 this decline can be explained by the constant change in yield. From this observation we can conclude that the excess return does not directly relate to the yield spread. However, under the expectations hypothesis the excess return should be equal to zero.

2.1.1 The expectations hypothesis and the term structure

The expectations hypothesis has historically been the most widely used analytical tool to understand the shape of the yield curve. Let's assume an investor who buys a zero–coupon bond in time t. He knows with certainty that the bond's price at maturity $t+k$ wil be equal to its nominal value. This logically means that any unexpected change in price that may occur sometimes in near future must be automatically compensated by the opposite change in price in more distant future in order to revert the price back to its nominal value. The variation in nominal returns on a given bond must therefore be negatively serially correlated and if the expectations hypothesis were true, the slope of the yield curve could be used to forecast the future path of the interest rate.[4][28]

[4] For ilustration consider an investor who faces a 3.5 percent yield on a 1–year bond and 10 percent yield on a 20–year bond.[6] Naive investor would go for the 20–year bond because he believes that this bond is superior investment. The investor ignores the fact that the 1–year bond's yield is known return over one year while the 20–year yield is the return over 20 years. A brighter investor will realize that if the bond will be sold before maturity, the holding period return won't be equal to the initial yield to maturity. In the example above, if the investor will sell the 20–year bond after one year. It's return will be 10 percent but the excess return will be 6.5 percent only if the yield stays unchanged at 10 percent. On the other hand, the 1–year return on the two bonds could be the same if the price of the 20–year bond rises during the year by 3.5 percent instead of 10 percent implied by its yield. This price increase will happen only if the 20–year bond's yield rises to 10.3 percent. If the investor will pursue strategy of rolling over a 1–period bond his average return will be 3.5 percent if the 1–year yields remain unchanged at 4 percent on average. The sequence of 1–year bonds can deliver the same return as the 20–year bond only if the 1–year yield over next 20 years average 10 percent. Since the 1–year yield over the first year is 3.5 percent the next 19 1–year yields have to average 10.3 percent which is negligibly higher than than today's 10 percent yield on 20–year bond but a lot higher than today's 3.5 percent 1–year yield.[6]

In theory there are two versions of expectations hypothesis. The pure expectations hypothesis of the terms structure of interest rates is the theory that interest rates are expected to move in exactly such a way, to equalize expected returns on short–term and long–term investment strategies. The expectations hypothesis is weaker expression which states that the difference between the expected short–term and long–term investment strategies is constant, and need not necessarily equal to zero as required by pure expectational hypothesis.

The pure expectations hypothesis actually says that the expected m–period holding return on a 1–period bond must be equal to the expected m–period holding return on a m–period bond.[5] It also says that the expected 1–period holding return on a 1–period bond is equal to expected 1–period holding return on a m–period bond.

We can express the first statement in simple algebra. Suppose that m is equal to 2, then it must hold

$$(1 + Y_{1,t})\,(1 + Y_{1,t+1}) = (1 + Y_{2,t})^2 \tag{2.4}$$

after the substitution and rearangement we get following[6]

$$\frac{y_{1,t} + \ldots + y_{1,t+m-1}}{m} = y_{m,t} \tag{2.5}$$

[5] In simple words: whenever a long bond yield exceeds a short yield, the yield on the long bond subsequently tends to rise over the life of the short bond; this generates expected capital losses on the long bond, which offset current yield advantage.[6]

[6] The derivation is provided in the mathematical appendix 4.2.2. [47]

We can clearly see that under the expectations hypothesis, the long yield is the average of the (expected) short yields. It is also true that the yield spread is the average of (expected) long run changes in short yields[7]

$$\Rightarrow y_{m,t} - y_{1,t} = \frac{y_{1,t+1} + \ldots + y_{1,t+m-1} - (m-1)\, y_{1,t}}{m} \tag{2.6}$$

after rearanging we get[8]

$$\frac{m}{m-1}\,(y_{m,t} - y_{1,t}) = \frac{y_{1,t+1} + \ldots + y_{1,t+m-1}}{m-1} - y_{1,t}$$

$$\Rightarrow \frac{y_{1,t+1} + \ldots + y_{1,t+m-1}}{m-1} - y_{1,t} = \alpha + \beta\,\frac{m}{m-1}\, y_{m,t} - y_{1,t} + \eta_t \tag{2.7}$$

As I mentioned above the pure expectations hypothesis requires that the expected 1–period holding return on a 1–period bond is equal to expected 1–period holding return on a m–period bond.[9]

The 1–period holding return on m-period bond can be written as follows

$$\frac{P_{m-1,t+1}}{P_{m,t}} = \frac{(1+Y_{m,t})^{m}}{(1+Y_{m-1,t+1})^{m-1}} \tag{2.8}$$

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[7] [48]

[8] The derivation is provided in mathematical appendix.

[9] It says that whenever a long yield exceeds a short yield, short yields tends to rise to equate returns over the life of the long bond as in example above.[6]

simple substitution followed by rearangment we obtain[10]

$$y_{m,t} - y_{1,t} = (m-1)\ (y_{m-1,t+1} - y_{m,t}) \tag{2.9}$$

This equation basicaly says that the yield spread in current period is equal to the $(m-1)$ times the change in (expected) long yields in the next period. While $y_{m-1,t+1}$ is unknown in period t so the yield spread predicts short run changes in long yields

$$y_{m-1,t+1} - y_{m,t} = \alpha + \beta\,\frac{y_{m,t} - y_{1,t}}{m-1} + \epsilon_t \tag{2.10}$$

Under the expectations hypothesis in equation 2.7 and 2.10, β should equal to unity. However, the empirical evidence does not support the expectations hypothesis. By using regression equation 2.10 Campbell (1995) found that if the yield spread is high the long yields tends to fall, which enhances the yield differential between long–term and short–term bonds, rather than rising to offset the yield spread. This founding, however was already presented in Macaulay (1938) where he wrote: "The yields of bonds of the highest grade should fall during a period in which short–term rates are higher than the yields of the bonds and rise during a period in which short–term rates are lower. Now experience is more nearly the opposite."[6]

From Campbell's results it is clear that the expectations hypothesis fails. Nevertheless, the expectations hypothesis is a joint hypothesis which consists of two parts

[10] The derivation is provided in the mathematical appendix 4.2.2. [48]

1. Perfect Substituability

2. Rational Expectations[11]

In the next section we will concentrate on rationale behind the first leg of expectations hypothesis, the perfect substituability.

2.1.2 Pefrect substituability

Perfect substituability hypothesis states that bonds with different time to maturity have identical expected holding returns over a certain period of time. Hence two consecutive short–term bonds have the same k–period holding return as one corresponding long–term bond

$$\left(1+I_t^j\right)^j \left(1+I_{t+j}^{(k-j)}\right)^{(k-j)} = \left(1+I_t^k\right)^k \tag{2.11}$$

[11] To be exact we are working with rational expectations in the sense of Muth. In theoretical economics the rational expectations were originally proposed by John F. Muth (1961) of Indiana University and later developed by Robert E. Lucas. It is used to model how economic agents forecast future economic situations depending to some extent upon what they expect to happen.[60] Expectations are vital and important part in economic models especially in finance. Rational expectations theory defines this kind of expectations as being identical to the best guess of the future, optimal forecast, that uses all available information. "However, without further assumptions, this theory of expectations determination makes no predictions about human behavior. That is why, rational expectations do not differ systematically from the equilibrium result. That is that agents in the model do not make systematic errors when predicting the future, and deviations from the perfect foresight are only random."[64] The standard economic assumption is that agents behave in such a way that maximize their utility or profit.[60]–Thomas J. Sargent

after algebraical adjustments we get a regression equation[12]

$$i_{t+j}^{(k-j)} - i_t^j = \alpha + \beta \, \frac{k}{k-j} \, \left(i_t^k - i_t^j\right) + \kappa_{t+j} \tag{2.12}$$

This equation states that the current spread between long and short yield is proportional to the expected difference between two consecutive short yields. Of course there are many ways to test the expectations hypothesis. Froot(1989) uses following regression to test the expectations hypothesis[13]

$$i_{t+j}^{(k-j)} - {i_t}^j = \alpha + \beta \, fp_t^{(j,k)} + \kappa_{t+j} \tag{2.13}$$

where

$$fp_t^{(j,k)} = \frac{k}{k-j} \left(i_t^k - i_t^j\right) \tag{2.14}$$

which states that the forward premium on borrowing between period $t+j$ and period $t+k$ is proportional to yield spread between maturities j and k.

Yet, empirical evidence suggests that β in equation 2.13 is statistically less than one.[14] For short maturities it is a common finding that β in this equation is not sta-

[12] The derivation is provided in the mathematical appendix 4.2.4.[47] Please note that the equation 2.11 is the generalized form of equation 2.4. In order to follow the regression equations presented in Froot (1989) we use the same notation as the ones presented in his paper. In this equation $\left(I_t^j\right)^j$ and $\left(I_{t+j}^{(k-j)}\right)^{(k-j)}$ represent two consecutive short term bonds in time t held for j and $k-j$ periods respectively. The term on the right hand side of the equation represents long term bond bought at time t and held until $t+k$.

[13] The test of expectations hypothesis is often performed with forward premium instead of yield spread. However, these tests are equvalent because it holds that $f_t^{(j,k)} - {i_t}^j = \frac{k}{k-j}\left(i_t^k - i_t^j\right) = fp_t^{(j,k)}$. The derivation is provided in the mathematical appendix. The null hypothesis is that $\alpha = 0$, $\beta = 1$, and the residual κ_{t+j} is purely random term.

[14] Once the results are adjusted in terms of equation 2.13. See Campbell(1995), Fuhrer(1996), Froot(1989). Similar results were obtained in foreign exchange markets Frankel and Froot(1987) [29] or commodity markets Fama and French(1989). [32]

tistically different from zero, which implies that forward premium does not help to forecast future changes in short–term interest rates. Sometimes even negative coefficients are reported indicating that interest rates on average move in the opposite direction to that predicted by movements in the slope of the yield curve.

A finding that β equates zero in 2.13 has two contradictory interpretations. On one hand, it is consistent with a model where all changes in the slope of the yield curve reflect changes in risk. This view rejects the expectations hypothesis and states that expected interest rate changes are uncorrelated with the slope of the yield curve.[32] The finding of Mankiw and Miron (1986) is supported by this view. They state that if interest rates would follow a random walk the expected interest rate changes would be zero. [45]

On the other hand, β in equation 2.13 equal to zero could be approving the expectations theory in which all movements in the slope of the yield curve reflect changes in expected future rates. This represents the perspective that an increase in the yield spread infers the expectation of future rate increases which on average are not realized ex–post.[32] In fact, investors would improve their forecasts if they would reduce their expectations of future interest rate changes toward zero.[15][7]

[15] But this need not to reject the rational expectations hypothesis. Learning about "peso problem" can also be consistent with a repeated tendency to mispredict interest rates. Peso problem might explain the rejection of the Expectations hypothesis of term structure of interest rates in Germany and United Kingdom. [42] The term derives its name from the Mexican peso, which over the past 20 years, has experienced lenghty periods of stability interupted by short periods of extreme turbulence. Peso problem has been advocated in the financial literature to explain historically puzzling high risk premium.[43] Peso problem also refers to appeerent irrationality of investors in certain samples. This occurs when investors expect a certain event to happen (such as depreciation) and they act upon this expectation. However, when this event happens later than they expected, it looks as if the traders were "irrational" for some period of time. It may be due to small sample bias where the expected event falls outside the (small) sample, and then β will be different from 1. The peso problem can be solved by enlarging the whole sample or rolling the sample forward so it contains the event. That is why the peso problem can also be seen as a small sample bias in uncovered interest rate parity test. [47]

There is no doubt that expectations hypothesis fails. As we noted above this could mean that either rational expectations or perfect substituability does not hold, or at worst that both parts of expectations hypothesis does not hold. Fortunately, the expectations hypothesis can be easily modified. While the future short rate $I_{t+j}^{(k-j)}$ is unknown in period t the expectations $I_{t+j}^{(k-j),e}$ are formed. Moreover, since bond prices do fluctuate over time, there exists uncertainty regarding the return from holding a long–term bond over the next period.[16] Aditionally, the amount of uncertainty increases with the maturity of the bond. The term premium $\Theta_t^{(j,k)}$ is introduced as investors require higher return for taking higher amounts of risk. The equation 2.11 can be restated as follows

$$\left(1 + I_t^j\right)^j \left(1 + I_{t+j}^{(k-j)}\right)^{(k-j)} \left(1 + \Theta_t^{(j,k)}\right)^{(k-j)} = \left(1 + I_t^k\right)^k \tag{2.15}$$

After algebraical adjustments we can deduct that the long rate is weighted average of the current short rate and the expected future short rate plus the term premium

$$i_t^k = \frac{j\ i_t^j + (k-j)\ i_{t+j}^{(k-j),e}}{k} + \frac{k-j}{k}\ \theta_t^{(j,k)} \tag{2.16}$$

But investors do not forecast interest rates precisely. In fact, different respondents report different answers, which suggests that if there is a single true expectation, it

[16] Even for default–free bonds there exists some uncertainty.

is measured with error. Let us define expectations error as follows

$$\eta_t^{(j,k)} = i_{t+j}^{(k-j)} - i_{t+j}^{(k-j),e} \quad \Rightarrow \quad i_{t+j}^{(k-j),e} = i_{t+j}^{(k-j)} - \eta_t^{(j,k)} \tag{2.17}$$

where $\eta_t^{(j,k)}$ represents the expectations error, which we get by substracting expected interest rate $i_{t+j}^{(k-j),e}$ from its actual value $i_{t+j}^{(k-j)}$ and $\theta_t^{(j,k)}$ stands for $\log\left(1+\Theta_t^{(j,k)}\right)$.[17]

After incorporating this result back into 2.16 we get[18]

$$i_t^k = \frac{j\, i_t^j + (k-j)\, i_{t+j}^{(k-j)}}{k} - \frac{k-j}{k}\,\eta_t^{(j,k)} + \frac{k-j}{k}\,\theta_t^{(j,k)} \tag{2.18}$$

By substituting this result back to the 2.13, the equation can be reestimated in order to test for deviations from the null hypothesis.[19]

These empirical deviations from expectations hypothesis can be explained by term premium and expectations error that vary systematically with the forward premium over time.[20] Usual empirical tests of expectations hypothesis reveal only joint deviation, that is $\beta = \beta_{TP} + \beta_{EE} - 1$.[32] Froot (1989) came with decomposition of the deviation from the null hypothesis into component attributable to the term premium

[17] [48].

[18] The derivation is provided in mathematical appendix.

[19] Again the expectations hypothesis requires α equal to zero and β equal to one. Moreover, the expectations hypothesis will hold only if κ_{t+j} is random, thus if both $\eta_t^{(j,k)}$ and $\theta_t^{(j,k)}$ are random terms. Note that $\kappa_{t+j} = \eta_t^{(j,k)} + \theta_t^{(j,k)}$. [48]

[20] It may well vary with yield spread as well. If the risk premium were constant, than changes in the slope of the yield curve would forecast changes in the future path of the interest rates. When the risk premia increase, so does the slope of the of the yield curve even though expectations are unchanged.[28]

β_{TP} and the component attributable to the expectations error β_{EE}.[21] The β_{TP} and β_{EE} are defined as follows[22]

$$\beta_{TP} = -\frac{cov\left(\theta_t^{(j,k)}, fp_t^{(j,k)}\right)}{var\left(fp_t^{(j,k)}\right)} \tag{2.19}$$

$$\beta_{EE} = -\frac{cov\left(\eta_t^{(j,k)}, fp_t^{(j,k)}\right)}{var\left(fp_t^{(j,k)}\right)} \tag{2.20}$$

Yet, obtaining separate estimates of β_{TP} and β_{EE} requires direct observations of $\eta_t^{(j,k)}$ and $\theta_t^{(j,k)}$. Froot (1989) used survey data which allow such a direct observation. He found out that expectational error contributes to the failure of expectatios hypothesis, especially for longer maturities.[32]

Fuhrer (1996) explored the posibility that failure to account for changes in monetary policy regime explains much of the failure of the expectations hypothesis.[33] He concetrated on that how the expectational error is caused by monetary policy surprises. Fuhrer(1996) used a VAR model to opreationalize rational expectations with implicit Taylor rule.[23] However, Fuhrer's expression for the monetary policy reaction function was backward looking type of the Taylor rule.[24] Also agents in Fuhrer's model violate the rational expectations hypothesis because they use current period reaction

[21] See [32] and [29]. By inspection, β_{TP} equals zero if there are no systematic prediction errors in the sample, and β_{EE} equals zero if there is no risk premium (weaker statement is that the risk premium is uncorrelated with the forward premium).

[22] [32].

[23] Taylor rules are monetary policy rules that describe how a central bank should adjust its interest rate policy instrument in a systematic manner in response to developments in economic activity and inflation.[49]

[24] See Fuhrer (1996) p. 1191 for further details. The function uses a smoothing factor, lagged value of short–term interest rate and deviations of output from potential.[33]

function parameters to forecast short–term interest rate over the horizon of the long bond. If there is a predictable movement in these parameters, than this information should be used to improve the forecasts of short rates.[25]

Cox, Ingersoll and Ross (1981) re–examine the expectations hypothesis and they show that the theory has five following versions: the unbiased expectation hypothesis, the local version of expectations, the globally equal expected holding–period return modification, the yield to maturity version and return to maturity expectations hypothesis.[26][11] They found that these version are not equivalent and only the local expectations hypothesis is consistent with the equilibrium.[12]

In order to find the role of expectations hypothesis as a major factor influencing the yield curve they developed a dynamic model (CIR model). The CIR model is a single–factor model which bases its analysis on mean–reverting parameter called adjustment coefficient.[27] Individuals are maximizing utility by choosing they optimal level of consumtion in this model. Hovever, the model has a major flaw dispite its

[25] Unfortunately it is not. The agents do not take into account changes in parameters of reaction function and thus they do not expect the monetary policy regime to change.

[26] The unbiased expectations hypothesis states that forward rates are equal to the expected future spot rates. This corresponds with equation 2.5. The globally equal expected holding–period return modification states that no matter how long the maturity of all securities is, the expected total returns should be equal when holding them for the same period of time. The local version of expectations is less extensive than global modification. It declares that total period returns should be equal if starting at present. This means if investor buys two different long–term bonds they will produce the same return in the short term. The yield to maturity deals with periodic returns whereas the return to maturity hypothesis states that holding a 5–period bond will produce the same return as rolling over a 5 consecutive 1–period bonds.

[27] The single–factor models are the most simple models of interest rates. The whole time structure is given only by single factor. Traditionally it is the short–term interest rate. However, in reality such a interest rate does not exist, that is why overnight interest rate is used. Let us suppose that r_t is continuos Markov stochastic process, which is described by stochastic differential equation of following kind: $dr = \mu(r,t)\,dt + \sigma(r,t)\,dw$, where $\mu(r,t)$ is the drift and represents the deterministic part and $\sigma(r,t)$ is the volatility. Nowadys, single–factor models are largely criticized. First of all, the returns on bonds are perfectly correlated with maturity which is inconsistent with empirical findings. Cintalova(2007) showed that two–factor model is able to capture the different shapes of yield curves better. See [8]

mathematical felicity. It works with pre–defined yield curve and does not bring understanding why the yield curve has the particular shape.[28]

2.1.3 The yield curve and the expectations hypothesis

As we mentioned above the yield curve depicts the spot yield $Y_{t,m}$ as a function of time to maturity m. It is the average cost of borrowing between period t and $t+m$. The yield curve reflets spot rates because nonzero cashflows start in current period t. However by combinig bonds with different times to maturity, we can construct assests that pay a fixed income between two future periods, e.g. $t+m$ and $t+m+1$, with no cost in current period t. The cost of borrowing is expressed in a forward rate. We can construct the m–period ahead 1–period forward rate[29]

$$1 + F_{m,t} = \frac{P_{m,t}}{P_{m+1,t}} \tag{2.21}$$

where $P_{m,t} = \frac{V}{(1+Y_{m,t})^m}$ is the price of the m–period bond in time t and the $P_{m+1,t} = \frac{V}{(1+Y_{m+1,t})^{m+1}}$ is the price of the $m+1$–period bond in time t. By substituting 2.1 into equation above we obtain[30]

$$1 + F_{m,t} = \frac{(1+Y_{m+1,t})^{m+1}}{(1+Y_{m,t})^m} \tag{2.22}$$

[28] This model also rules out financial capital.[12]
[29] [47].
[30] The derivation is provided in the mathematical appendix.

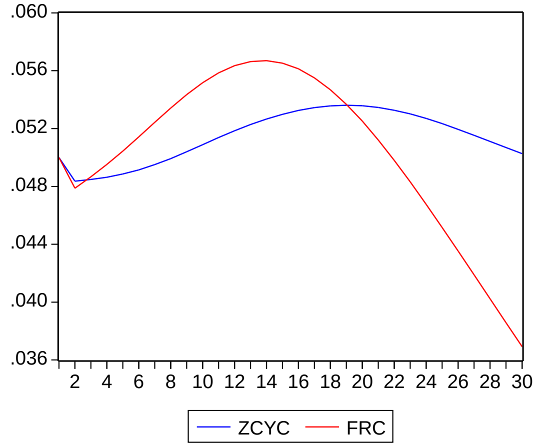

Figure 2.1: The picture depicts the the zero–coupon Yield curve and Forward Rate Curve for May 2006 for United States.

which can be restated as

$$f_{m,t} = y_{m,t} + (m+1)\left(y_{m+1,t} - y_{m,t}\right) \tag{2.23}$$

In simple words the equation above says that the forward rate is equal to spot rate plus $(m+1)$ times slope of the yield curve. If the forward rate is bigger than the spot rate, then yield curve has a positive slope and vice–versa; if the forward rate is smaller than the spot rate, the yield curve has a negative slope.[31] This relationship is depicted in figure 2.1.

[31] The yield curve depicts the spot yield $Y_{t,m}$ as a function of time to maturity m. It is the average cost of borrowing between period t and $t+m$. The forward rate curve depicts the forward rate $F_{t,m}$ as a function of time to maturity m. It is a marginal cost of borrowing between period $t+m$ and $t+m+1$.[47]

Chapter 3

Models and data description

3.1 Binary choice model

Following Estrella and Mishkin (1995), in order to quantify the predictive power of the examined variables with respect to the future recessions, we use a statistical regression technique–the binary choice model. This particular form of the model used, the probit equation, is due to the fact that the predicted variable takes only two possible values. Particularly, the explanatory variable in this model takes on values

1. $R_t = 1$ if the economy is in the recession in period t, or
2. $R_t = 0$ otherwise.

The probabilities (P) of recessions (R) for k–quarters ahead are obtained from a probit equation that takes the following forms[1]

$$P\left(R_{t+k} = 1\right) = F\left(\alpha_0 + \alpha_1 \, yield\, spread_t\right) \tag{3.1}$$

$$P\left(R_{t+k} = 1\right) = F\left(\alpha_0 + \alpha_1 \, inflation_t\right) \tag{3.2}$$

[1] All equations we estimated in the Eviews 5.0 programm.

$$P\left(R_{t+k}=1\right)=F\left(\alpha_0+\alpha_1\; index_t\right) \tag{3.3}$$

$$P\left(R_{t+k}=1\right)=F\left(\alpha_0+\alpha_1\; yield\, spread_t+\alpha_2\; inflation_t\right) \tag{3.4}$$

$$P\left(R_{t+k}=1\right)=F\left(\alpha_0+\alpha_1\; yield\, spread_t+\alpha_2\; 10\,Y\; GBY_t\right) \tag{3.5}$$

where $10Y\,GBY_t$ stands for 10–year governement bond yield, the α coefficients are statistically estimated and where F is the normal cumulative distribution function.[22] A weighted sum of one or more explanatory variables in period t is used to forecast whether k–quarters ahead R_{t+k} will be equal to zero or one. Applying the normal cummulative distribution function F to weighted sum will convert the result into a probability. The probability close to one indicates a intense prediction of recession and vice–versa.

The test of the hypothesis that all coefficients β_i, except the constant, are zero ($H_0 : \beta_1 = \beta_2 = \beta_3 = ... = \beta_K = 0$) is one of the model quality tests. This hypothesis can by tested by likelihood ratio $\lambda = \frac{L_C}{L_U}$, where L_C denotes likelihood function of constrained model and L_U likelihood function of unconstrained model. If tested hypothesis satisfies, then $-2ln\lambda$ is asymptotic χ–squared distributed variable with K degrees of freedom.

The principal measure is a pseudo–R^2 which is a simple measure of goodness of fit that corresponds intuitively to the wide used coefficient of determination - R^2 - in a standard linear regression.

Following measures of goodness of fit were used

$$R_E^2 = 1 - \left(\frac{L_U}{L_C}\right)^{-\frac{2}{n}L_C} \qquad Estrella(1995) \tag{3.6}$$

$$R_{CU_1}^2 = 1 - \left(\frac{L_C}{L_U}\right)^{\frac{2}{n}} \qquad Cragg - Uhler\ (1970)^2 \tag{3.7}$$

$$R_{CU_2}^2 = \frac{1 - \left(\frac{L_C}{L_U}\right)^{\frac{2}{n}}}{1 - L_C^{\frac{2}{n}}} \qquad Cragg - Uhler\ (1970)^3 \tag{3.8}$$

$$R_{VZ}^2 = \frac{\ln L_U - \ln L_C}{2\left(\ln L_U - \ln L_C\right) + n} \frac{2 \ln L_C - n}{2 \ln L_C} \qquad Veall - Zimmermann\ (1992)^4 \tag{3.9}$$

However, all measures produced very similar results therefore only results from 3.6, originally proposed by Estrella (1995), are reported in the appropriate sections. For the in–sample results the pseudo–R^2 takes on values between zero and one. A value that is close to zero indicates that the variable or variables in the model have little explanatory power while value close to one indicates very close fit. Yet, for the out–of–sample results there is no certainty that the values of pseudo–R^2 will lie in the inteval between zero and one, which is also the case in the standard linear regression. But still, the values of pseudo–R^2 for out–of–sample results are useful as simple measure of fit.

As in the case of linear regression, the psuedo–R^2 is not sufficient for testing statistical hypothesis. Therefore two additional measures are presented in tables that are associated with valid statistical tests. First there are standard errors, which are

[2] [41]
[3] [41]
[4] [41]

reported under the coefficient values. Second, we also report * and ** whether the variable is significant at 5 percent and 1 percent levels respectively.

3.2 Linear models

Following Haubrich and Dombrosky (1996) we used following forecasting equations. We use the yield spread in–sample and out–of–sample in basic OLS regression

$$\frac{RGDP_{t+k} - RGDP_t}{RGDP_t} = \alpha + \beta\, yield\ spread_t \tag{3.10}$$

We also used the same model including different variables

$$\frac{RGDP_{t+k} - RGDP_t}{RGDP_t} = \alpha + \beta\, X_t \tag{3.11}$$

We tested OLS models including yield spread and different variables

$$\frac{RGDP_{t+k} - RGDP_t}{RGDP_t} = \alpha + \beta\, yield\ spread_t + \gamma X_t \tag{3.12}$$

and finally we used model with yield spread and lagged values of GDP growth

$$y_t^k = \alpha_0 + \beta_1 aysq_t + \beta_2 y_{t-1} + \beta_3 y_{t-2} + \beta_3 y_{t-3} + \beta_4 y_{t-4} \tag{3.13}$$

where $y_t^k = \frac{RGDP_{t+k} - RGDP_t}{RGDP_t}$ represents measure of annualized real GDP growth k–quarters ahead, $aysq_t$ stands for yield spread in time t and y_{t-i} denotes the lagged GDP growth starting in quarter $t - i$. The equation 3.10 is designed to predict GDP growth k–quarters ahead. Consequently, it is compared with the forecasting ability of different variables which includes inflation, stock price indices of corresponding countries, 10–year government bond yield and 3–month treasury bill yield.

The next two models used were models following Kim and Hamilton (2002).[5] Firstly, we re–examined the prediciting ability of the yield spread in forecasting year–over–year real GDP growth using following forecasting equation

$$\frac{(y_{t+k} + y_{t+k-1} + y_{t+k-2} + y_{t+k-3})}{4} = \alpha_0 + \beta_1 aysq_t + \epsilon_t \tag{3.14}$$

where y_{t+k} is quarterly real GDP growth.

Secondly, we decomposed the yield spread, in order to better understand its forecasting contribution, into an expectations effect (EE) and term premium effect (TP), to see which mechanism accounts for the historical correlation. With respect to Kim and Hamilton (2002) we estimated following equation

$$y_t^k = \alpha + \beta \left(\frac{1}{n} \sum_{i=0}^{n-1} i_{t+i}^1 - i_t^1 \right) + \gamma \left(i_t^n - \frac{1}{n} \sum_{i=0}^{n-1} i_{t+i}^1 \right) + \epsilon_t \tag{3.15}$$

[5] Many various studies already adressed this problem, including Estrella and Hardouvelis (1991), Haubrich and Dombrosky (1996) and Estrella and Mishkin (1997).

where $\left(\frac{1}{n}\sum_{i=0}^{n-1} i^1_{t+i} - i^1_t\right)$ represents the expectations effect and $\left(i^n_t - \frac{1}{n}\sum_{i=0}^{n-1} i^1_{t+i}\right)$ stands for term premium effect.[6] This equation was estimated using Instrumental Variables method with constant, long–term interest rate i^n_t and short–term interest rate i^1_t as instruments. We performed the Wald test in order to investigate whether the coefficients of future expected change in short–term interest rates is equal to that of the term premium. The test statistics is χ^2 distributed with one degree of freedom and stars * and ** represent 5% and 1% significance levels in two tailed test respectively.

The forecasting horizon, k, varies from one to eight or four to twelve quarters ahead depending on regression equation used. The overlapping of forecasting horizons creates special econometric problem which was already adressed by Hansen and Hodrick (1980). [21] The data behave in such a way that the overlapping of forecating horizons creates a moving average error term of order $k-1$, where k denotes forecasting horizon. The moving average does not affect the consistency of the OLS regression coefficients but does affect the consistency of the OLS standard errors. But to draw the correct conslusions the standard errors have to be adjusted. We used the Newey and West (1987) correction method for standard errors. It may happen that the non–overlapping data will have autocorrelated errors, we allow for a moving average error term longer than $k-1$. However the observation of the values of Newey and West corrected residuals were not sensitive to the length of choosen lag.

[6] The derivation is shown in mathematical appendix.

3.2.1 Variables and data used

The primary focus of this thesis is to test simple financial variables whether they may be useful predictors of future recession. The variables examined are the yield spread, inflation[7], stock indices of corrensponding countries, 3–month Treasury bond yield and 10–year treasury bond yield.

The stock indices were chosen with respect to corresponding countries.[8] We have examined whether the quarter–to–quarter growth in stock prices is able to predict the probability of recesion.

For selection of spread, different yields can be used. However, choosing a particular spread is no trivial matter. Among the ten most closely watched interest rates, 45 possible combinations do exist.[9] When forecasting real activity, in contrast, the best results are obtained empirically by taking the difference between two treasury yields whose maturities are far apart.[19] At the long end of the yield curve, the evident choice seems to be 10–year government bond, which is also the longest maturity available in most countries over a long sample period on a consistent basis. At the short end, fewer choices can be made. An overnight rate is the extreme of maturity spectrum.[8] Yet, background research in connection with Estrella and Mishkin (1998) suggested that the 3–month treasury bond yield, when used with combina-

[7] By inflation we mean q–o–q growth in Harmonized Consumer Price Index (HCPI).

[8] FTSE 100 for United Kingdom, DAX for Germany, CAC 40 for France, BUX for Hungary, WIG 20 for Poland, PX for Czech Republic and SAX for Slovankia.

[9] The ten most watched interest rates are overnight rate, three–month, six–month, and one–, two–, three–, five–, seven–, ten– and thirty–year treasury rates. [19] If there are n yields, there are $\frac{n}{2(n-1)}$ spreads. This is basic formula for combinations.

tion with 10–year treasury rate, provides reasonable combination of robustness and accuracy over the long time periods. In the end, most term spreads are highly correlated and provide similar information about the real economy, so the particular choices with regard to maturity amount mainly to fine tunnig rather than to reversal of results.[10][19]

Together with the yield spread we have estimated also combinations of yield spread and each one of the other variables. The results are sumarised in chapter 4.

Another important aspect in consideration was selection of time period. Some variables, such as interest rates and stock prices were available even on intra–day basis. On the other hand the data on GDP were available only on quarterly basis. To place all variables on equal footing, all variables were adjusted to quarterly basis.[11]

In this thesis following countries were examined: United Kingdom, Germany, France, Slovak Republic, Czech Republic, Hungary and Poland. The time period varies from country to country depending on the availability of the data.

In order to forecast future recessions we need to define what we actually mean by recession. For our real GDP growth data we had created a trend line by using

[10]The catch is that a benchmark that works for one spread may not be working for another. The spread between 10–year governemnt bond yield and 3–month treasury bill inverts later than e.g. if two–year treasury bond yield was used instead.

[11]The data were adjusted by taking the average of three consecutive average monthly figures. This process should smoothen the anomalous rates that appaer at the turn of each month. A priori there is no presumption that the GDP should correlate better with a particular date's spread.[40] We decided to convert all data to quarterly basis, rather than decomposing quarterly GDP to monthly data because of computational difficulties. Also for this reason our sampling period is quarterly.

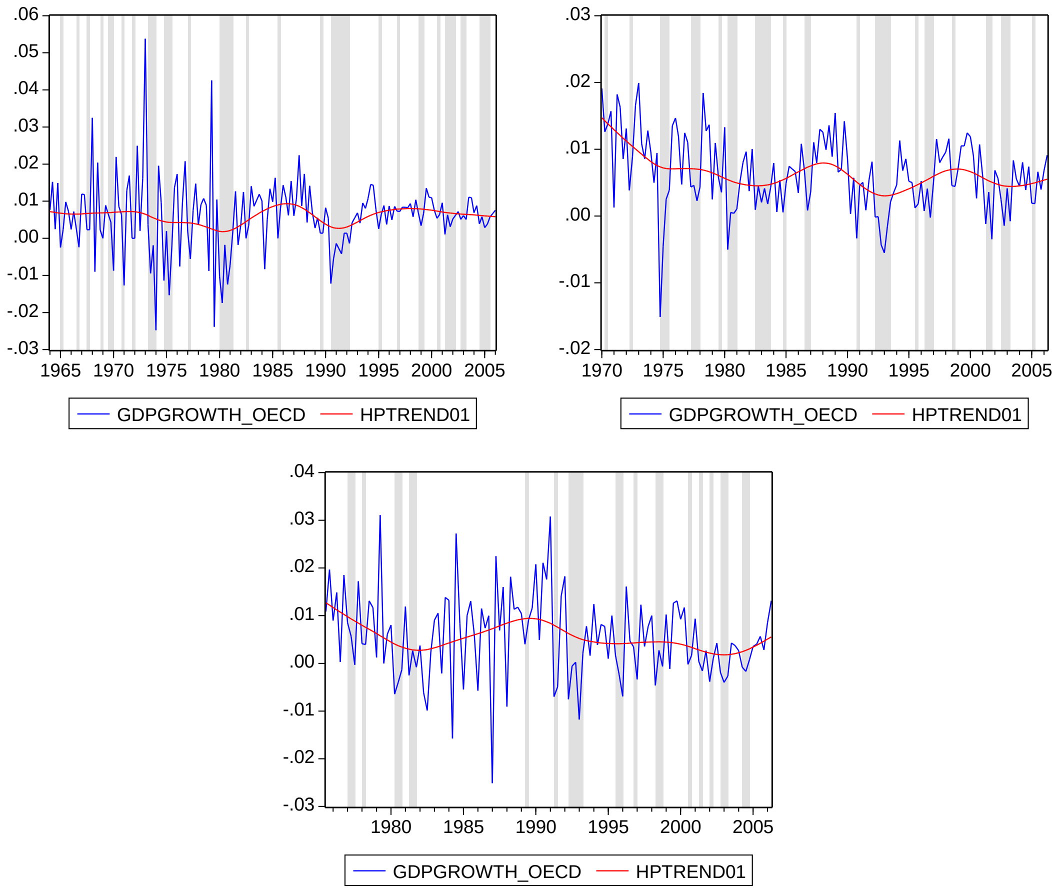

Figure 3.1: The graph depicts the real GDP growth clockwise starting from upper left corner the United Kingdom, France and Germany and the trend line generated by Hodric-Prescott filter. The shaded areas represent at least two consecutive quarters of negative GDP growth which was calculated as real GDP growth less value of trend line.

a Hodrick–Prescott filter. The Hodrick–Prescott filter is a widely used smoothing method to obtain smooth estimate of the long–term trend component of a series.[12]

We had defined recession as at least two consecutive quarters when the real GDP growth was below the trend line. As a result the number 1 was assigned to each quarter meaning that in this quarter the recession had occured. The results of this procedure are depicted in figures 3.1 and 3.2. If the real GDP growth did not fall below this trend line or fell only for one quarter, we attached number 0 to this observation. By doing this we arrived at the vector of ones and zeros which was further used in probit regressions.[13]

All data were acquired from the International Financial Statistics (IFS), OECD, European Central Bank, Eurostat, National Bank of Slovak Republic databases.

[12] The method was first used in a working paper by Hodrick and Prescott to analyze postwar U.S business cycles. Basicaly, the Hodrick–Prescott filter (HPF) is a two–sided linear filter that computes the smoothed series s of y by minimizing the variance of y around s, subject to a penalty that constraints the second difference of s. That is, HPF chooses to minimize $\sum_{t=1}^{T}(y_t - s_t)^2 + \lambda \sum_{t=2}^{T-1}((s_{t+1} - s_t) - (s_t - s_{t-1}))^2$. The penalty parameter λ controls the smoothness of the series σ. The larger the λ, the smoother the σ. As $\lambda = \infty$, s approaches a linear trend. For quarterly data λ equals 1600.[61]

[13] The eviews code is available upon request.

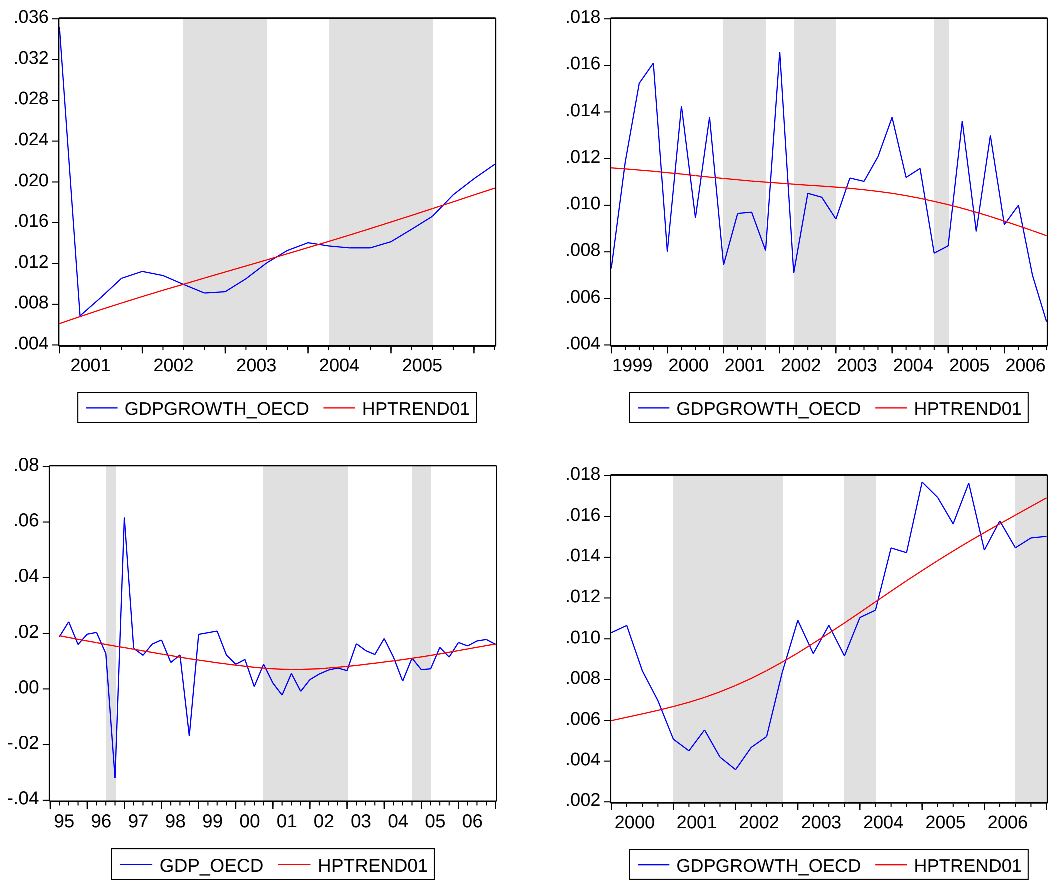

Figure 3.2: The graph depicts the real GDP growth clockwise starting from upper left corner for Slovak Republic, Hungary, Czech Republic and Poland and their trend lines generated by Hodric-Prescott filter. The shaded areas represents at least two consecutive quarters of negative GDP growth which was calculated as real GDP growth less value of trend line.

Chapter 4

Forecasting results

4.1 In–sample results

Does the yield curve predict future GDP growth? First, let us take a look at the data. Figures 4.1 and 4.2 depict the real GDP growth vs. lagged yield spread between 10–year governemnt bond and 3–month treasury bill for all examined countries. We can see that a decline in the GDP growth is usually preceded by an increase in the yield spread. A negative yield spread usually procedes recessions, but not every time. Even from the first look it is evident that the relationship is positive for most countries, that is that positive real GDP growth is associated with a positive lagged yield spread, and vice–versa.[1] By simply plotting the data we get an impression that the yield spread can predict real GDP growth. In order to generate the real GDP predictions we used an in–sample regression to generate each predicted data point. The general strategy of our analysis is the following. The probit equation,

$$P\left(R_{t+k}=1\right)=F\left(\alpha_0+\alpha_1\,X_t\right) \tag{4.1}$$

[1] The exceptions are Slovakia and Hunagry depicted in figure 4.2.

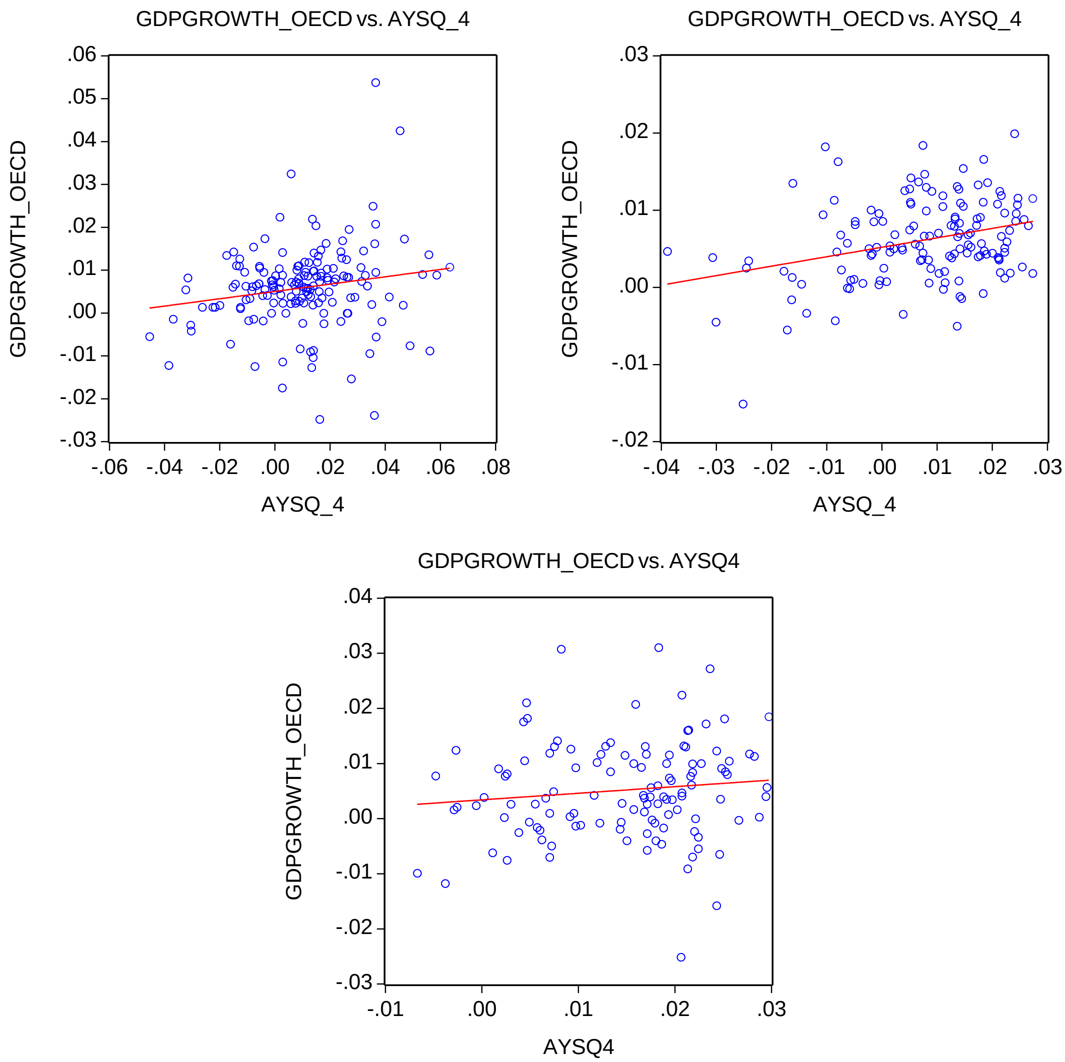

Figure 4.1: The graph depicts the relationship between real GDP growth versus the yield spread lagged four quarters. The countries are depicted in clockwise order starting from upper left corner: United Kingdom, France and Germany. The data were obtained from the IFS database.

where X_t denotes yield spread, inflation and stock indices respectively, is estimated using each variable separately.

Our findings are presented in subsections below.

4.1.1 In–sample results from probit equations

In–sample results for the United Kingdom are based on equations estimated over the entire sample period. The sample period ranges from 1964:Q1 to 2006:Q1 which counts 169 observations. The predictions of fitted values are then compared with the actual recession dates. For each estimation, four types of results are provided: the actual regression coefficients, the standard error, the pseudo–R^2 and indicators of significance at the 5 and 1 percent level denoted by * and ** respectively.

The results for United Kingdom are presented in table 4.7. The q–o–q growth rate of FTSE 100 together with yield spread emerge as significant predictors in the probit model.[2] The significance and fit of the yield spread increases up to the fourth quarter, in which they peak. The coefficients have negative signs which is consistent with the results of Estrella and Mishkin (1995). The negative signs of the coefficients mean that whenever the yield spread is positive the cumulative distribution function (CDF) returns lower probability of recession. However, if the yield spread turns negative, the value of CDF becomes positive, returning higher probability of recession.[3] That means

[2] The q–o–q stands for quarter to quarter growth.

[3] That means the more negative yield spread the higher probability of recession.

that negative yield spread predicts recession. What is more, the yield spread produces consistently strong results up to six quarters ahead, that is why equations are also run containing the spread and other selected variables.[4] From the results provided in the tables 4.27 and 4.28 we can deduct that the inclusion of inflation into the regression model increases its significance up to two quarters. Unfortunately, this is not true for the 10–year government bond yield. Including this variable into the regression does not produce significant results, what is more it reduces the significance of the yield spread. The estimate of q–o–q growth rate of stock index FTSE 100 produce values with correct sign and is significant up to two quarters ahead. The eight–quarter ahead estiamte is also significant with the best model fit having pseudo–R^2 equal to 0.631.[5]

The sampling period for France is a little bit shorter, starting from 1970:Q1 until 2006:Q2, which counts 146 observations. Comparing table 4.8 to the United Kingdom, the results are again significant only for the yield spread and q–o–q growth of stock price index CAC 40. Please note that the yield spread bears the correct sign again and it is significant from 2^{nd} to 4^{th} quarter ahead with most significnat being the 2 quarters ahead result.

Results for Germany are summarized in table 4.9. The data sample counts 124 observations and ranges from 1975:Q3 to 2006:Q2. Similarly to France or United Kingdom, the yield spread is being significant but only up to 2 quarters ahead. The high negative regression coefficients suggest that if the yield on 3–month treasury bill

[4] "The yield spread should never be tested alone." [19]

[5] The pseudo–R^2 values for all stock price indices are considerably higher compared to the rest of the variables

is only slighty bigger than the yield on 10–month treasury bond, this indicates high probability of recession.[6] For Germany, the inflation is also significant up to 2 quarters ahead. However, including additional variables does not make much difference. On top of that, it decreases the signifficance of the yield spread in case of including the inflation into the regression equation.

Further countries examined were Visegrad countries.[7] With respect to these countries we have to say that the sample range is different compared to France or United Kingdom. Namely the sample range is much smaller. The reason for this difference is number of historical events and political decisions that were taken in the past decades. We have to admit that not all results are complete because of computational problems.[8] Some of the results are presented in table 4.10. By looking at these results we have to conclude that yeild spread in equation 4.1 does not produce significant results in case of Czech Republic. However, the opposite is true for q–o–q growth rate of Prague stock exchange index (PSE). It emerges as significant predictor of recession from three to five quarters ahead, but the fit of the model is very weak compared to the United Kingdom or France.

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[6] The constant term for Germany from equation 4.1 is less than 0.5 in absolute terms.

[7] Data samples for Visegrad countries varies from country to country. For Poland from 1995:Q1 to 2007:Q1 (49 observations), Slovak Republic from 2001:Q1 to 2006:Q2 (22 observations), Czech Republic from 2000:Q1 to 2007:Q1 (29 observations) and Hungary from 1999:Q1 to 2006:Q4 (32 observations).

[8] Literally, for Slovak Republic, there was a non–positive likelihood value detected for observation 2004 Q2. Similar problem occured to Hungary. The only country for which we were unable to produce any results from probit equation 4.1 was Poland due to the similar problems. It is due to the fact that these problems occured at the begining of the sample.

4.1.2 In–sample results from linaer models

First linear model that we estimated were equations 3.1 and 3.2. It seems that for most of the countries these models does not produce significant resuts. However, results for Hungary in table 4.15 are surprising. Not only are the coefficients negative, which indicates that the more positive spread the lower real GDP growth, but are also significant up to two quarters ahead. Czech Republic is the only country, for which almost all variables are significant at one percent level up to eight quarters ahead.[9]

In order to compare our results with previous studies we decided to follow recent work of Kim and Hamilton (2002). We have estimated models 3.13, 3.14 and 3.15 for United Kingdom, France and Germany.[10]

Because current and lagged rates of growth of real GDP may be useful for forecasting future, these growth rates are included in the estimated equation 3.13.[11] Tables 4.25, 4.26 and 4.24 show the result for United Kingdom, Germany and France respectively. Again these results are qualitatively similar to previous studies. The values of estimated coefficients on the yield spread are significant at conventional levels. Thus the yield spread provides additional information beyond that contained in current and lagged growth rates.

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[9] Please note that the yield spread coefficient bears the correct sign.

[10] Please note that the equation 3.13 is constructed as to measure the mearginal effect on y–o–y GDP growth for a horizon k–quarters in the future. The same equation was earlier estimated by Estrella and Hardouvelis (1991) or Dotsey (1998). See [21] and [15].

[11] This equation was also estimated in Haubrich and Domborsky (1996) or Dotsey (1998). However, Dotsey (1998) shows that the information contained in the yield spread differs across sample periods and the yield spread does not appear to be statistically significant over some subperiods.[15]

The estimates for equation 3.14 are presented in tables 4.18, 4.20 and 4.19 are consistent with previous studies. They confirm that yield spread makes positive contribution to y–o–y growth rates,in case of France up to six quarters ahead.

The yield spread is determined by the expectations of future interest rates in financial markets and by the term premium. We can explaine the relationship between the yield spread and future economic growth either by in terms of the spread's role as a signal of future expected short rates, the expectations effect, or as a signal of the change in the term premium, the term premium effect. We have estimated the equation 3.15 for United Kingdom, Germany and France in order to investigate which factor contributes more to predicting real GDP growth.[12] Our results are presented in tables 4.21, 4.22 and 4.23. However, they are not in line with results presented in Kim and Hamilton (2002). We find that only for United Kingdom the model is able to produce significant results. Moreover, only the expectations part is significant up to five quarters ahead. The term premium part is not significant, which is however in line with Barnanke's speech before Economic club in New York.[4] The Wald test, whose null hypothesis states that the coefficient on the expected change in short–term interest rates over k periods is equal the coefficient of term premium, was rejected only up to four quarters ahead.

[12] Note that for the rest of the countries we do not have enough observations.

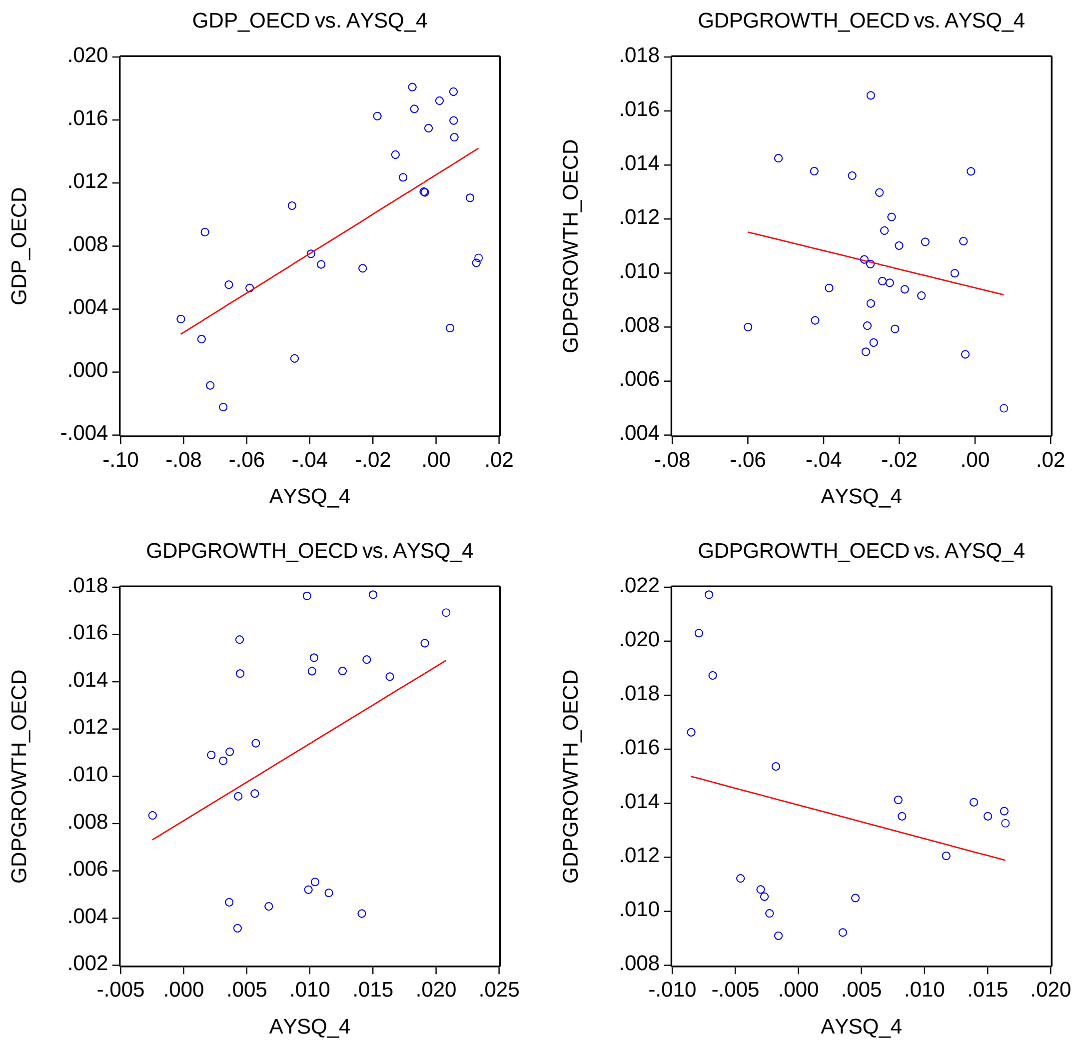

Figure 4.2: The graph depicts the relationship between real GDP growth versus the yield spread lagged four quarters. The countries are depicted clokwise starting from upper left corner: Poland, Hungary, Slovak Republic and Czech Republic. The data were obtained from the IFS database.

4.2 Out–of–sample forecasting

Each estimated variable is based on the regression using data only before the predicted data point. That is, the predicted GDP growth rate for, lets say 2001:Q1 is based on the data sample ranging from 1964:Q1 to 2000:Q4. Hence this regression generates a true forecast because it uses available data to predict future real GDP growth. To evaluate the predictive power of different variables we use the root mean square error (RMSE) and mean absolute error (MAE) criteria.

We run the models through the out–of–sample data set one observation at a time while each time forecasting the target variable k–periods ahead. In our case we do not have specific loss function in order to describe the losses connected with forecast errors, it is always possible to adopt the standard average loss functions, the MAE and RMSE. The mean absolute error average loss function is defined as $MAE = \sum_{t=t_0+k}^{N} |e_t| / (N - k + 1)$. The root mean square error average loss function is defined as $RMSE = \sqrt{\sum_{t=t_0+k}^{N} e_t^2 / (M - h + 1)}$ where N is the total number of observations, the M is the number of observations reserved for the out–of–sample data set and the k is the forecasting horizon. Following the notation above the $N - M$ is the number of data in the in–sample set so we can forecast $M - k + 1$–times when rolling the forecasting models through the out–of–sample data set and with the choosen forecast horizon being k–steps ahead.

In order to evaluate our findings according to the MAE and RMSE criteria we have decided that the forecast with the smallest MAE and RMSE values at the same time

variable	RMSE	MAE	time period
aysq	0.459908	0.442419	1982:Q1–2006:Q2
inflation	0.464332	0.435767	1982:Q1–2006:Q2
dax	0.472766	0.447982	1982:Q1–2006:Q2

Table 4.1: RMSE and MAE for Germany. These results were obtained from the probit model 4.1 using the out–of–the sample forecasting.

in the out–of–sample forecasting will be the superior forecasting method. However, if one of the forecasting methods have the smallest RMSE value while the other forecasting method has the smallest MAE value than we have to determine between the two forecasting models and choose one of the average loss functions as our base choice criterion.

Moreover, we have been able to perform the out–of–sample forecasts only for Germany, United Kingdom and France due to fact that there is not enough observations available for the rest of the countries in our study.

4.2.1 Out–of–sample results

Out–of–sample results from the probit model 4.1 for Germany are summarized in table 4.1. The yield spread emerged as the best out–of–sample forecast variable for Germany. It produces the best forecast by returnig the lowest values of RMSE and

variable	RMSE	MAE	time period
aysq	0.008861	0.006819	1982:Q1–2006:Q2
inflation	0.008876	0.006916	1982:Q1–2006:Q2
dax	0.008847	0.006878	1982:Q1–2006:Q2
trbill	0.008991	0.007075	1982:Q1–2006:Q2
gby	0.008970	0.007072	1982:Q1–2006:Q2

Table 4.2: RMSE and MAE for Germany. These results were obtained from the out–of–sample estimation of the OLS model 3.11 estimating GDP growth 4–quarters ahead.

MAE compared to other variables.[13] The same holds for United Kingdom. From the table 4.3 we can see that both of the loss function values produced by the yield spread forecast are the lowest again. However, different results are produced in the case of France. In table 4.5 the lowest values of RMSE and MAE where obtained by using the stock index CAC 40 as forecasting variable.

For the same countries we have also run the out–of–sample forecast using regression equation 3.11. Tables 4.2, 4.4 and 4.6 show the results. However, the results are not as obvious as in the previous case. While non of the variables has both lowest loss function values we decided to choose RMSE as our refence measure. According to this criterion, the stock index DAX from the table 4.2 is the variable with the lowest RMSE. As for United Kingdom the stock index FTSE 100 has the lowest RMSE as well. Finally, results for France presented in table 4.6 are in favour of yield spread.

[13] The aysq stands for average yield spread on quarterly basis.

variable	RMSE	MAE	time period
aysq	0.490577	0.469699	1972:Q3–2006:Q1
inflation	0.490642	0.484455	1972:Q3–2006:Q1
ftse 100	0.492596	0.483250	1972:Q3–2006:Q1

Table 4.3: RMSE and MAE for United Kingdom. These results were obtained from the probit model 4.1 using the out–of–the sample forecasting.

variable	RMSE	MAE	time period
aysq	0.009557	0.006277	1972:Q3–2006:Q1
inflation	0.009230	0.005935	1972:Q3–2006:Q1
ftse 100	0.005059	0.003607	1972:Q3–2006:Q1
trbill	0.009242	0.006084	1972:Q1–2006:Q1
gby	0.009365	0.006114	1972:Q1–2006:Q1

Table 4.4: RMSE and MAE for United Kingdom. These results were obtained from the out–of–sample estimation of the OLS model 3.11 estimating GDP growth 4–quarters ahead.

variable	RMSE	MAE	time period
aysq	0.487175	0.461587	1977:Q1–2006:Q2
inflation	0.485467	0.460249	1977:Q1–2006:Q2
cac 40	0.475360	0.448116	1977:Q1–2006:Q2

Table 4.5: RMSE and MEA for France. These results were obtained from the probit model 4.1 using the out–of–the sample forecasting.

variable	RMSE	MAE	time period
aysq	0.004510	0.003620	1977:Q1–2006:Q2
inflation	0.004603	0.003713	1977:Q1–2006:Q2
cac 40	0.004543	0.003564	1977:Q1–2006:Q2
trbill	0.004628	0.003694	1977:Q1–2006:Q2
gby	0.004557	0.003717	1977:Q1–2006:Q2

Table 4.6: RMSE and MAE for France. These results were obtained from the out–of–sample estimation of the OLS model 3.11 estimating GDP growth 4–quarters ahead.

Conclusion

In this thesis, we have examined the predictive power of the yield spread and other variables for United Kingdom, Germany, France, Slovak Republic, Czech Republic, Poland and Hungary. We tried to find the answer to the following question: can the yield spread predict recession or real GDP growth? However, answer to this simple question requires a staggering amount of preliminary work. For each country above we estimated four in–sample models, but not all results were produced due to computational difficulties. However, the models produce mixed results. While for the probit models we found little support, the results from the equation 3.13 suggest that the slope of the yield curve has some extra predicitve power over the predictive power contained in the lagged output growth. Surprisingly, the analytical results for Hungary show contraintuitive results. We found that the bigger the 4 quarters lagged difference between 3–month Treasury bill yield and 10–year Treasury bond yield is, the higher real GDP growth. We also decomposed the yield spread into a expectations part and term premium part and found that the term premium has no significant predictive power for future real economic activity.

More important, for out–of–sample predictions the yield spread together with corresponding stock price indices emerges as most accurate variable to predict real GDP growth and economic recession.

Nevertheless, the slope of the yield curve is not indubitable indicator of future economic activity. The recent evidence suggests that inverted yield curve is necessary though not sufficient condition to indicate future recession. Our results are to the bigger extent influenced by our definition of recession, but also encourage further research into the reasons behind the worsening predictive power of the yield curve.

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Appendix

4.2.2 The Expectations Hypothesis

The pure expectations hypothesis actually says that the expected m–period holding return on a 1–period bond must be equal to the expected m–period holding return on a m–period bond.

Suppose that m = 2, then it must hold

$$(1+Y_{1,t})\,(1+Y_{1,t+1}) = (1+Y_{2,t})^2$$

by substitution we get

$$(1+Y_{1,t})\,(1+Y_{1,t+1}) = (1+Y_{2,t})^2$$

$$(1+Y_{1,t})\,(1+Y_{1,t+1})\,(1+Y_{1,t+2}) = (1+Y_{3,t})^3$$

$$\vdots$$

$$(1+Y_{1,t})\,(1+Y_{1,t+1})\ldots(1+Y_{1,t+m-1}) = (1+Y_{m,t})^m$$

by taking logarithms

$$\log\{(1+Y_{1,t})\,(1+Y_{1,t+1})\ldots(1+Y_{1,t+m-1})\} = \log\{(1+Y_{m,t})^m\}$$

we can rewrite this as follows

$$\log{(1+Y_{1,t})} + \ldots + \log{(1+Y_{1,t+m-1})} = m\ \log{(1+Y_{m,t})}$$

this equals to[14]

$$y_{1,t} + \ldots + y_{1,t+m-1} = m\, y_{m,t}$$

which is the same as

$$\frac{y_{1,t} + \ldots + y_{1,t+m-1}}{m} = y_{m,t}$$

So under the expectations hypothesis the long yield is the average of (expected) short yields. After substracting $y_{1,t}$ from both sides of the equation above we yield

$$y_{m,t} - y_{1,t} = \frac{y_{1,t} + \ldots + y_{1,t+m-1}}{m} - \frac{m}{m}\, y_{1,t}$$

after adjusting for the same denominator we yield

$$y_{m,t} - y_{1,t} = \frac{(y_{1,t} - y_{1,t}) + \ldots + (y_{1,t+m-1} - y_{1,t})}{m}$$

From this equation it is evident that under the expectations hypothesis the yield spread is the average of (expected) long run changes in short yields.

$$y_{m,t} - y_{1,t} = \frac{y_{1,t+1} + \ldots + y_{1,t+m-1} - (m-1)\, y_{1,t}}{m}$$

after rearanging we get

$$\frac{m}{m-1}\ (y_{m,t} - y_{1,t}) = \frac{y_{1,t+1} + \ldots + y_{1,t+m-1}}{m-1} - y_{1,t}$$

[14] For small X the following holds: $\log(1+X) \approx X$

$$\frac{y_{1,t+1} + \ldots + y_{1,t+m-1}}{m-1} - y_{1,t} = \alpha + \beta\,\frac{m}{m-1}\,y_{m,t} - y_{1,t} + \eta_t$$

This is the basic regression equation tested in Campbell (1995).[15]

The expectations hypothesis also requires that expected 1-period holding return on a 1-period bond is equal to expected 1-period holding return on a m-period bond. We can write the bond in period t with time to maturity m as

$$P_{m,t} = \frac{V}{(1+Y_{m,t})^m}$$

and the same for period $t+1$ and maturity $m-1$

$$P_{m-1,t+1} = \frac{V}{(1+Y_{m-1,t+1})^{m-1}}$$

by dividing the two expressions above we arrive at the 1-period holding return on an m-period bond

$$\frac{P_{m-1,t+1}}{P_{m,t}} = \frac{(1+Y_{m,t})^m}{(1+Y_{m-1,t+1})^{m-1}}$$

after taking logarithms and rearengement we get

$$\log\left(P_{m-1,t+1}\right) - \log\left(P_{m,t}\right) = m\,\log\left(1+Y_{m,t}\right) - (m-1)\,\log\left(1+Y_{m-1,t+1}\right)$$

$$y_{1,t} = p_{m-1,t+1} - p_{m,t} = m\,y_{m,t} - (m-1)\,y_{m-1,t+1}$$

by rearanging we obtain

$$y_{m,t} - y_{1,t} = (m-1)\,\left(y_{m-1,t+1} - y_{m,t}\right)$$

[15] See 2.7.

This equation basicaly says that the yield spread in current period is equal to the (m–1) times the change in (expected) long yields in the next period. While $y_{m-1,t+1}$ is unknown in period t so the yield spread predicts short run changes in long yields.

Rearangement implies

$$y_{m-1,t+1} - y_{m,t} = \alpha + \beta \, \frac{y_{m,t} - y_{1,t}}{m-1} + \epsilon_t \tag{4.2}$$

Again if the expectations hypothesis holds the β should be equal to unity.[16]

4.2.3 The forward rate

The cost of borrowing is expressed in a forward rate. We can construct the m–period ahead 1–period forward rate[47]

$$1 + F_{m,t} = \frac{V}{-\left(\left(\frac{P_{m+1,t}}{P_{m,t}}\right)(-V)\right)} = \frac{P_{m,t}}{P_{m+1,t}}$$

where $P_{m,t} = \frac{V}{(1+Y_{m,t})^m}$ is the price of the m bond in time t and the $P_{m+1,t} = \frac{V}{(1+Y_{m+1,t})^{m+1}}$ is the price of the $m+1$ period bond in time t.

Now we take the logarithms

$$\log\left(1 + F_{m,t}\right) = (m+1)\log\left(1 + Y_{m+1,t}\right) - m\log\left(1 + Y_{m,t}\right)^{17}$$

---

[16] See Campbell(1995), table 2 row1 (short run changes in long yields).[6]

[17] For small X the following holds: $\log(1 + X) \approx X$

The forward rate at time t is the expected one–period spot rate at time t.

$$f_{m,t} = (m+1)\, y_{m+1,t} - m y_{m,t}$$

which is the same as

$$f_{m,t} = y_{m,t} + (m+1)\,(y_{m+1,t} - y_{m,t})$$

This means that forward rate is equal to spot rate plus (m+1) times slope of the yield curve.

Forecasting with the yield curve. the slope of the yield curve reflects expected long run changes in short yields. The slope of the yield curve reflects expected short run changes in long yields. The expectations hypothesis is a joint hypothesis. It consists of the perfect substituability hypothesis which states that bonds with different time to maturity have identical expected holding returns over a certain period and rational expectaions hypothesis under which the investors do not make systematic forecast errors with respect to yields which are unknown at time t.

It is clear that the expectations hypothesis fails. That is why the expectations hypothesis have to be decomposed in terms of two legs of the joint hypothesis: the rational expectations and perfect substituability.

Variable: Yield Spread	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
value	-10.78*	-13.88**	-15.44**	-16.57**	-14.11**	-11.01*	-9.18	-5.35
st. error	5.07	5.16	5.26	5.35	5.29	5.22	5.13	5.01
R^2	0.0272	0.0440	0.0532	0.0598	0.0438	0.0272	0.0193	0.0067
Variable: Inflation								
value	2.36	2.06	1.01	0.04	-0.64	-1.32	-2.07	-3.01
st. error	1.83	1.82	1.83	1.85	1.86	1.88	1.90	1.95
R^2	0.0098	0.0075	0.0018	3.14E-06	0.0007	0.0029	0.0071	0.0144
Variable: FTSE 100								
value	-4.73*	-4.48*	-2.83	-1.66	-0.19	0.71	-1.95	-5.06*
st. error	2.063	2.04	2.00	2.00	2.04	2.07	2.06	2.29
R^2	0.624	0.622	0.609	0.604	0.602	0.604	0.609	0.631

Table 4.7: Regression results for United Kingdom. These results were obtained from model $P(R_{t+k} = 1) = F(\alpha_0 + \alpha_1 X_t)$ where X_t denotes yield spread, inflation and stock index FTSE 100 respectively. The k denotes number of leads – quarters ahead. The variables were obtained from International Financial Statistics and OECD databases and were adjusted to quarterly basis by taking the average of the 3 consecutive monthly average annualized figures. The model was estimated by Maximum Likelihood – Binary Probit (Quadratic Hill Climbing) method. The R^2 is the coefficient of determination and measures goodness of fit and stars * and ** represent 5% and 1% significance levels respectively.

Variable: Yield Spread	**k=1**	**k=2**	**k=3**	**k=4**	**k=5**	**k=6**	**k=7**	**k=8**
value	-9.99	-16.75*	-16.41*	-17.37*	-12.73	-2.65	6.16	14.71
st. error	7.96	8.10	8.02	8.05	8.00	8.02	8.26	8.64
R^2	0.0109	0.0300	0.0291	0.0325	0.0174	0.0007	0.0038	0.0207
Variable: Inflation								
value	3.69	4.34	4.56	4.32	3.85	3.00	2.61	2.72
st. error	2.54	2.55	2.56	2.56	2.57	2.57	2.58	2.58
R^2	0.0146	0.0200	0.0219	0.0195	0.0154	0.0093	0.0070	0.0076
Variable: CAC 40								
value	-4.13*	-2.37	-1.66	-1.34	0.07	-0.63	0.81	0.06
st. error	2.05	2.02	2.04	2.08	2.11	2.08	2.12	2.08
R^2	0.53	0.51	0.51	0.51	0.51	0.51	0.51	0.51

Table 4.8: Regression results for France. These results were obtained from model $P\left(R_{t+k}=1\right)=F\left(\alpha_0+\alpha_1\,X_t\right)$ where X_t denotes yield spread, inflation and stock index CAC 40 respectively. The k denotes number of leads – quarters ahead. The variables were obtained from International Financial Statistics database and were adjusted to quarterly basis by taking the average of the 3 consecutive monthly average annualized figures. The model was estimated by Maximum Likelihood – Binary Probit (Quadratic Hill Climbing) method. The R^2 is the coefficient of determination and measures goodness of fit and stars * and ** represent 5% and 1% significance levels respectively.

Variable: Yield Spread	**k=1**	**k=2**	**k=3**	**k=4**	**k=5**	**k=6**	**k=7**	**k=8**
value	-60.19**	-37.54**	-25.14	-15.13	-6.91	-0.35	-2.05	-5.44
st. error	15.04	14.02	13.72	13.50	13.45	13.58	13.55	13.59
R^2	0.1392	0.0591	0.0273	0.0101	0.0021	5.59E-06	0.0001	0.0012
Variable: Inflation								
value	15.09*	15.39*	17.14	13.59	9.62	6.18	1.90	-1.94
st. error	6.96	6.99	7.03	6.94	6.87	6.84	6.86	6.91
R^2	0.0386	0.0399	0.0491	0.0314	0.0159	0.0065	0.0006	0.0006
Variable: DAX								
value	-2.15	-2.99*	-1.40	0.03	0.33	-1.21	-4.44*	-2.42
st. error	1.40	1.41	1.38	1.43	1.42	1.39	1.56	1.43
R^2	0.259	0.285	0.268	0.260	0.270	0.285	0.349	0.295

Table 4.9: Regression results for Germany. These results were obtained from model $P(R_{t+k} = 1) = F(\alpha_0 + \alpha_1 X_t)$ where X_t denotes yield spread, inflation and stock index DAX respectively. The k denotes number of leads – quarters ahead. The variables were obtained from International Financial Statistics database and were adjusted to quarterly basis by taking the average of the 3 consecutive monthly average annualized figures. The model was estimated by Maximum Likelihood – Binary Probit (Quadratic Hill Climbing) method. The R^2 is the coefficient of determination and measures goodness of fit and stars * and ** represent 5% and 1% significance levels respectively.

Variable: Yield Spread	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
value	-24.58	-28.73	-58.45	-46.59	-33.44	-56.95	-51.97	-12.02
st. error	43.48	43.64	46.32	46.62	45.00	47.42	48.35	46.91
R^2	0.0019	0.0026	0.0101	0.0062	0.0033	0.0090	0.0072	0.0003
Variable: Inflation								
value	18.24	26.64	26.51	25.55	15.73	11.18	17.96	34.26
st. error	16.09	16.58	16.35	16.63	16.14	15.87	16.16	17.91
R^2	0.0079	0.0164	0.0165	0.0149	0.0058	0.0029	0.0076	0.0254
Variable: PSE								
value	-3.11	-4.87	-7.35*	-7.93*	-13.58*	-5.09	-1.38	0.11
st. error	2.90	3.15	3.65	3.71	5.17	2.92	2.87	2.88
R^2	0.044	0.043	0.051	0.045	0.096	0.028	0.010	0.009

Table 4.10: Regression results for Czech Republic. These results were obtained from model $P(R_{t+k}=1) = F(\alpha_0 + \alpha_1\, X_t)$ where X_t denotes yield spread, inflation and stock index PSE respectively. The k denotes number of leads – quarters ahead. The variables were obtained from International Financial Statistics database and were adjusted to quarterly basis by taking the average of the 3 consecutive monthly average annualized figures. The model was estimated by Maximum Likelihood – Binary Probit (Quadratic Hill Climbing) method. The R^2 is the coefficient of determination and measures goodness of fit and stars * and ** represent 5% and 1% significance levels respectively.

4.2.4 Perfect substituability

Bonds of different maturities are perfectly substituable, hence two consecutive short–term bonds have the same k–period holding return as one corresponding long–term bond

$$\left(1+I_t^j\right)^j \left(1+I_{t+j}^{(k-j)}\right)^{(k-j)} = \left(1+I_t^k\right)^k$$

after taking logarithms we get

$$j\ \log\left(1+I_t^j\right) + (k-j)\ \log\left(1+I_{t+j}^{(k-j)}\right) = k\ \log\left(1+I_t^k\right)$$

this is the same as

$$j\ i_t^j + (k-j)\ i_{t+j}^{(k-j)} = k\ i_t^k$$

by rearanging we get that long–term rate is weighted average of the two consecutive short term rates

$$i_t^k = \frac{j\ i_t^j + (k-j)\ i_{t+j}^{(k-j)}}{k}$$

substracting i_t^j from both sides we get

$$i_t^k - i_t^j = \frac{j\ i_t^j + (k-j)\ i_{t+j}^{(k-j)}}{k} - \frac{k}{k}\ i_t^j = \frac{(j-k)\ i_t^j + (k-j)\ i_{t+j}^{(k-j)}}{k} =$$

$$= \frac{k-j}{k}\ \left(i_{t+j}^{(k-j)} - i_t^j\right)$$

This equation states that the current spread between long and short yield is proportional to the expected difference between two consecutive short yields. Further-more the yield spread predicts the interest rate change[18]

$$i_{t+j}^{(k-j)} - i_t^j = \alpha + \beta \, \frac{k}{k-j} \, \left(i_t^k - i_t^j\right) + \kappa_{t+j}$$

This regression equation is used to test the expectations hypothesis under which $\alpha = 0$ and $\beta = 1$. By generalizing this equation we get regression equation adjusted for coupon–bearing bonds.[19]

$$i_{t+j}^{(k-j)} - i_t^j = \alpha + \beta \, \frac{D_k}{D_k - Dj} \, \left(i_t^k - i_t^j\right) + \kappa_{t+j}$$

The test for expectations hypothesis is often performed with forward premium instead of yield spread. These test are equivalent because

$$f_t^{(j,k)} - i_t^j = \frac{D_k}{D_k - Dj} \, \left(i_t^k - i_t^j\right)$$

[18] [48]

[19] For zero–coupon bonds: D_k = k and D_j = j, where D_K stands for duration of the bond.[32]

Variable: Yield Spread	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
value	0.092	0.128**	0.133**	0.122**	0.060	0.012	-0.005	-0.025
st. error	0.050	0.042	0.043	0.042	0.049	0.042	0.040	0.038
Variable: Inflation								
value	-0.006	-0.007	-0.005	-0.0004	0.002	0.004	0.005	0.004
st. error	0.016	0.015	0.014	0.013	0.013	0.013	0.012	0.012
Variable: CAC 40								
value	0.013	0.010	0.009	0.011	-0.002	0.011*	0.006	0.004
st. error	0.009	0.006	0.008	0.005	0.008	0.005	0.007	0.009
Variable: GBY								
value	-0.0004*	-0.0004*	-0.0004*	-0.0004*	-0.0004	-0.0004	-0.0004	-0.0004
st. error	0.0002	0.0002	0.0002	0.0002	0.00024	0.00024	0.00024	0.00026
Variable: Tr. Bill								
value	-0.024	-0.030*	-0.031*	-0.027*	-0.015	-0.007	-0.002	0.001
st. error	0.015	0.015	0.014	0.013	0.012	0.012	0.013	0.013

Table 4.11: Regression results for France. These results were obtained from model $\frac{RGDP_{t+k}-RGDP_t}{RGDP_t} = \alpha + \beta\, X_t$ where X_t denotes yield spread, inflation, stock index CAC 40 and 10 – year government bond respectively. The k denotes number of leads – quarters ahead. The standard errors are Newey and West (1987) corrected standard errors that take into account the moving average created by the overlaping of horizons as well as conditional heteroskedasticity. The variables were obtained from International Financial Statistics database and were adjusted to quarterly basis by taking the average of the 3 consecutive monthly average annualized figures. The model was estimated by Maximum Likelihood – Binary Probit (Quadratic Hill Climbing) method. The R^2 is the coefficient of determination and measures goodness of fit and stars * and ** represent 5% and 1% significance levels respectively.

Variable: Yield Spread	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
value	0.206*	0.158	0.131	0.119	0.082	0.087	0.130	0.166
st. error	0.088	0.086	0.089	0.088	0.089	0.075	0.075	0.087
Variable: Inflation								
value	-0.013	-0.044	-0.064	-0.048	-0.023	-0.031	-0.027	-0.020
st. error	0.052	0.046	0.048	0.043	0.041	0.039	0.040	0.045
Variable: DAX								
value	0.003	0.011	0.012	0.005	-0.010	0.012	0.007	0.007
st. error	0.010	0.010	0.008	0.008	0.009	0.007	0.007	0.007
Variable: GBY								
value	0.028	-8.22E-05	-0.0126	-0.026	-0.020	-0.012	0.0008	0.010
st. error	0.055	0.052	0.050	0.049	0.047	0.042	0.039	0.041
Variable: Tr. Bill								
value	-0.0122	-0.024	-0.029	-0.036	-0.026	-0.022	-0.021	-0.021
st. error	0.047	0.045	0.043	0.039	0.036	0.032	0.032	0.035

Table 4.12: Regression results for Germany. Regression results for Germany. These results were obtained from model $\frac{RGDP_{t+k}-RGDP_t}{RGDP_t} = \alpha + \beta X_t$ where X_t denotes yield spread, inflation, stock index DAX and 10 – year government bond respectively. The k denotes number of leads – quarters ahead. The standard errors are Newey and West (1987) corrected standard errors that take into account the moving average created by the overlaping of horizons as well as conditional heteroskedasticity. The variables were obtained from International Financial Statistics and ECB databases and were adjusted to quarterly basis by taking the average of the 3 consecutive monthly average annualized figures. The model was estimated by Maximum Likelihood – Binary Probit (Quadratic Hill Climbing) method. The R^2 is the coefficient of determination and measures goodness of fit and stars * and ** represent 5% and 1% significance levels respectively.

Variable: Yield Spread	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
value	-0.177	-0.161	-0.146	-0.140	-0.106	-0.060	0.003	0.123
st. errors	0.107	0.106	0.111	0.130	0.153	0.176	0.182	0.143
Variable: Inflation								
value	-0.044	-0.040	-0.040	-0.035	-0.022	0.015	0.037	0.057
st. errors	0.049	0.054	0.056	0.053	0.043	0.025	0.032	0.037
Variable: SAX								
value	0.019**	0.016**	0.018**	0.017**	0.025**	0.026**	0.023**	0.025**
st. errors	0.005	0.004	0.005	0.006	0.007	0.006	0.005	0.006
Variable: GBY								
value	-0.198**	-0.197**	-0.196**	-0.198**	-0.198**	-0.199**	-0.200**	-0.199**
st. errors	0.034	0.035	0.041	0.049	0.060	0.064	0.064	0.057
Variable: Tr. Bill								
value	-0.210**	-0.219**	-0.239**	-0.260**	-0.314**	-0.383**	-0.423**	-0.447**
st. errors	0.015	0.019	0.032	0.040	0.054	0.074	0.073	0.070

Table 4.13: Regression results for Slovak Republic. These results were obtained from model $\frac{RGDP_{t+k}-RGDP_t}{RGDP_t} = \alpha + \beta X_t$ where X_t denotes yield spread, inflation, stock index SAX and 10 – year government bond respectively. The k denotes number of leads – quarters ahead. The standard errors are Newey and West (1987) corrected standard errors that take into account the moving average created by the overlaping of horizons as well as conditional heteroskedasticity. The variables were obtained from International Financial Statistics database and were adjusted to quarterly basis by taking the average of the 3 consecutive monthly average annualized figures. The model was estimated by Maximum Likelihood – Binary Probit (Quadratic Hill Climbing) method. The R^2 is the coefficient of determination and measures goodness of fit and stars * and ** represent 5% and 1% significance levels respectively.

Variable: Yield Spread	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
value	0.343**	0.348**	0.403**	0.326**	0.291*	0.236*	0.105	0.103
st. error	0.113	0.114	0.108	0.116	0.132	0.130	0.144	0.139
Variable: Inflation								
value	-0.114	-0.253*	-0.405**	-0.563**	-0.744**	-0.921**	-1.084**	-1.241**
st. error	0.065	0.069	0.070	0.065	0.059	0.053	0.046	0.037
Variable: PSE								
value	0.029**	0.031**	0.035**	0.038**	0.035**	0.031**	0.031**	0.024**
st. error	0.006	0.004	0.004	0.004	0.006	0.008	0.009	0.007
Variable: GBY								
value	-0.234**	-0.262**	-0.269**	-0.287**	-0.285**	-0.281**	-0.293**	-0.288**
st. error	0.065	0.053	0.043	0.038	0.038	0.041	0.042	0.042
Variable: Tr. Bill								
value	-0.254**	-0.278**	-0.293**	-0.296**	-0.292**	-0.287**	-0.284**	-0.267**
st. error	0.053	0.039	0.030	0.031	0.038	0.041	0.044	0.045

Table 4.14: Regression results for Czech Republic. These results were obtained from model $\frac{RGDP_{t+k}-RGDP_t}{RGDP_t} = \alpha + \beta X_t$ where X_t denotes yield spread, inflation, stock index PSE and 10 – year government bond respectively. The k denotes number of leads – quarters ahead. The standard errors are Newey and West (1987) corrected standard errors that take into account the moving average created by the overlaping of horizons as well as conditional heteroskedasticity. The variables were obtained from International Financial Statistics and ECB databases and were adjusted to quarterly basis by taking the average of the 3 consecutive monthly average annualized figures. The model was estimated by Maximum Likelihood – Binary Probit (Quadratic Hill Climbing) method. The R^2 is the coefficient of determination and measures goodness of fit and stars * and ** represent 5% and 1% significance levels respectively.

Variable: Yield Spread	**k=1**	**k=2**	**k=3**	**k=4**	**k=5**	**k=6**	**k=7**	**k=8**
value	-0.056*	-0.075**	-0.045	-0.034	-0.057	-0.022	-0.035	0.055*
st. errors	0.022	0.025	0.037	0.040	0.038	0.034	0.036	0.022
Variable: Inflation								
value	0.018	0.020	0.019	0.009	0.017	0.008	0.010	-0.0008
st. errors	0.019	0.021	0.021	0.021	0.022	0.022	0.020	0.015
Variable: BUX								
value	0.011*	-0.002	0.006	-0.003	-0.005	-0.004	-0.009*	0.006
st. errors	0.003	0.003	0.003	0.003	0.003	0.003	0.004	0.007
Variable: GBY								
value	0.061	0.051	0.035	0.023	0.057	0.012	0.012	-0.044
st. errors	0.045	0.041	0.042	0.047	0.050	0.045	0.044	0.032
Variable: Tr. Bill								
value	0.033*	0.039*	0.023	0.016	0.031	0.010	0.014	-0.028
st. errors	0.016	0.018	0.021	0.023	0.023	0.020	0.021	0.014

Table 4.15: Regression results for Hungary. These results were obtained from model $\frac{RGDP_{t+k}-RGDP_t}{RGDP_t} = \alpha + \beta\, X_t$ where X_t denotes yield spread, inflation, stock index BUX and 10 – year government bond respectively. The k denotes number of leads – quarters ahead. The standard errors are Newey and West (1987) corrected standard errors that take into account the moving average created by the overlaping of horizons as well as conditional heteroskedasticity. The variables were obtained from International Financial Statistics database and were adjusted to quarterly basis by taking the average of the 3 consecutive monthly average annualized figures. The model was estimated by Maximum Likelihood – Binary Probit (Quadratic Hill Climbing) method. The R^2 is the coefficient of determination and measures goodness of fit and stars * and ** represent 5% and 1% significance levels respectively.

Variable: Yield Spread	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
value	0.100*	0.115**	0.126**	0.124**	0.133**	0.120**	0.116**	0.108**
st. error	0.038	0.033	0.033	0.034	0.032	0.033	0.030	0.034
Variable: Inflation								
value	0.022	0.020	0.013	0.013	0.006	-0.001	0.004	0.042
st. error	0.015	0.016	0.015	0.015	0.017	0.023	0.029	0.0313
Variable: WIG20								
value	-0.007	0.012	0.003	0.013	0.013*	-0.001	-0.001	0.005
st. error	0.012	0.007	0.007	0.012	0.006	0.008	0.007	0.009
Variable: GBY								
value	-0.150**	-0.180**	-0.189**	-0.175**	-0.165**	-0.145**	-0.100	-0.084
st. error	0.033	0.025	0.017	0.027	0.038	0.046	0.053	0.055
Variable: Tr. Bill								
value	0.007	-0.002	-0.007	-0.007	-0.006	-0.005	-0.001	0.027
st. error	0.018	0.018	0.018	0.018	0.020	0.023	0.026	0.032

Table 4.16: Regression results for Poland. These results were obtained from model $\frac{RGDP_{t+k}-RGDP_t}{RGDP_t} = \alpha + \beta\, X_t$ where X_t denotes yield spread, inflation, stock index WIG20 and 10 – year government bond respectively. The k denotes number of leads – quarters ahead. The standard errors are Newey and West (1987) corrected standard errors that take into account the moving average created by the overlaping of horizons as well as conditional heteroskedasticity. The variables were obtained from International Financial Statistics and OECD databases and were adjusted to quarterly basis by taking the average of the 3 consecutive monthly average annualized figures. The model was estimated by Maximum Likelihood – Binary Probit (Quadratic Hill Climbing) method. The R^2 is the coefficient of determination and measures goodness of fit and stars * and ** represent 5% and 1% significance levels respectively.

Variable: Yield Spread	**k=1**	**k=2**	**k=3**	**k=4**	**k=5**	**k=6**	**k=7**	**k=8**
value	0.065	0.075	0.082	0.085*	0.068	0.040	0.030	-0.004
st. error	0.043	0.046	0.044	0.037	0.038	0.045	0.048	0.052
Variable: Inflation								
value	-0.042	-0.041	-0.033	-0.023	-0.017	-0.011	-0.008	-0.010
st. error	0.017	0.019	0.018	0.015	0.013	0.010	0.009	0.009
Variable: FTSE 100								
value	0.004	0.019	0.006	0.003	0.004	0.004	0.007	0.005
st. error	0.009	0.007	0.008	0.006	0.007	0.008	0.007	0.006
Variable: GBY								
value	-0.053*	-0.056*	-0.048*	-0.036	-0.034	-0.024	-0.010	-0.009
st. error	0.023	0.023	0.022	0.023	0.022	0.020	0.020	0.021
Variable: Tr. Bill								
value	-0.076**	-0.082**	-0.077**	-0.067*	-0.058*	-0.038	-0.021	-0.007
st. error	0.023	0.024	0.025	0.025	0.025	0.023	0.022	0.020

Table 4.17: Regression results for UK. These results were obtained from model $\frac{RGDP_{t+k}-RGDP_t}{RGDP_t} = \alpha + \beta\, X_t$ where X_t denotes yield spread, inflation, stock index FTSE 100 and 10 – year government bond respectively. The k denotes number of leads – quarters ahead. The standard errors are Newey and West (1987) corrected standard errors that take into account the moving average created by the overlaping of horizons as well as conditional heteroskedasticity. The variables were obtained from International Financial Statistics and OECD databases and were adjusted to quarterly basis by taking the average of the 3 consecutive monthly average annualized figures. The model was estimated by Maximum Likelihood – Binary Probit (Quadratic Hill Climbing) method. The R^2 is the coefficient of determination and measures goodness of fit and stars * and ** represent 5% and 1% significance levels respectively.

4.2.5 Decomposition of Expectations Hypothesis

In period t we buy one k–period bond at price ${P_t}^k$ and sell $\frac{{P_t}^k}{{P_t}^j}$ j–period bonds at a price ${P_t}^j$ so that cashflow: $-{P_t}^k + \frac{{P_t}^k}{{P_t}^j}\,{P_t}^j = 0$. My cashflow in period $t+j$ is equal to $(-V)\left(\frac{{P_t}^k}{{P_t}^j}\right)$ and in period $t+k$ is V. The forward rate between period $t+j$ and $t+k$ is

$$\left(1+F_t^{(j,k)}\right)^{(k-j)} = \frac{V}{\left(-(-V)\left(\frac{P_t^k}{P_t^j}\right)\right)}$$

after substitution, the rearangement implies

$$\left(1+F_t^{(j,k)}\right)^{(k-j)} = \frac{\left(1+I_t^k\right)^k}{\left(1+I_t^j\right)^j}$$

taking logarithms and rearanging again

$$f_t^{(j,k)} = \frac{k\,i_t^k - j\,i_t^j}{k-j}$$

now we can add i_t^j to both sides of the equation and rearange

$$fp_t^{(j,k)} = \frac{k\,i_t^k - j\,i_t^j - (k-j)\,i_t^j}{k-j} = \frac{k}{k-j}\left(i_t^k - i_t^j\right)$$

the equation above states that the forward premium on borrowing between period $t+j$ and $t+k$ is proportional to yield spread between maturities j and k. In regression equation 2.13 to test the expectations hypothesis we can use forward premium instead of $\frac{k}{k-j}\left(i_t^k - i_t^j\right)$.

Now imagine that you have the possibility of buying a k–period bond and j–period treasury bill in period t.[20] Instead of investing into k–period bond you decide to purchase a j–period treasury bill that pays return r_t. Then at maturity you decide to invest you proceeds in the $(k-j)$–period treasury bill that pays r_{t+1}.

If the return on k–period bond is bigger than on the j–period treasury bill plus $(k-j)$–period treasury bill the investors will be better off and vice–versa. However the investors don't know at time t what strategy will be better. While the future short rate $I_{t+j}^{(k-j)}$ is unknown hence the investors form some expectations, let us define $I_{t+j}^{(k-j),e}$. If the perfect substituability hypothesis holds than both strategies must be equal otherwise arbitrage opportunity will arrise. What is more, investors expect higher return for taking on higher amounts of risk. Let us define the term premium $\theta_t^{(j,k)}$ such that

$$\left(1+I_t^j\right)^j \ (1+I_{t+j})^{(k-j),e}\left(1+\theta_t^{(j,k)}\right)^{(k-j)}=\left(1+I_t^k\right)^k$$

Again after taking logarithms and rearanging we obtain

$$i_t^k=\frac{j\ i_t^j+(k-j)\ i_{t+j}^{(k-j),e}}{k}+\frac{k-j}{k}\,\theta_t^{(j,k)}$$

From equation above we can clearly see that the long rate is weighted average of the current short rate and the expected future short rate plus a term premium. But investors does not predict the future interest rate precisely. Let us define expectational

[20] where $k>j$

error as follows

$$\eta_t^{(j,k)} = i_{t+j}^{(k-j)} - i_{t+j}^{(k-j),e} \quad \Rightarrow \quad i_{t+j}^{(k-j),e} = i_{t+j}^{(k-j)} - \eta_t^{(j,k)}$$

after incorporating this into 2.15 we get

$$i_t^k = \frac{j\ i_t^j + (k-j)\ i_{t+j}^{(k-j)}}{k} - \frac{k-j}{k}\,\eta_t^{(j,k)} + \frac{k-j}{k}\,\theta_t^{(j,k)}$$

This means that the long rate is actually a weighted average of current and ex–post future short rate plus a disturbance caused by expectational error and a term premium.[21]

After adding i_t^j to the both sides of the equation

$$i_t^k - i_t^j = \frac{k-j}{k}\ \left(i_{t+j}^{(k-j)} - i_t^j\right) - \frac{k-j}{k}\,\eta_t^{(j,k)} + \frac{k-j}{k}\,\theta_t^{(j,k)}$$

$$\Rightarrow \left(i_{t+j}^{(k-j)} - i_t^j\right) = \frac{k}{k-j}\ \left(i_t^k - i^j t\right) + \eta_t^{(j,k)} + \theta_t^{(j,k)}$$

from 2.14 we can write:

$$\left(i_{t+j}^{(k-j)} - i_t^j\right) = \alpha + \beta\, fp_t^{(j,k)} + \omega_{t,j}{}^{22}$$

Empirical deviations from expectations hypothesis can be explained by term premium and expectational error that vary systematically with the forward premium (or yield spread) over time.[47]

$$\beta = 1 + \beta_{TP} + \beta_{EE}$$

[21] Let us note that under expectations hypothesis the $\eta_t^{(j,k)}$ is a random error and $\theta_t^{(j,k)}$ is equal to 0.

[22] The expectations hypothesis requires α=0 and β=1. The expectations hypothesis will however hold only if $\omega_{t,j}$ is a random error thus if both $\eta_t^{(j,k)}$ and $\theta_t^{(j,k)}$ are random errors.

$$\beta_{TP} = -\frac{cov\left({\theta_t}^{j,k}, {fp_t}^{j,k}\right)}{var\left({fp_t}^{j,k}\right)}$$

$$\beta_{EE} = -\frac{cov\left({\eta_t}^{j,k}, {fp_t}^{j,k}\right)}{var\left({fp_t}^{j,k}\right)}$$

Obtaining separate estimates of β_{TP} and β_{EE} requires direct observation of ${\eta_t}^{j,k}$ and ${\theta_t}^{j,k}$. Survey data on ${i_{t+j}}^{(k-j),e}$ allow direct observation. See Froot (1989)[23]

4.2.6 Yield Spread Decomposition

As noted above, obtaining separate estimates of β_{TP} and β_{EE} requires direct observation of ${\eta_t}^{j,k}$ and ${\theta_t}^{j,k}$. Survey data on ${i_{t+j}}^{(k-j),e}$ allow direct observation. However, most of the time the survey data are not available. Kim and Hamilton (2002) provide such a decomposition using ex-post short-term interest rates as a substitute for ex-ante expected rates. Following Kim and Hamilton (2002), let i_t^n and i_t^1 denote long–term and short–term interest rates respectively. Now consider this definition of time–varying risk premium[24]

$$i_t^n = \frac{1}{n}\sum_{j=0}^{n-1} E_t i_{t+j}^1 + TP_t$$

[23] He found out tha expectation hypothesis is rejected for all maturities – Table 1 both term premium and expectational error contribute to the failure of expectational hypothesis – Table 2 it is mostly expectational error for long bonds – Table 2

[24] hamilton

where $E_t i^1_{t+j}$ defines market expectations at time t about short–term interest rate in time $t+j$. By substracting short–term interest rate at time t we get[25]

$$i^n_t - i^1_t = \left(\frac{1}{n} \sum_{j=0}^{n-1} E_t i^1_{t+j} - i^1_t \right) + TP_t$$

Now, the first term on the right–hand side of the equation above is the expectational error and the second term is the time–varying term premium. Thus if fall in the spread predicts upcomming recession it could either be because the temporarily high short–term rate suggests a future recession or a fall in the term premium on long–term bonds relative to short–term bonds suggests a economic recession. The final equation can be rewriten[26]

$$y^k_t = \alpha + \beta \left(\frac{1}{n} \sum_{i=0}^{n-1} i^1_{t+i} - i^1_t \right) + \gamma \left(i^n_t - \frac{1}{n} \sum_{i=0}^{n-1} i^1_{t+i} \right) + \epsilon_t$$

where $\epsilon_t = e_t + (\beta - \gamma)\mu_{t+n}$. Under the rational expectations, the error term ϵ_t should be uncorrelated with any variable known in time t. Thus the last equation mentioned can be estimated by the instrumental variables method (IV) with any variable dated t or earlier as instruments.[27][35]

[25] hamilton
[26] hamilton
[27] For further details see Kim and Hamilton (2002) or Favero et al. (2004).

k (quarters ahead)	α_0	β_1	R^2
4	0.0065** (0.001)	0.088 (0.040)	0.094181
5	0.0064** (0.001)	0.094* (0.042)	0.109257
6	0.0065** (0.001)	0.088* (0.040)	0.094883
7	0.0066** (0.001)	0.077 (0.041)	0.072137
8	0.0068** (0.001)	0.055 (0.045)	0.036980
9	0.0071** (0.001)	0.023 (0.051)	0.006761
10	0.0074** (0.001)	-0.005 (0.056)	0.000322
11	0.0077** (0.001)	-0.028 (0.059)	0.009844
12	0.008** (0.0009)	-0.054 (0.060)	0.035573

Table 4.18: Prediciting the year–over–year real GDP growth using the yield spread of United Kingdom. These results were obtained from model $1/4(y^1_{t+k} + y^1_{t+k-1} + y^1_{t+k-2} + y^1_{t+k-3}) = \alpha_0 + \beta_1 aysq_t + \epsilon_t$ where $aysq_t$ denotes yield spread. The k denotes number of leads–quarters ahead. In the parentheses are Newey and West (1987) heteroskedasticity and autocorelation consistent standard errors corrected with twelve lags. The R^2 is the coefficient of determination and measures goodness of fit and stars * and ** represent 5% and 1% significance levels respectively.

k (quarters ahead)	α_0	β_1	R^2
4	0.0034 (0.001)	0.212** (0.088)	0.121445
5	0.0038* (0.001)	0.180* (0.089)	0.088305
6	0.0042* (0.001)	0.148 (0.084)	0.061543
7	0.0043** (0.0016)	0.140 (0.077)	0.055343
8	0.0041* (0.001)	0.146 (0.078)	0.061450
9	0.003* (0.001)	0.162 (0.086)	0.074810
10	0.003* (0.001)	0.182* (0.094)	0.096083
11	0.003* (0.001)	0.207* (0.098)	0.123931
12	0.003* (0.001)	0.215 (0.100)	0.133670

Table 4.19: Prediciting the year–over–year real GDP growth using the yield spread of Germany. These results were obtained from model $1/4(y_{t+k} + y_{t+k-1} + y_{t+k-2} + y_{t+k-3}) = \alpha_0 + \beta_1 aysq_t + \epsilon_t$ where $aysq_t$ denotes yield spread. The k denotes number of leads–quarters ahead. In the parentheses are Newey and West (1987) heteroskedasticity and autocorelation consistent standard errors corrected with twelve lags. The R^2 is the coefficient of determination and measures goodness of fit and stars * and ** represent 5% and 1% significance levels respectively.

k (quarters ahead)	α_0	β_1	R^2
4	0.0068** (0.0007)	0.128** (0.044)	0.148518
5	0.0067** (0.000753)	0.134** (0.042)	0.164747
6	0.0068** (0.0007)	0.114** (0.042)	0.122201
7	0.0069** (0.0008)	0.081 (0.043)	0.062721
8	0.0072** (0.0008)	0.041 (0.043)	0.016376
9	0.0074** (0.0008)	0.0012 (0.042)	0.00001
10	0.007** (0.0008)	-0.018* (0.041)	0.003325
11	0.007** (0.0008)	-0.022 (0.042)	0.005166
12	0.007** (0.0008)	-0.017 (0.046)	0.003115

Table 4.20: Prediciting the year–over–year real GDP growth using the yield spread of France. These results were obtained from model $1/4(y_{t+k} + y_{t+k-1} + y_{t+k-2} + y_{t+k-3}) = \alpha_0 + \beta_1 aysq_t + \epsilon_t$ where $aysq_t$ denotes yield spread. The k denotes number of leads–quarters ahead. In the parentheses are Newey and West (1987) heteroskedasticity and autocorelation consistent standard errors corrected with twelve lags. The R^2 is the coefficient of determination and measures goodness of fit and stars * and ** represent 5% and 1% significance levels respectively.

k (quarters ahead)	α	β	γ	test: $\chi^2_1, H_0 : \beta = \gamma$
1	0.007**	0.070*	-0.135	7.099**
	(0.001)	(0.033)	(0.077)	
2	0.007**	0.080*	-0.137	7.545**
	(0.001)	(0.034)	(0.082)	
3	0.006**	0.090*	-0.096	6.206*
	(0.001)	(0.036)	(0.076)	
4	0.006**	0.097**	-0.039	3.327*
	(0.001)	(0.030)	(0.076)	
5	0.006**	0.077*	-0.049	2.705
	(0.001)	(0.037)	(0.083)	
6	0.006**	0.046	-0.034	1.436
	(0.001)	(0.047)	(0.080)	
7	0.005**	0.038	0.014	0.141
	(0.001)	(0.052)	(0.084)	
8	0.005**	-0.0008	-0.013	0.037
	(0.001)	(0.057)	(0.094)	

Table 4.21: Prediction of the real GDP growth using decomposed Yield Spread of United Kindom. These results were obtained from model $y_t^k = \alpha + \beta\left(\frac{1}{n}\sum_{i=0}^{n-1} i_{t+i}^1 - i_t^1\right) + \gamma\left(i_t^n - \frac{1}{n}\sum_{i=0}^{n-1} i_{t+i}^1\right) + \epsilon_t$. The equation was estimated with Instrumental Variables method with a constant, long–term and short–term yield as instruments. The k denotes number of leads–quarters ahead. In the parentheses are Newey and West (1987) heteroskedasticity and autocorelation consistent standard errors corrected with twelve lags. In the last column are χ^2 test statistics of one degree of freedom and stars * and ** represent 5% and 1% significance levels in two tailed test respectively. These stars indicates rejection of null hypothesis that the value of estimated coefficients the term premium is equal to the future expected change of short term interest rate. The sample range is 1964:Q1 through 1996:Q1 (shorter due to the cummulative sum of short term interest rate).

k (quarters ahead)	α	β	γ	test: χ^2_1, $H_0 : \beta = \gamma$
1	0.001	0.264*	0.299	0.091
	(0.003)	(0.122)	(0.217)	
2	0.006	0.106	0.016	0.6953
	(0.003)	(0.109)	(0.197)	
3	0.008**	0.024	-0.115	2.341
	(0.003)	(0.096)	(0.164)	
4	0.011**	-0.052	-0.268	5.695*
	(0.003)	(0.110)	(0.184)	
5	0.012**	-0.116	-0.354	4.787*
	(0.004)	(0.139)	(0.234)	
6	0.010*	-0.071	-0.259	3.607
	(0.003)	(0.099)	(0.181)	
7	0.006	0.060	-0.033	0.850
	(0.003)	(0.091)	(0.173)	
8	0.003	0.160	0.156	0.001
	(0.003)	(0.104)	(0.191)	

Table 4.22: Prediction of the real GDP growth using decomposed Yield Spread of Germany. These results were obtained from model $y_t^k = \alpha + \beta \left(\frac{1}{n} \sum_{i=0}^{n-1} i_{t+i}^1 - i_t^1 \right) + \gamma \left(i_t^n - \frac{1}{n} \sum_{i=0}^{n-1} i_{t+i}^1 \right) + \epsilon_t$. The equation was estimated with Instrumental Variables method with a constant, long–term and short–term yield as instruments. The k denotes number of leads–quarters ahead. In the parentheses are Newey and West (1987) heteroskedasticity and autocorelation consistent standard errors corrected with twelve lags. In the last column are χ^2 test statistics of one degree of freedom and stars * and ** represent 5% and 1% significance levels in two tailed test respectively. These stars indicates rejection of null hypothesis that the value of estimated coefficients the term premium is equal to the future expected change of short term interest rate. The sample range is 1975:Q3 through 1996:Q2 (shorter due to the cummulative sum of short term interest rate).

k (quarters ahead)	α	β	γ	test: χ^2_1, $H_0: \beta = \gamma$
1	0.009	0.069	-0.000315	1.82E-05
	(2.457)	(16.731)	(0.298)	
2	0.007	0.073	-6.60E-05	1.79E-05
	(2.606)	(17.745)	(0.316)	
3	0.005	0.078	0.0001	1.85E-05
	(2.716)	(18.493)	(0.329)	
4	0.001	0.086	0.0005	1.92E-05
	(2.930)	(19.948)	(0.355)	
5	0.004	0.058	0.0001	6.63E-06
	(3.405)	(23.182)	(0.413)	
6	0.007	0.030	-0.0001	1.69E-06
	(3.488)	(23.751)	(0.423)	
7	0.006	0.020	-0.0001	7.93E-07
	(3.441)	(23.428)	(0.417)	
8	0.003	0.028	0.0002	1.75E-06
	(3.189)	(21.70)	(0.387)	

Table 4.23: Prediction of the real GDP growth using decomposed Yield Spread of France. These results were obtained from model $y_t^k = \alpha + \beta\left(\frac{1}{n}\sum_{i=0}^{n-1} i_{t+i}^1 - i_t^1\right) + \gamma\left(i_t^n - \frac{1}{n}\sum_{i=0}^{n-1} i_{t+i}^1\right) + \epsilon_t$. The equation was estimated with Instrumental Variables method with a constant, long–term and short–term yield as instruments. The k denotes number of leads–quarters ahead. In the parentheses are Newey and West (1987) heteroskedasticity and autocorelation consistent standard errors corrected with twelve lags. In the last column are χ^2 test statistics of one degree of freedom and stars * and ** represent 5% and 1% significance levels in two tailed test respectively. These stars indicates rejection of null hypothesis that the value of estimated coefficients the term premium is equal to the future expected change of short term interest rate. The sample range is 1964:Q1 through 1996:Q1 (shorter due to the cummulative sum of short term interest rate).

k (quarters ahead)	α_0	β_1	β_2	β_3	β_4	β_5	$\overline{R}^2$
1	0.002**	0.110**	0.250**	0.144*	-0.127	0.112	0.171
	(0.0009)	(0.041)	(0.055)	(0.066)	(0.080)	(0.086)	
2	0.003**	0.158**	0.172**	-0.127	0.084	0.106	0.172
	(0.001)	(0.042)	(0.062)	(0.076)	(0.082)	(0.087)	
3	0.004**	0.170**	-0.112	0.068	0.069	0.060	0.152
	(0.001)	(0.042)	(0.090)	(0.078)	(0.091)	(0.081)	
4	0.004**	0.139**	0.023	0.019	0.033	-0.027	0.098
	(0.001)	(0.042)	(0.094)	(0.078)	(0.083)	(0.089)	
5	0.004**	0.096*	0.0005	-0.029	-0.138	0.204	0.057
	(0.001)	(0.048)	(0.076)	(0.086)	(0.090)	(0.067)	
6	0.004**	0.055	-0.041	-0.198	0.117	0.220	0.053
	(0.001)	(0.047)	(0.088)	(0.096)	(0.076)	(0.095)	
7	0.004**	0.027	-0.216*	0.083	0.170	0.086	0.041
	(0.001)	(0.043)	(0.104)	(0.078)	(0.093)	(0.073)	
8	0.004**	-0.011	0.016	0.124	0.019	0.027	-0.009
	(0.0009)	(0.037)	(0.102)	(0.090)	(0.059)	(0.092)	

Table 4.24: Regression results for France. These results were obtained from model $y_t^k = \alpha_0 + \beta_1 aysq_t + \beta_2 y_{t-1} + \beta_3 y_{t-2} + \beta_3 y_{t-3} + \beta_4 y_{t-4}$ where y_t^k denotes real GDP growth k–quarters ahead, y_{t-i} is quarterly real GDP growth beginning in quarter $t-i$ and $aysq_t$ stands for Yield Spread. The $\overline{R}^2$ is the coefficient of determination and measures goodness of fit and stars * and ** represent 5% and 1% significance levels respectively. In the parentheses are the Newey and West (1987) autocorrelation and heteroscedasticity consistent standard errors. The whole sample ranges from 1971:Q1 through 2004:Q2.

k (quarters ahead)	α_0	β_1	β_2	β_3	β_4	β_5	$\overline{R}^2$
1	0.003*	0.064	0.056	0.176	-0.008	0.021	0.023
	(0.001)	(0.035)	(0.117)	(0.094)	(0.119)	(0.092)	
2	0.003**	0.082*	0.141	-0.025	0.039	0.124	0.039
	(0.001)	(0.039)	(0.094)	(0.121)	(0.091)	(0.083)	
3	0.004**	0.093**	-0.049	0.026	0.148	0.035	0.023
	(0.001)	(0.046)	(0.118)	(0.098)	(0.086)	(0.092)	
4	0.005**	0.074*	0.052	0.136	0.006	-0.171	0.046
	(0.001)	(0.037)	(0.091)	(0.078)	(0.086)	(0.079)	
5	0.005**	0.055	0.130	-0.003	-0.172*	-0.019	0.032
	(0.0008)	(0.039)	(0.087)	(0.084)	(0.082)	(0.087)	
6	0.007**	0.035	-0.003	-0.173*	-0.008	-0.060	0.010
	(0.001)	(0.048)	(0.078)	(0.085)	(0.083)	(0.092)	
7	0.006**	0.034	-0.185*	-0.017	-0.060	0.046	0.011
	(0.0009)	(0.054)	(0.094)	(0.084)	(0.096)	(0.084)	
8	0.007**	-0.011	0.010	-0.069	-0.003	-0.099	-0.017
	(0.001)	(0.052)	(0.093)	(0.092)	(0.083)	(0.111)	

Table 4.25: Regression results for United Kindom. These results were obtained from model $y_t^k = \alpha_0 + \beta_1 aysq_t + \beta_2 y_{t-1} + \beta_3 y_{t-2} + \beta_3 y_{t-3} + \beta_4 y_{t-4}$ where y_t^k denotes real GDP growth k–quarters ahead, y_{t-i} is quarterly real GDP growth beginning in quarter $t-i$ and $aysq_t$ stands for Yield Spread. The $\overline{R}^2$ is the coefficient of determination and measures goodness of fit and stars * and ** represent 5% and 1% significance levels respectively. In the parentheses are the Newey and West (1987) autocorrelation and heteroscedasticity consistent standard errors. The whole sample ranges from 1964:Q1 through 2006:Q2.

k (quarters ahead)	α_0	β_1	β_2	β_3	β_4	β_5	$\overline{R}^2$
1	0.007**	0.083**	0.493**	-0.033	-0.075	-0.015	0.421
	(0.001)	(0.024)	(0.135)	(0.170)	(0.081)	(0.087)	
2	0.010**	0.105**	0.414	-0.145	0.002	-0.138**	0.543
	(0.001)	(0.027)	(0.071)	(0.080)	(0.084)	(0.049)	
3	0.013**	0.125**	-0.031	-0.022	-0.097*	0.007	0.023
	(0.002)	(0.034)	(0.136)	(0.094)	(0.048)	(0.073)	
4	0.015**	0.125**	-0.025	-0.154**	0.056	-0.173*	0.046
	(0.001)	(0.033)	(0.141)	(0.040)	(0.075)	(0.080)	
5	0.016**	0.131*	-0.223	0.002	-0.128	-0.041	0.481
	(0.001)	(0.028)	(0.082)	(0.092)	(0.059)	(0.076)	
6	0.015**	0.125**	-0.271**	-0.172**	-0.006	0.127	0.497
	(0.001)	(0.025)	(0.087)	(0.055)	(0.073)	(0.091)	
7	0.016**	0.125**	-0.388**	-0.056	0.177	-0.033**	0.479
	(0.002)	(0.025)	(0.089)	(0.102)	(0.050)	(0.140)	
8	0.015**	0.115**	-0.438**	0.141	0.002	0.141*	0.483
	(0.002)	(0.026)	(0.110)	(0.074)	(0.082)	(0.071)	

Table 4.26: Regression results for Germany. These results were obtained from model $y_t^k = \alpha_0 + \beta_1 aysq_t + \beta_2 y_{t-1} + \beta_3 y_{t-2} + \beta_3 y_{t-3} + \beta_4 y_{t-4}$ where y_t^k denotes real GDP growth k–quarters ahead, y_{t-i} is quarterly real GDP growth beginning in quarter $t-i$ and $aysq_t$ stands for Yield Spread. The $\overline{R}^2$ is the coefficient of determination and measures goodness of fit and stars * and ** represent 5% and 1% significance levels respectively. In the parentheses are the Newey and West (1987) autocorrelation and heteroscedasticity consistent standard errors. The whole sample ranges from 1975:Q3 through 2006:Q2.

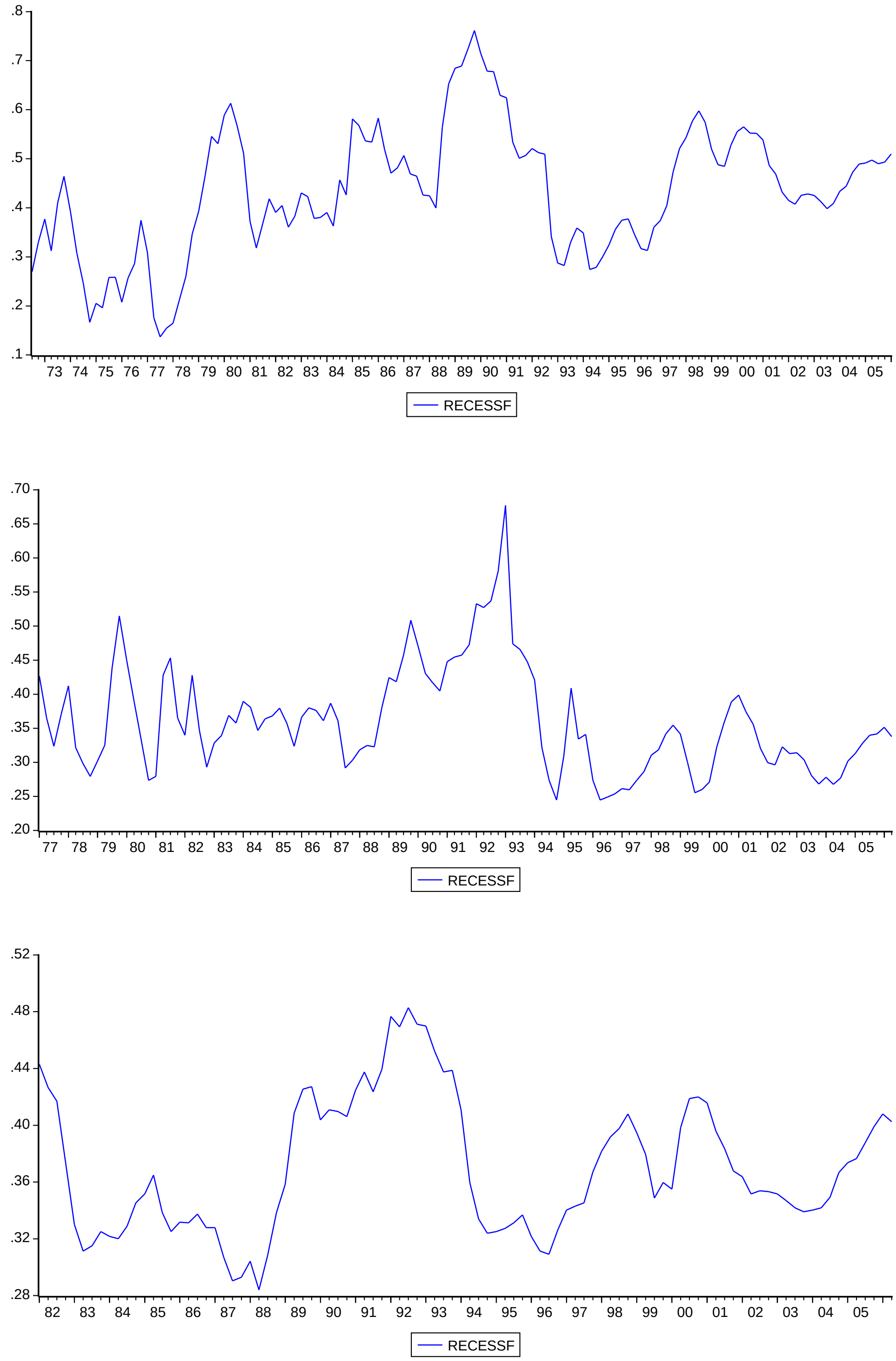

Figure 4.3: The graph depicts out–of–sample forecasted probabilities of recession using 4.1. From the top: United Kingdom, France and Germany.

Variable: Yield Spread	**k=1**	**k=2**	**k=3**	**k=4**	**k=5**	**k=6**	**k=7**	**k=8**
value	-14.12**	-17.22**	-17.76**	-17.96**	-14.65**	-10.81*	-8.31	-3.53
st. error	5.39	5.48	5.53	5.57	5.47	5.38	5.31	5.23
Variable: Inflation								
value	3.99*	4.06*	2.96	1.87	0.77	-0.29	-1.26	-2.65
st. error	1.980434	2.011150	2.017688	2.013299	1.976380	1.966541	1.978691	2.030259
R^2	0.051414	0.068172	0.065862	0.064938	0.044797	0.027417	0.021818	0.017187

Table 4.27: Regression results for United Kingdom. These results were obtained from model $P\left(R_{t+k}=1\right)=F\left(\alpha_0+\alpha_1\ yield\ spread_t+\alpha_2\ inflation_t\right)$ where k denotes number of leads – quarters ahead. The variables were obtained from International Financial Statistics and OECD databases and were adjusted to quarterly basis by taking the average of the 3 consecutive monthly average annualized figures. The model was estimated by Maximum Likelihood – Binary Probit (Quadratic Hill Climbing) method. The R^2 is the coefficient of determination and measures goodness of fit and stars * and ** represent 5% and 1% significance levels respectively.

Variable: Yield Spread	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
value	-11.23*	-14.50**	-15.63**	-16.01**	-12.87*	-8.99	-6.37	-2.07
st. error	5.28	5.37	5.46	5.51	5.45	5.39	5.32	5.23
Variable: GBY								
value	0.97	1.35	0.41	-1.36	-3.15	-5.37	-7.53*	-8.62*
st. error	3.28	3.32	3.35	3.39	3.41	3.46	3.53	3.56
R^2	0.027742	0.045083	0.053335	0.060851	0.049055	0.041956	0.047408	0.043193

Table 4.28: Regression results for United Kingdom. These results were obtained from model $P\left(R_{t+k}=1\right)=F\left(\alpha_0+\alpha_1\ yield\ spread_t+\alpha_2\ 10\,Y\ GBY_t\right)$ where k denotes number of leads – quarters ahead and GBY represents 10 – year government bond yield. The variables were obtained from International Financial Statistics and OECD databases and were adjusted to quarterly basis by taking the average of the 3 consecutive monthly average annualized figures. The model was estimated by Maximum Likelihood – Binary Probit (Quadratic Hill Climbing) method. The R^2 is the coefficient of determination and measures goodness of fit and stars * and ** represent 5% and 1% significance levels respectively.

Variable: Yield Spread	**k=1**	**k=2**	**k=3**	**k=4**	**k=5**	**k=6**	**k=7**	**k=8**
value	-58.32**	-29.38	-12.29	-3.78	2.01	6.23	-0.54	-8.72
st. error	17.15	15.74	15.36	15.18	15.13	15.15	15.00	15.12
Variable: Inflation								
value	1.87	8.67	14.38	12.73	10.08	7.58	1.79	-3.82
st. error	8.39	7.98	7.86	7.75	7.69	7.67	7.59	7.63
R^2	0.139616	0.068515	0.054325	0.031967	0.016044	0.007959	0.000634	0.003318

Table 4.29: Regression results for Germany. These results were obtained from model $P\left(R_{t+k}=1\right)=F\left(\alpha_0+\alpha_1\ yield\ spread_t+\alpha_2\ inflation_t\right)$ where k denotes number of leads – quarters ahead. The variables were obtained from International Financial Statistics database and were adjusted to quarterly basis by taking the average of the 3 consecutive monthly average annualized figures. The model was estimated by Maximum Likelihood – Binary Probit (Quadratic Hill Climbing) method. The R^2 is the coefficient of determination and measures goodness of fit and stars * and ** represent 5% and 1% significance levels respectively.

Variable: Yield Spread	**k=1**	**k=2**	**k=3**	**k=4**	**k=5**	**k=6**	**k=7**	**k=8**
value	-63.83**	-35.50*	-20.69	-10.02	-2.54	2.29	-3.23	-9.55
st. error	16.00	14.73	14.41	14.24	14.20	14.25	14.17	14.32
Variable: GBY								
value	-5.01	3.07	7.14	8.32	7.10	4.47	-2.12	-7.50
st. error	7.05	6.99	7.11	7.19	7.22	7.27	7.38	7.54
R^2	3.59E-06	0.020795	0.034987	0.018892	5.70E-05	0.000135	0.002753	0.008018

Table 4.30: Regression results for Germany. These results were obtained from model $P\left(R_{t+k}=1\right)=F\left(\alpha_0+\alpha_1\ yield\,spread_t+\alpha_2\ 10\,Y\ GBY_t\right)$ where k denotes number of leads – quarters ahead and GBY represents 10 – year government bond yield. The variables were obtained from International Financial Statistics database and were adjusted to quarterly basis by taking the average of the 3 consecutive monthly average annualized figures. The model was estimated by Maximum Likelihood – Binary Probit (Quadratic Hill Climbing) method. The R^2 is the coefficient of determination and measures goodness of fit and stars * and ** represent 5% and 1% significance levels respectively.

Projekt Diplomovej práce

Termín magisterskej skúšky: **Zimný semester 2007/2008**
Autor diplomovej práce: **Jozef Jamriška**
Vedoucí diplomovej práce: **PhDr. Michal Hlaváček, Ph.D.**

Téma: Výnosová krivka a jej schopnost předikovat recesiu

Cieľ práce: Cieľom práce je popísať predikčnú schopnosť výnosovej krivky a niektorých vybraných makroekonomických veličín (GDP), peňažných agregátov (Peňažná báza, M1, M2) a burzových indexov (Dow Jones a S&P 500) na krajinách Veľkej Británie, Českej a Slovenskej Republiky, Maďarska a Poľska. V práci budem hľadať odpoveď na otázku či výnosová krivka môže predikovať recesiu vo vyššie spomenutých krajinách.

V práci bude hľadaná odpoveď na nasledujúce otázky:

- Čo je výnosová krivka?
- Prečo sa zaoberáme výnosovou krivkou?
- Aká je predikčná schopnosť výnosovej krivky a aká je predikčná schopnosť ostatných premenných
- Môže výnosová krivka predikovať recesiu vo vybraných krajinách
- Výnosová krivka a hypotéza racionálnych očakávaní
- Zlyhanie hypotézy očakávaní a jej rozklad.

Osnova:

1. Základné pojmy a metodológia výnosovej krivky
2. Popis dát
3. Popis ekonometrických metód pri odhadovaní predikčnej schopnosti výnosovej krivky
4. Porovnanie jednotlivých výsledkov analýzy
5. Zhodnotenie
6. Záver

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V Prahe dňa ……………

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