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DIPLOMOVÁ PRÁCE

The Applicability of Merton's Credit Risk Model in the Czech Republic

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Prohlašuji, že jsem diplomovou práci vypracoval samostatně a použil pouze uvedené prameny a literaturu.

Hereby I declare that I compiled this thesis independently, using only the listed literature and resources.

V Praze dne 15.1.2007 Martin Peška

Abstrakt

Kreditní riziko je nejdůležitějším rizikem pro finanční instituce na celém světě. Nejdůležitější a zároveň nejhůře měřitelnou složkou kreditního rizika je pravděpodobnost defaultu (nedodržení smluvních závazků) a z ní plynoucího bankrotu firmy.

Ve své diplomové práci „Mertonův model kreditního rizika a jeho použitelnost v České republice" se podrobněji zabývám různými metodami měření kreditního rizika. Hlavním cílem této práce je představit v Evropě méně známý Mertonův model kreditního rizika, který vyvozuje pravděpodobnost defaultu na základě volatility a tržní hodnoty akcií a kriticky ohodnotit jeho použitelnost v České republice. Aby bylo možné tento model správně aplikovat, věnuje se podstatná část této práce teoretickému odvození tohoto modelu, diskusi předpokladů, na kterých model stojí, možnosti jeho vylepšení a celkovým výhodám a nevýhodám modelů založených na vývoji hodnoty firmy v čase. Aplikace tohoto modelu spočívá ve vypočítání pravděpodobností bankrotu pro patnáct nejlikvidnějších nefinančních firem kótovaných na Pražské burze cenných papírů. Výsledky jsou porovnány s dalšími měřítky kreditního rizika, kterými jsou Altmanovo *Z* a Ohlsonovo *O* skóre, rankingy a ratingy přidělené mezinárodními ratingovými agenturami. Výsledky tohoto srovnání naznačují, že tradiční metody měření kreditního rizika založené na finančních výkazech dokáží postihnout skutečnou finanční situaci podniku lépe nežli Mertonův model.

Aby bylo tyto závěry ze srovnávací analýzy možné kvantifikovat a určit vypovídací schopnost jednotlivých modelů, aplikuji „ordered logit" regresi, kdy vysvětlovanou proměnnou jsou Czech Sector Awards rankingy a nezávislými proměnnými jsou jednotlivé vypočítané ukazatele kreditního rizika. Vzorek analyzovaných společností je příliš malý a rankingy jsou příliš špatné odhady „skutečného" kreditního rizika firem než aby mohly být výsledky této analýzy považovány za spolehlivé. Nicméně se výsledky regrese shodují se závěry učiněnými na základě srovnávací analýzy.

Hlavní závěr této diplomové práce je konstatování, že Mertonův model kreditního rizika v podmínkách České republiky s mladým a poměrně málo likvidním akciovým trhem sice obsahuje některé informace o míře kreditního rizika firem, avšak v současné době je nedostatečným ukazatelem pravděpodobnosti defaultu.

Abstract

Credit risk is the most important risk that financial institutions all around the world have to face. Even though the credit risk consists of several components, none are more important and more difficult to measure than the probability of default.

In my diploma thesis "The Applicability of Merton's Credit Risk Model in the Czech Republic" I take a closer look at several methods of measurement of default probability. I start with the traditional accounting-based methods (Altman's *Z* and Ohlson's *O*) and present the methodology of credit ratings. But the main focus of this work lies on the Merton model, which derives the probabilities of default for publicly traded companies mainly from the prices and volatility of equity. I discuss the model's assumptions, derive the key formulas, give step-by-step directions for its actual implementation and discuss thoroughly the model's advantages, limitations, improvements and previous empirical tests of model quality.

Building on this theoretical ground, I compute the Merton-implied probabilities of default for Czech companies that are listed (and actively traded) on the Prague Stock Exchange. I compare the obtained results with the traditional indicators of credit risk, Altman's *Z*- and Ohlson's *O- Scores* with both original and updated coefficients, and with credit ratings from external rating agencies and Czech Sector Awards rankings. Based on these comparisons, I find that the traditional accounting-based measures are better predictors of the "real" situation of the company's credit risk than the Merton model.

I discuss the possibilities to test the quality of the respective credit risk measures and perform an ordered logit regression on the company rankings using these measures of credit risk as explanatory variables. Because of small sample size and lack of dependable ground for model quality assessment, the results of the test are not statistically reliable. Nevertheless, the results obtained from the regression match the conclusions of the qualitative analysis.

The bottom line of this work is that the Merton model can under the conditions of a young and rather less liquid Czech capital market potentially be used as a source of information about the underlying credit risk but these default probabilities are, as for now, an insufficient measure of credit risk and some other models for the assessment of default probability should be used instead of or in addition to the Merton model.

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1. Introduction

All types of financial institutions around the world are exposed to many different types of risk. These risks can be generally categorized as market, operational, liquidity, systemic and credit risk. For banks and other lending institutions around the world, **credit risk** has always been the **most important risk** ever since the first credit had been extended.

In the Czech Republic, credit risk is also the main risk factor for financial stability, especially with respect to the growing share of loans on the banks' total assets.[1] The total volume of loans granted by banks in the Czech Republic has reached by the end of 2005 a total of CZK[2] 1,186 billion, which is about 40% of GDP.[3] Out of this volume, 11.73% were marked as classified loans and 4.3% as non-performing loans.[4] This relatively high share of classified and non-performing loans on the banks' portfolios only underlines the importance of correct measurement and management of credit risk, especially to banks.[5] But before going further into the techniques of credit risk assessment, it is important to look at what credit risk actually is.

Credit risk can be defined as the **risk of loss on a financial or non-financial contract due to the counterparty's failure to meet its obligations on that contract**.[6] Because there are many types of counterparties (from individuals to companies and sovereign governments) and many different types of obligations (from consumer loans, bank loans, bonds to derivatives transactions) credit risk takes on many forms. In this paper I focus on the credit risk associated with companies. To give a better idea about what credit risk on the corporate level actually means, let's consider a bank which is extending credit to a company. From the bank's perspective, credit risk is the risk of not having the loan repaid in full. As it is clear, credit risk has many different elements. The first element is the probability that such event will happen- the probability of default[7]. Another factor is the amount of money lost when such default occurs, because default can occur both with the first as with the last

[1] CNB, Financial Stability Report (2005)
[2] CZK is an abbreviation for Czech crown. As of the beginning of January 2007 the exchange rates were CZK/EUR=27.5 and CZK/USD=20.75.
[3] Out of which CZK 526 billion (18% of GDP) were loans to the non-financial corporate sector.
[4] CNB, Banking supervision (2005)
[5] E.g. Pirner (2003) estimates that credit risk represents 60-70% of the banks' risk profile.
[6] http://www.riskglossary.com/link/credit_risk.htm
[7] The term "Default" means that a debtor has not met its legal obligations according to the debt contract, which may occur if the debtor is either unwilling or unable to pay this debt (or has violated a covenant). Therefore, the term default should be distinguished from the terms insolvency and bankruptcy. "Bankruptcy" is a legal finding that imposes court supervision over the financial affairs of those who are insolvent or in default. But for the scope of this paper, default immediately implies bankruptcy, and therefore default and bankruptcy are considered to be equivalent terms.

repayment of the loan. The probability and value impact of changes in default probability- the migration risk (e.g. a loan to company that is approaching financial distress is losing on its value) has to be considered as well. Moreover, when dealing with a portfolio of companies, other factors, such as default correlations (the degree to which the default risks of the individual borrowers are related to each other), and exposure of the portfolio (the size, or proportion, of the portfolio exposed to the default risk) have to be taken into account as well.

But even though all of these elements are critical to the management of credit portfolios, none are more important or more difficult to determine, than the **default probability**.[8] There exists a large number of methods, how to assess the likelihood of such event. The first formalized and widespread methods have been the scoring methods building on accounting data, such as the Altman's *Z-* or Ohlson's *O- Scores*, which remain popular until today. Other, more recent methods, such as the credit ratings, methods based on Value-at-Risk (e.g. CreditMetrics) or models built on macroeconomic variables (e.g. CreditPortfolio View) have evolved. One method, which is popular especially in the Anglo-Saxon countries, builds on the original **Merton's model**, proposed by Robert C. Merton (1974), and is currently promoted by Moody's KMV.[9] This method determines the default probability mainly from **market price and volatility of its equity**. The idea to view the company's equity as a call option on the company's assets and to predict default from the value and volatility of assets (inferred from the value and volatility of equity) in relation to a predetermined Default Point (e.g. the face value of a zero-coupon bond or the firm's total liabilities), has been revolutionary and is very economically intuitive and appealing.

In my diploma thesis, I decided to explore this asset-based approach and **discuss thoroughly the Merton model**. The goal is to bring this, in continental Europe, little undervalued model of credit risk closer to the Czech Republic and **try to question its applicability** there. Using the share prices of 15 actively traded, non-financial **Czech companies** listed on the Prague Stock Exchange (PSE), I **compute the Merton-implied probabilities of default** for these companies, as well as the theoretical credit spreads.

The Prague Stock Exchange is developing fast in the last years in terms of market capitalization of traded stock as well as trading volumes.[10] Nevertheless, I anticipate that the

[8] Crosbie and Bohn (2003)
[9] KMV was founded in 1989 by Stephen Kealhofer, John McQuown and Oldrich Vasicek. In 2002, KMV was acquired by Moody's and was renamed Moody's KMV.
[10] As of 31 December 2005, the market capitalization of stocks traded on PSE was CZK 1,1331 billion with the trading volume reaching CZK 1,574.4 billion, which is a 36.4%, respectively 34.3%, year-on-year increase. [CNB, Czech Capital Market Report (2005)]

Czech stock market is too young and small[11] **to give sufficient information about the default probability of the companies listed on the PSE**.

I **compare the default probabilities of the Merton model with** traditional accounting-based measures, the **Altman's *Z*- and Ohlson's *O- Scores*** with both original and updated coefficients. I also confront the computed probabilities of default with rankings and credit ratings from external rating agencies.

I discuss the possibilities of testing **the explanatory power of the calculated Merton-implied default probabilities in comparison to** the explanatory power of **traditional scores**. Despite the fact that no solid ground for such test exists and the extremely small sample size, I carry out an **ordered logit regression** using the computed measures of probability as explanatory variables and the Czech Sector Awards rankings as discrete response variable. The main purpose of this test is to give a **practical illustration** of the theoretical concepts of rating models. In other words, the empirical test in this paper is more of a guide to how such a test should be carried out (e.g. when a sufficient dataset of listed companies and external ratings is available) rather than a source of reliable results.

The diploma thesis is structured as follows:

Chapter 2 presents the traditional accounting-based scoring functions of Altman and Ohlson together with a brief discussion of their limitations. The next Chapter 3 sheds some light on how the credit ratings are assigned and what are the reasons of their popularity. The following and pivotal Chapter 4 presents the assumptions and derivation of the Merton model. Especially the step-by-step approach to obtaining the required model inputs and to computing the default probabilities can be found very useful. The discussion on the advantages and problems of this model and some model improvements, such as those of Moody's KMV, are included as well. This chapter also gives a brief overview of the previous empirical tests of the model quality. The actual calculations of the model-implied probabilities of default and the comparison with other measures of credit risk form Chapter 5. Chapter 6 starts with a discussion on various tests of model quality and an ordered logit regression is carried out. Chapter 7 summarizes and concludes the paper.

All of the necessary model inputs as well as the calculations of the accounting measures of credit risk can be found on the Excel spreadsheets in Attachment 1. I include the

[11] The Czech stock market is in international comparison relatively small with the total market capitalization by the end of 2005 amounting to only 45% GDP. In the developed countries of "western" Europe, these values range between 80-100% GDP [CNB, Financial Stability Report (2005)]

SAS codes and datasets used for calculating the Merton model's default probabilities and for the ordered logit regressions in the folder called Attachment 2 so that the interested reader can replicate my work.

2. Accounting based measures

Credit risk models based on accounting measures (i.e. measures derived from the firm's financial statements) adopt **fundamental analysis** and try to pre-identify, which factors such as cash flow adequacy, asset quality, earning performance, or capital adequacy, are important in explaining the credit risk of a company.[12] They evaluate the significance of these factors, mapping a reduced set of accounting variables, financial ratios and other information into a quantitative score- a scoring function of credit risk. These traditional, accounting-based models have been used for a long time and are undoubtedly the most popular and intuitive means to measure credit risk on the academic grounds.[13] The reasons for their popularity are, besides the economic intuition, the **simplicity in terms of technical implementation and the availability of data.** Sources of accounting data are for larger companies usually publicly available and ratios are easy to compute and interpret. The main characteristic that differentiates the individual traditional models is the econometric method which was applied for their estimation.

2.1. Beaver's financial ratios

Beaver[14] was the first scholar, who had performed an essential study of financial ratios as bankruptcy indicators. He analyzed 30 different financial ratios aggregated into six groups: cash-flow ratios, net income ratios, debt to total asset ratios, liquid asset to total asset ratios, liquid asset to current debt ratios, and turnover ratios. All 30 ratios were tested for their ability to predict bankruptcy. As a result, seven ratios, which exhibited the best performance, were identified. Among them were six accounting ratios and one accounting measure.[15]

[12] Benos and Papanastasopoulos (2005)
[13] Chartkou et al. (2006) citing Penman (2003)
[14] Beaver (1966)
[15] These include: 1) Cash flow/Total debt; 2) Net income/Total assets; 3) Total debt/Total assets; 4) Working capital/Total assets, i.e. (Current assets – Current liabilities)/Total assets; 5) Current ratio,

The most important contribution of Beaver's study was the development of methodology employing accounting data for company's failure prediction. Beaver has introduced the univariate approach of discriminant analysis in bankruptcy prediction by examining the predictive ability of ratios one at a time.[16] But the practice revealed that this method suffers from a number of deficiencies, namely, there are too many ratios to be considered and combination of different ratios can have different implications.

This issue called for a method of **combining ratios into one composite score** that would indicate the overall creditworthiness of the firm. Several composite measures that combine different accounting variables were introduced. The most popular and still frequently used measures are the Altman's *Z-Score* and the *O-Score* derived from Ohlson's model.[17]

2.2. Altman's *Z-Score*

Edward Altman introduced in the year 1968[18] a composite credit score model (called *Z-Score*) based on multiple discriminant analysis. He considered various combinations of 22 variables before choosing the five with the highest predictive power. The resulting model takes the following form:

$$(1) \qquad Z\text{-}Score = 1.2\left(\frac{WC}{TA}\right) + 1.4\left(\frac{RE}{TA}\right) + 3.3\left(\frac{EBIT}{TA}\right) + 0.6\left(\frac{E}{TL}\right) + 0.999\left(\frac{Sales}{TA}\right)$$

where:

WC – working capital, TA – total assets, RE – retained earnings, EBIT – earnings before interest and taxes, E– market value of equity[19], TL – total liabilities.

The result of the model is the predictor, *Z-Score*, that is a linear function of several explanatory variables. This predictor classifies the likelihood of bankruptcy or non-bankruptcy as follows:

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i.e. Current assets/Current liabilities; 6) No credit interval, i.e. (Defensive assets – Current liabilities)/Expenditures for operations; 7) Total assets

[16] Beaver's results indicated that not all ratios predicted equally well. The ability of failure prediction was the strongest in Cash flow/Total debt ratio and Net income/Total assets ratio predicted second best. The result was expectable because both ratios are flow based and they show high correlation with the firm's performance.

[17] Some of the studies that utilize the Z-Score and/or O-Score are: Begley et al. (1996), Berger et al. (1996), Burgstahler et al. (1989), Dichev (1998), Francis (1990), Griffin and Lemmon (2002), Han et al. (1992), Stone (1991), Subramanyam and Wild (1996), and Hillegeist et al. (2004).

[18] Altman (1968)

[19] Even though the market value of equity, *E*, is a market based variable, for the purpose of this paper, the *Z-Score* is still referred to as accounting based.

Z-Score > **3.0** - The company is considered to be healthy- bankruptcy is unlikely

1.8< *Z-Score* <**3.0** - Gray area- inconclusive result

Z-Score < **1.80** - Probability of bankruptcy is high.

Obviously, as the score decreases, the probability of bankruptcy increases and vice versa.
The *Z-Score* is a powerful diagnostic tool that forecasts the probability of a company entering bankruptcy within a 2 year period. Studies measuring the effectiveness of the *Z-Score* have shown that the model has a 70%-80% reliability.[20]

2.3. Ohlson's *O-Score*

The next model that achieved a worldwide impact was the credit scoring model by Ohlson.[21] Ohlson used logit methodology[22] to derive a default risk model known as *O-Score*. According to Ohlson, four factors affected the choice of financial ratios, which are: size of the company, measure of financial structure, measure of performance, and measure of current liquidity. Ohlson chose nine different kinds of accounting measures, which reflected these factors. Moreover, all of the nine accounting-based variables employed in the model were found statistically significant. The probability of default is increasing as the *O-Score* increases.

$$\begin{aligned} O\text{-}Score = & -1.32 - 0.407(Size) + 6.03\left(\frac{TL}{TA}\right) - 1.43\left(\frac{WC}{TA}\right) + 0.08\left(\frac{CL}{CA}\right) - 2.37\left(\frac{NI}{TA}\right) - \\ & -1.83\left(\frac{EBITDA}{TL}\right) + 0.285(INTWO) - 1.72(OENEG) - 0.52\left(\frac{NI_t - NI_{t-1}}{|NI_t| + |NI_{t-1}|}\right) \end{aligned} \tag{2}$$

where:
Size is inflation adjusted total assets[23], WC – working capital, TA – total assets, TL – total liabilities, CL – current liabilities, CA – current assets, NI – net income, EBITDA – pre-tax income plus depreciation and amortization, INTWO – indicator variable equal to 1 if

[20] http://en.wikipedia.org/wiki/Edward_Altman
[21] Ohlson (1980)
[22] Probit (Logit) methodology weights the independent variables and assigns scores in a form of failure and survival probability using the normal (logistic) cumulative function. These models can be also used as a classification system and place the potential borrower into either a good or a bad group according to a cut-off point. [Benos and Papanastasopoulos (2005)]
[23] I.e. ln(TA/GDP price level index), where GDP price level index is the ratio of GDP by current exchange rate to GDP by PPP.

cumulative net income was negative for the last two years or 0, if otherwise, OENEG – indicator equal to 1 if book value of (owner's) equity is negative or 0, if otherwise.

2.4. *Z-* and *O- Scores* with updated coefficients

Hillegeist et al. (2004) recently updated the coefficients of the original Altman *Z-Score* and Ohlson *O-Score* models using an expanding rolling window approach.[24]

(3) $$Z\text{-}Score^{U} = 4.34 + 0.08\left(\frac{WC}{TA}\right) - 0.04\left(\frac{RE}{TA}\right) + 0.1\left(\frac{EBIT}{TA}\right) + 0.22\left(\frac{E}{TL}\right) - 0.06\left(\frac{Sales}{TA}\right)$$

(4) $$\begin{aligned} O\text{-}Score^{U} &= -5.91 + 0.04(Size) + 0.08\left(\frac{TL}{TA}\right) + 0.01\left(\frac{WC}{TA}\right) - 0.01\left(\frac{CL}{CA}\right) + 1.20\left(\frac{NI}{TA}\right) + \\ &+0.18\left(\frac{EBITDA}{TL}\right) + 0.01(INTWO) + 1.59(OENEG) - 1.10\left(\frac{NI_t - NI_{t-1}}{|NI_t| + |NI_{t-1}|}\right) \end{aligned}$$

Surprisingly, Hillegeist found that only two of the Altman variables, VE/TL and EBIT/TA, were statistically significant. For the Ohlson model, eight of the nine updated coefficients were statistically significant, but five of the eight significant coefficients had different signs than their original counterparts and these changed signs were not intuitive.

Hillegeist then compared the original Altman *Z-Score* and Ohlson *O-Score* models with *Z-Score* and *O-Score* models with updated coefficients using a relative information content test. He found that *O-Score* outperformed *Z-Score*, updated $O\text{-}Score^{U}$ was superior to *O-Score* and original *Z-Score* was better than updated $Z\text{-}Score^{U}$. His results also implied that studies using *Z-Score* and/or *O-Score* may lack sufficient statistical power to yield reliable results.

2.5. Rankings

One way to assess and easily compare the solvency of more companies is the method of ranking. Ranking is generally defined[25] as the process of positioning items (such as individuals, groups or businesses) on an ordinal scale in relation to others. In the economic sense, ranking is a method that comprises of creating a list of placings (rankings) of

[24] These updated coefficients were estimated using a database of 78,100 firm-year observations (representing 14,303 individual industrial firms) with 756 initial bankruptcies.
[25] http://en.wikipedia.org/wiki/Ranking

companies within a group (usually either a country or industry) subject to some predefined criteria. Assigning rankings is a **purely quantitative method** that uses the companies' financial statements to create a scoring function and according to the resulting score, the companies are sorted in rank order.

In the Czech Republic, a nice example of such sorting can be seen within the yearly Czech Sector Awards (CSA) assigned by Čekia.[26] The CSA ranking evaluates the solvency and investment attractiveness of the Czech companies based on financial indicators (profitability, liquidity, indebtedness and trade activity) and gives these companies a ranking (in form of an index) depending on how they placed within their given industry sector.[27] The scale of CSA ranking is depicted in the following Table 1:

Table 1: *Czech Sector Award ranking scale*

RANKING	DESCRIPTION OF RANKING CATEGORY	INVESTMENT PROFILE
A-1	**Excellent within industry**	**INVESTEMENT ZONE OF THE INDUSTRY INDEX**
A-2	**Very suitable within industry**	
A-3	**Suitable within industry**	
A-4	**Strong within industry**	
A-5	**Above-average within industry**	
A-6	**Average within industry**	
B-1	**Slightly below-average within industry**	**SPECULATIVE ZONE OF THE INDUSTRY INDEX**
B-2	**Less suitable within industry**	
B-3	**Weak within industry**	
B-4	**Risky within industry**	
B-5	**Unsuitable within industry**	
B-6	**Not investment-worth**	

Source: www.ranking.cz

The main advantage of rankings is that they **enable an easy comparison** of otherwise heterogeneous companies within the ranking group and give a basic idea about the financial health of these companies. However, as ranking is based on the financial statements, this method inherits the drawbacks of the accounting based models.

[26] Čekia (Česká kapitálová informační agentura, a.s., i.e. Czech Capital Information Agency) is a leading provider of corporate databases and economic information in the Czech Republic.
[27] For more information about the CSA ranking, see the webpage www.ranking.cz, which is currently unfortunately available only in Czech.

2.6. Problems of accounting-based models[28]

Even though accounting-based measures of credit risk are very popular and economically intuitive, using them brings several problems. These models employ financial statements data that **measures past performance** of the firm. On the other hand, the estimates of default probability are statements about the likelihood of future events and relying on historical data may generate misleading results when projecting them into the future.

Financial statements are formulated under the **going-concern principle**, which assumes that firms will not default. Thus, their ability to accurately and reliably assess the probability of bankruptcy will be limited already by the design of these models.

Accounting conservatism distorts the real picture as well, because under the conservatism principle the market value of assets (especially intangible and fixed assets) is often undervalued. Such underestimation will cause accounting-based leverage measures to be overstated, which leads to overestimation of the probability of default.

Another important deficiency of accounting models is their **inability to capture a measure of asset volatility**. Volatility is a crucial variable in bankruptcy prediction because it incorporates the likelihood of default – the chance that the firm's asset value will drop below its debt value. Other things being equal, the higher the asset volatility, the higher is the possibility that the value of assets will cross the threshold triggering default. Two firms with identical financial ratios and leverage may have substantially different probability of bankruptcy depending on their asset volatilities. Therefore, volatility is an important omitted variable in both the Altman's Z and Ohlson's O bankruptcy prediction models.

As a result of these problems, the quality of accounting models has been critically questioned by a number of academic papers.[29] Therefore, relying solely on financial statements for predicting the default probability is not sufficient. As a natural consequence, other methods for the assessment of credit risk have evolved. Among these, the credit ratings play a dominant role.

[28] Partially based on Hillegeist et al. (2004)
[29] E.g. Hao (2006)

3. Credit ratings

Credit ratings have evolved as successors of the traditional models and are the most common and popular measure of credit risk among investors and practitioners. They enhance the traditional accounting models by **taking into account the qualitative factors** as well.

A rating can be defined as a formal opinion, given by a credit rating agency, of the creditworthiness of an obligor.[30] This opinion is expressed in the form of a "mark", which is usually expressed by letters or numbers (or a combination of both). The credit rating is applied not to an organization itself, but to its debt securities. However, it is usual to refer to the creditworthiness of companies themselves in the terms of the credit rating of their debt.[31]

3.1. Public sources of credit ratings

External credit ratings are provided by specialist rating agencies. The most frequently used and worldwide available credit rating systems come from the major rating agencies Standard & Poor's, Moody's or Fitch IBCA. An illustration of the long-term rating scale from Standard & Poor's is depicted in Table 2.

Table 2: *Standard & Poor's long-term rating scale*

RATING GRADE	EXPLANATION OF THE "LETTER" RATING	INVESTMENT PROFILE
AAA	Extremely strong capacity to meet financial commitments. Highest rating.	**INVESTEMNT GRADE**
AA	Very strong capacity to meet financial commitments	
A	Strong capacity to meet financial commitments, but somewhat susceptible to adverse economic conditions and changes in circumstances	
BBB	Adequate capacity to meet financial commitments, but more subject to adverse economic conditions	
BBB-	This is the lowest rating before non-investment grade	
BB	Less vulnerable in the near-term but faces major ongoing uncertainties to adverse business, financial and economic conditions	**SIGNIFICANT SPECULATIVE CHARACTERISTICS**
B	More vulnerable to adverse business, financial and economic conditions but currently has the capacity to meet financial commitments	
CCC	Currently vulnerable and dependent on favorable business, financial and economic conditions to meet financial commitments	
CC	Currently highly vulnerable	
C	A bankruptcy petition has been filed or similar action taken but payments or financial commitments are continued	
D	Payment default on financial commitments	

Note: Ratings from 'AA' to 'CCC' may include a plus (+) or minus (-) sign to show relative standing within the major rating categories.

Source: Standard & Poor's

[30] The rated entity could be e.g. a government, government agency or a city, but for the scope of this paper, only the ratings of companies and financial institutions are taken into account.
[31] Coyle (2000) pp. 25

In the Czech Republic, the most active rating agency is the CRA Rating Agency (CRA)[32] with almost 80 assigned international ratings. Their rating scale is depicted in Table 3 for both international[33] and local ratings and gives an interesting insight into the different interpretations of rating categories.

Table 3: *CRA Rating Agency's long-term international and local ratings*

Long-term international CRA Rating		Long-term local (Czech Republic) CRA Rating	
Aaa	First-class with small grade of risk	**czAaa**	First class subjects with very small risk and max. ability to fulfill their debts
Aa	High quality, with moderate grade of risk in a longer period	**czAa**	Very good subjects with high quality of repayments, small risk in long-term horizon
A	Above average, with realities which could cause small grade of risk in the future	**czA**	Sound subjects with above-average ability to pay back their liabilities, future risk is small and could be changed by additional characteristics
Baa	Medium grade risk, with a stable present and realities, which could influence future risk level	**czBaa**	Medium-grade subjects with acceptable present ability to pay back its liabilities, some realities could change future ability
Ba	Already speculative, with uncertain future grade of risk	**czBa**	Subject with risk of repayment in the future and able to fulfill its present obligations
B	Less suitable for investment, with problematic grade of risk	**czB**	Subjects with speculative present ability to pay back its obligations with risky future
Caa	Under average with problems in meeting its obligations	**czCaa**	Not defined ability to repay its present liabilities with problematic future
Ca	With high grade of risk and high measure of failing in its obligations	**czCa**	Very poor ability to fulfill its present obligations with high grade of risk in the future
C	Highly speculative without investment recommendations	**czC**	High risk and non stable subjects not able to repay its liabilities

Note: Ratings from 'Aa' to 'Ca' may be modified by the addition of a plus (+) or minus (-) sign to show relative standing within the major rating categories.

Source: CRA Rating Agency

3.2. The purpose of credit ratings

The main purpose of credit ratings from external rating agencies is to **provide information to investors**. It works as a guide to the investment risk. Relying on ratings, especially from the prestigious rating agencies, allows the investors to significantly lower the costs associated with carrying out their own company analyses. Small and individual investors often rely solely on these ratings and can on that account easily diversify their personal

[32] CRA is the biggest rating agency operating in Central Europe since 1998, and is the only regional rating agency that has been officially recognized by the Commission for Securities in the Czech Republic. CRA was bought by Moody's in 2006 and recently renamed Moody's Central Europe.
[33] The international CRA rating scale is similar to that of Moody's, except that Moody's uses instead of the +/- signs the numbers 1,2,3 to show the relative standing within the category (e.g. Ba3).

portfolios and match their personal risk profile. Over the years, credit ratings have achieved wide investor acceptance as convenient tools for differentiating credit quality.[34]

On the other hand, companies pay considerable amounts to acquire rating from prestigious rating agencies in order to attract investors and thus get the hands on cheap source of financing from the capital markets. Having a good rating also enables the company to get a significantly lower interest rate on a bank loan. Credit ratings can also have public relations implications, because top-rated companies can present themselves as elite organizations within their industry.[35]

The absolute majority of banks worldwide use ratings as the main instrument for assessing their clients' credit risk.[36] Banks can use, within the New Basel Capital Accord standardized approach, the ratings from renowned international rating agencies to determine the required regulatory capital. But as the majority of companies, especially in Europe, do not have such rating, banks most commonly develop their own "internal rating based" (IRB) approaches to evaluate the risk of the companies in their loan portfolios.[37]

But regardless whether the rating comes from an international rating agency, a local rating agency or from banks, the basic philosophy of credit rating is common to all of these institutions.

3.3. How the credit ratings originate

Rating agencies work out a firm's corresponding grade- credit rating, on the basis of information supplied by client (including private information obtained during regular discussions with the firm's representatives) as well as information drawn from public sources. A rating evaluation includes in itself a substantial part of analysis of the so called **soft factors**, i.e. assessment of the **qualitative parameters**.[38] Quantitative indicators are, naturally, also evaluated, especially those related to cash-flow and to dynamic coverage of obligations. The decisive importance for the level of assigned rating is the long-term ability of a company to generate sufficient amounts of cash-flow for covering all its current obligations.

[34] "Credit Rating Facts Sheet", Standard & Poor's (http://www2.standardandpoors.com/spf/pdf/media/credit_ratings_fact_sheet.pdf)
[35] Coyle (2000), pp.25
[36] Benos and Papanastasopoulos (2005)
[37] See e.g. consultative document "Overview of The New Basel Capital Accord" from the Bank for International Settlements for more details about this issue. (http://www.bis.org/bcbs/cp3ov.pdf)
[38] E.g. the market position of the company, support by the shareholders, management strategy, and/or the financial flexibility of the entity.

It is important to stress out that, in contrast to the standardized scoring methods, a universally applicable methodology of rating doesn't exist. It is because the specific methods vary from agency to agency and differ depending on the evaluated client. Moreover, the rating agencies tend to use different rating scales with varying number of rating categories. In addition to that, the ratings may also vary according to the degree to which other dimensions of default risk are considered.[39]

3.4. Advantages of ratings

The rating methodology has several significant advantages, which have made credit ratings the most popular measure of credit risk for investors. The biggest advantage is the fact that the **credit ratings are very easy to interpret** and allow an effortless **comparison of the rated companies**. Changes in the company's rating send a clear signal to all investors about the underlying shift in credit risk. Among the other advantages is the fact that as the process of assigning credit rating is not a formalized, model-based approach, no simplifying assumptions (e.g. about the efficiency of capital markets or the capital structure of the firm) have to be made. **Combining quantitative, qualitative and legal analysis** makes the rating method as **close to "real-life"** as possible.

3.5. Problems of ratings

By taking into account a high volume of relevant information sources about the credit risk of the evaluated company, assigning a rating is a rather **timely procedure**. It may take a team of credit analysts months before the rating becomes public.[40] Once a rating is assigned, an on-going review of material factors that could affect the rating (such as changes in the capital structure, an acquisition or other major economic developments) has to be maintained. Because of changes in or unavailability of information, ratings may be changed, suspended or withdrawn. Generally, an issuer credit rating is reviewed formally at least once a year.[41]

Credit ratings are discrete variables and are therefore **ordinal measures of firm's creditworthiness**- i.e. "categories" of credit risk exposure. This approach inherently implies the grouping of companies of potentially differing credit risks into same rating categories,

[39] E.g. Standard & Poor's risk ratings represent default probabilities only, while Moody's factors also include a measure of the extent of loss in the case of default (Crouhy et al. 2000)
[40] E.g. Typical time of assigning a rating by the CRA Rating Agency (for a medium-sized Czech company) takes approximately 10 weeks and it takes 12 weeks before the assigned rating becomes public. (www.crarating.cz)
[41] Standard & Poor's "Credit Rating Facts Sheet"

which naturally brings along several problems.[42] Within any rating class the default probabilities of issuers are clustered around the median. However, the average default rate for each class is considerably higher than the default rate of the typical firm. This is because each rating class contains a group of firms which have much higher probabilities of default, due to the approximate exponential change in default rates as default risk increases.[43] These are firms which should have been downgraded but haven't been downgraded yet and there are also firms that should have been upgraded. This happens because changing of the obligor's rating doesn't happen instantaneously.[44] The fact that the rating agencies change the company ratings with substantial delay and thus **fail to reflect newly available information as it arrives in the market** has three negative consequences. First, the historical frequency of staying in a rating class should overstate the true probability of keeping the same credit quality. Second, the average historical probability of default overstates the true probability of default for typical firms within each rating class, due to the difference between the mean and the median default rates. Third, if both the probability of staying in a given rating class, and the probability of default are too large, then the transition probabilities must be too small.[45]

Another disadvantage of credit ratings is that **not too many companies**, **especially in continental Europe**, **have a rating** from the prestigious rating agencies. And even though the number of rated companies worldwide is increasing rapidly,[46] most of these companies come from the United States of America. This fact is closely related to the differing functions of capital markets in continental Europe and USA. In USA, capital markets are very liquid and commonly used as a source of financing. Therefore, in order to attract these investors, the companies actively seek to be rated and are willing to pay relatively high fees to the rating agencies.[47] The **European companies** rely traditionally more on the **direct financing through bank loans and private placements of bonds**, with the banks carrying out their

[42] The rating agencies, such as S&P, may also include a plus or minus sign to show relative standing within the category, increasing thus the total number of rating categories but that doesn't solve the problem.
[43] Crouhy et al. (2000)
[44] Standard & Poor's outline the steps that lead to a change in rating as "When a rating change appears necessary, we undertake a preliminary review that may lead to a CreditWatch listing. The next step is a comprehensive analysis, including, if needed, a meeting with management and a presentation to the rating committee. The rating committee considers the circumstances, comes to a decision and notifies the issuer, subject to the appeal process noted above." (Standard & Poor's "Credit Rating Facts Sheet") This procedure is likely to take at least a few days.
[45] This has a negative impact especially on models that derive the credit risk from transition matrices, such as CreditMetrics.
[46] E.g. in 2005 alone, Standard & Poor's Ratings Services published more than 500,000 ratings, including new and revised ratings. (www.standardandpoors.com)
[47] E.g. Standard & Poor's charged their U.S. based corporate clients in the year 2005 up to 4 basis points for most transactions, with a minimum fee of $50k (http://www2.standardandpoors.com/portal/site/sp/en/eu/page.article/4,1,4,0,1113591451215.html)

own, internal credit risk assessments. As a result, when managing credit risk of a portfolio of companies, some other measures of credit risk than the credit ratings from international rating agencies have to be added or used instead. The local rating agencies may provide useful information about the credit risk of these companies but they lack the credibility of their international counterparts and by using different methods and rating scales, the comparison with the prestigious rating grades is difficult.

The quantitative part of assigning the credit ratings is, besides the qualitative analysis, largely derived from traditional accounting-based measures and financial analysis. The **stock market provides an alternative source of information regarding the probability of default.** In addition to the financial statements the stock market also **aggregates information from other sources**, which can substitute the lengthy qualitative analysis. While the potential for market-based variables to provide information about bankruptcy prediction has long been recognized[48], one difficulty with this approach has been how to extract the information related to default from market prices.

4. Merton‘s model for individual firms

In the year 1973, Black and Scholes,[49] in close cooperation with Robert C. Merton, introduced their world famous option pricing formula. Even though the BS formula is until now being extensively used in the field of pricing derivatives, the original purpose of developing this model was to acquire a **powerful tool to value corporate liabilities**.[50] It was Robert Merton, who proposed in his 1974 seminal paper[51] on valuation of corporate debt “On The Pricing Of Corporate Debt: The Risk Structure Of Interest Rates” the extension and possible **application of the Black-Scholes option pricing formula into the field of corporate finance.**

The Merton model uses an option pricing approach, which brings together systematic risk, probability of loss and recovery rate into a call option on the value of the firm. His model

[48] e.g. Beaver (1968) as cited by Landsman and Maydew (2001)
[49] Black and Scholes (1973)
[50] Shimko (1999), pp. 43
[51] Merton (1974)

was also the first structural model[52], because it uses the evolution of firms' capital structure, such as asset and debt values, to determine the time and probability of default. **Credit events are triggered by movements of the firm's value relative to some pre-defined threshold or barrier, called the Default Point.** As a result, the evolution of the firm's value and capital structure is the main issue of this approach. The resulting Merton model is often referred to as asset-based model or contingent claims approach, because equity is viewed as a contingent claim on the value of the firm's assets.

The Merton model is considered to be the first modern credit risk model and its concept is widely used until today on both academic and commercial ground.

4.1. Assumptions

The original Merton's model rests upon several **rather unrealistic and simplifying assumptions:**

- The stock market efficiently incorporates all publicly-available information about default probability into equity prices.
- The liabilities of the firm consist only of single, zero-coupon debt with face value F.[53]
- The debt structure remains within the time period static (i.e. the management doesn't change the debt structure at any case) and the behavior of the company, such as the riskiness of its investments, will not be impacted by how close it is to default.
- The firm can default only at time T and not before. If the firm's value falls down to minimal levels before the maturity of the debt but still is able to recover and meet the debt's payment at maturity, the firm will avoid the default (since there are no coupons to be paid).
- Firm's asset values follow log-normal distribution.
- Firm value process follows the geometric Brownian motion.
- Interest rate is constant.
- No intermediate payments, such as dividends, will be made to equity holders.
- Bankruptcy is costless.

[52] For the purpose of this paper, structural and Merton's model are considered to be equivalent terms

[53] The model does not distinguish among different types of debt according to their seniority, collaterals, covenants or convertibility.

- In bankruptcy, a strict priority of claims is preserved.

Besides these model assumptions, some assumptions about the Black-Scholes frictionless market have to be drawn as well:

- There is no arbitrage in the market.
- The access to short selling is unlimited and there are no indivisibilities of assets.
- Borrowing and lending through a money-market account can be done at the same riskless, continuously compounded rate r.
- Agents are price takers, that is, trading in assets has no effect on prices.
- There are no transaction costs and taxes.

4.2. The option theory

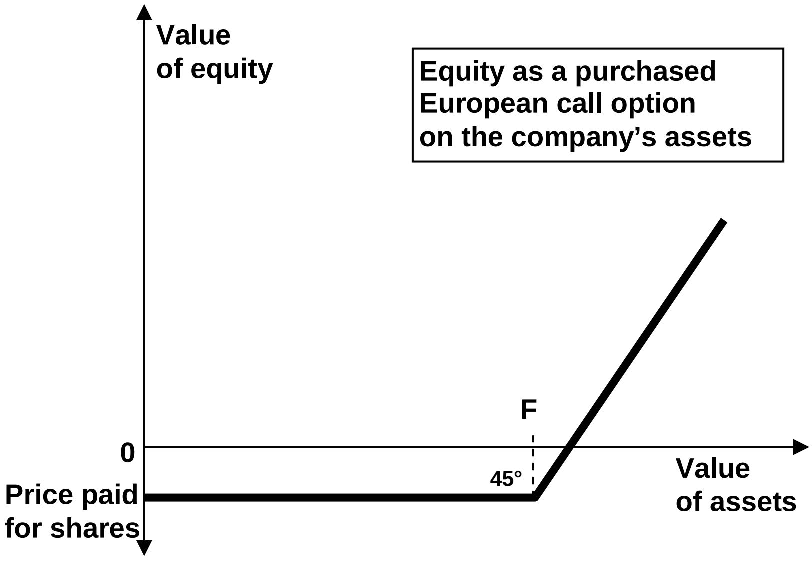
Starting with those assumptions, Merton introduced a contingent claims approach to valuing corporate capital categories using Black and Scholes' option pricing theory. At the beginning stood the recognition that the firm's **equity can be viewed as a European call option on the company's assets**, i.e. value of the firm (V). The strike price of this option is the book value of liabilities (F) and the option is exercised in time T (the time of maturity of the debt). This means that, at time T, the equity holders will exercise their option and pay off the debt holders if the value of the firm's assets is greater than the face value of its liabilities. If the value of the company's assets is lower than the nominal value of debt, the shareholders will let their call option expire. In such case, the firm files for bankruptcy and due to the assumption of strict priority of claims in bankruptcy, the shareholders receive nothing, since all the company's assets will be used to service the debt. The cost and maximum potential loss to these shareholders is the price paid for the purchase of these shares. As the value of assets grows above the threshold of debt value, the shareholders acquire the entire amount in excess of the debt.

Therefore, the value of equity to shareholders (E) at time T is:

(5) $$E_T = \max(0; V_T - F)$$

The following Figure 1 illustrates the idea that the equity can be viewed as a call option on value of company’s assets graphically.

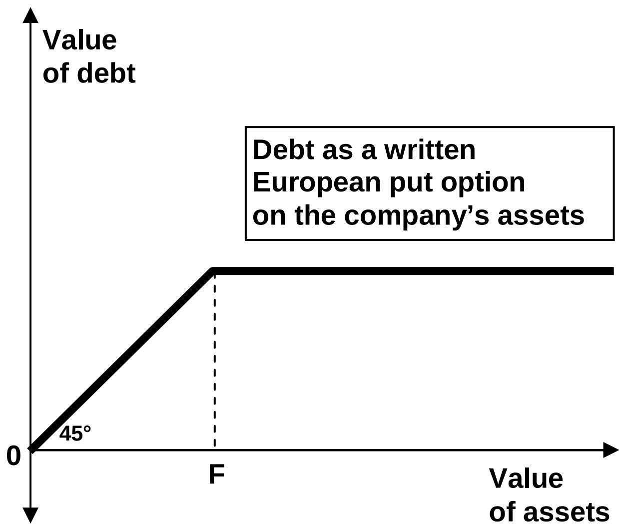
Figure 1: *Equity value at maturity as a function of the asset value of the firm*



Similarly, the payoff to holders of the company’s debt (the creditors) is analogous to the payoff from writing a European put. The value of debt to creditors (D) at time T is:

(6) $D_T = \min(V_T; F)$

Figure 2: *Debt value at maturity as a function of the assets value of the firm*



The Merton model is a type of a default-mode credit risk model, where at the end of the time period only two possible outcomes can arise: the company gets into default or it doesn't. The default occurs if at the time of servicing the debt, the company's terminal value of assets is below its outstanding debt. The following Table 4 summarizes this idea:

Table 4: *Payoffs at Maturity*

	Assets	**Debt**	**Equity**
No Default	$V_T \geq F$	F	$V_T - F$
Default	$V_T < F$	V_T	0

In order to compute the value of equity (i.e. the call option on the firm's total assets), it is assumed that the firm value process follows the geometric Brownian motion, that is:

$$(7) \qquad dV = \mu V dt + \sigma_V V dW$$

or often rewritten as

$$\frac{dV}{V} = \mu dt + \sigma_V dW$$

where V is again the value of firm's assets, μ is the asset drift (i.e. the expected continuously compounded return on V), σ_V is volatility of the firm value and dW is a standard Wiener process.

By applying the Ito's lemma on Equation (7), the following equation can be obtained[54],

$$(8) \qquad dE = \left(\frac{\partial E}{\partial t} + rV \frac{\partial E}{\partial V} + \frac{1}{2} \sigma_V{}^2 V^2 \frac{\partial^2 E}{\partial V^2} \right) dt + \sigma_V V \frac{\partial E}{\partial V} dW$$

and after several steps, the following partial differential equation for the value of equity, which is well known from option pricing theory is attained:

$$(9) \qquad \frac{\partial E}{\partial t} + rV \frac{\partial E}{\partial V} + \frac{1}{2} \sigma_V{}^2 V^2 \frac{\partial^2 E}{\partial V^2} - rE = 0$$

where t refers to time, r is the risk-free interest rate and σ_V is the volatility of the company's assets.

[54] See Appendix A for details on the Ito's lemma

This Equation (9) under the boundary condition (5):

$$E_T = \max(0; V_T - F)$$

can be solved[55] to obtain the Black and Scholes formula for the value of equity:

(10) $$\boxed{E = VN(d_1) - Fe^{-rT}N(d_2)}$$

where $N(\cdot)$ is the standard normal cumulative distribution function,[56]

(11) $$d_1 = \frac{\ln\left(\frac{V}{F}\right) + \left(r + \frac{\sigma_V{}^2}{2}\right)T}{\sigma_V\sqrt{T}}$$

and

(12) $$d_2 = d_1 - \sigma_V\sqrt{T} = \frac{\ln\left(\frac{V}{F}\right) + \left(r - \frac{\sigma_V{}^2}{2}\right)T}{\sigma_V\sqrt{T}}$$

To show the analogy of the valuation of company's equity (Equation(10)) to the usual stock option pricing, recall the formula of a call option on stock:

(13) $$C = SN(d_1) - Ke^{-rT}N(d_2)$$ with

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma_S{}^2}{2}\right)T}{\sigma_S\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma_S\sqrt{T} = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \frac{\sigma_S{}^2}{2}\right)T}{\sigma_S\sqrt{T}}$$

where S is the current price of the stock and K is the strike price of the option.

[55] See Nekula (2005)

[56] $N(y) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{y} e^{-\frac{u^2}{2}}\,du$

4.3. Probability of default[57]

The **probability of default at time T** is the **probability that the market value of firm's assets will be less than the book value of the firm's outstanding debt at time T**. To put it formally, at time zero, default probability (DP) is given by:

(14) $DP = \Pr(V_T \le F)$

and from the properties of natural logarithm:

(15) $DP = \Pr(\ln V_T \le \ln F)$

At this point, a little detour should be made to take a closer look at the evolution of firm's value in time. From Equation (8) comes the following formula:

(16) $d\ln V = \left(\mu - \frac{\sigma_V^{\ 2}}{2}\right)dt + \sigma_V dW$

The incremental changes in $\ln V$ follow a generalized Wiener process with drift $\left(\mu - \frac{\sigma_V^{\ 2}}{2}\right)$ and the diffusion coefficient σ_V. The following formula comes from the use of an approximation for the incremental change in $\ln V$ - from $\ln V_0$ (at time $t=0$) to $\ln V_T$:

$$\ln V_T - \ln V_0 \sim N\left[\left(\mu - \frac{\sigma_V^{\ 2}}{2}\right)T, \sigma_V^{\ 2}T\right]$$

or equivalently:

(17) $\ln V_T \sim N\left[\ln V_0 + \left(\mu - \frac{\sigma_V^{\ 2}}{2}\right)T, \sigma_V^{\ 2}T\right]$

Since the logarithm of V_T, as displayed in Equation (17), is normal, **the value of the firm at maturity, V_T, is log-normally distributed.** As Crouhy et al. (2000) claim, this assumption is quite robust and actual data confirm quite well to this hypothesis. Moreover, the distribution of asset returns is stable over time, i.e. the volatility of asset returns stays relatively constant. Because of the fact that the standard deviation of $\ln V_T$ is a linear function of $\sqrt{T}$, the uncertainty about the future development of the firm value grows with the time-to-maturity.

[57] The mathematical derivations of this section are mainly based on the works of Nekula (2005), Kulkarni et al. (2005), Crouhy et al. (2000), Vassalou and Xing (2004) and Crosbie and Bohn (2003).

From the properties of log-normal distribution, the moments for the value of the firm can be derived. The mean has the form:

(18) $$E(V_T) = V_0 e^{\mu T}$$

and the variance is

$$\mathrm{var}(V_T) = V_0^{\,2} e^{2\mu T}\left(e^{\sigma_V^{\,2} T} - 1\right)$$

Because of the fact that the value of assets follows the geometric Brownian motion as described in formula (7), and from the previous discussions, it is straightforward to show that the value of firm's assets at time T , V_T (given that the value at time 0 is V_0), is:

(19) $$V_T = V_0 \exp\left[\left(\mu - \frac{\sigma_V^{\,2}}{2}\right)T + \sigma_V \sqrt{T} Z_T\right]$$

and after rearranging:

$$\frac{V_T}{V_0} = \exp\left[\left(\mu - \frac{\sigma_V^{\,2}}{2}\right)T + \sigma_V \sqrt{T} Z_T\right]$$

(20) $$\boxed{\ln V_T = \ln V_0 + \left(\mu - \frac{\sigma_V^{\,2}}{2}\right)T + \sigma_V \sqrt{T} Z_T}$$

where

(21) $$\sqrt{T} Z_T = W_T - W_0$$

is normally distributed with zero mean and variance equal to T[58], and

(22) $Z_T = \dfrac{W_T - W_0}{\sqrt{T}} \sim N(0;1)$ [59] is the random component of the firm's return.

Coming back to Equation (15)

$$DP = \Pr(\ln V_T \le \ln F)$$

and substituting from Equation (20), the default probability can be rewritten as:

[58] Crouhy et al. (2000)
[59] Vassalou and Xing (2004)

(23) $$DP = \Pr\left(\ln V_0 + \left(\mu - \frac{\sigma_V{}^2}{2} \right) T + \sigma_V \sqrt{T} Z_T \leq \ln F \right)$$

$$DP = \Pr\left(\ln V_0 + \left(\mu - \frac{\sigma_V{}^2}{2} \right) T + \sigma_V \sqrt{T} Z_T - \ln F \leq 0 \right)$$

and after rearranging:

(24) $$DP = \Pr\left(-\frac{\ln\left(\frac{V_0}{F}\right) + \left(\mu - \frac{\sigma_V{}^2}{2} \right) T}{\sigma_V \sqrt{T}} \geq Z_T \right)$$

From Equation (22) it can be seen that the random component of the firm's asset returns, Z_T, follows the Normal distribution $Z_T \sim N(0,1)$. Therefore, the **default probability** can be defined in terms of the **cumulative normal distribution** as follows:

(25) $$\boxed{DP = N\left(-\frac{\ln\left(\frac{V}{F}\right) + \left(\mu - \frac{\sigma_V{}^2}{2} \right) T}{\sigma_V \sqrt{T}} \right)}$$

This equation gives the **probability of default (DP) for a company** at the time of maturity T (e.g. the DP in one year). Equation (25) shows that the probability of bankruptcy is a function of the distance between the current value of the firm's assets and the face value of its liabilities adjusted for the expected growth in asset value relative to the asset volatility.[60] This distance is called the **distance-to-default (DD).** It is the number of standard deviations that the firm is away from default. The higher the distance-to-default, the better for the company, since higher DD implies being further from the default. The distance-to-default can be expressed as:

(26) $$\boxed{DD = \frac{\ln\left(\frac{V}{F}\right) + \left(\mu - \frac{\sigma_V{}^2}{2} \right) T}{\sigma_V \sqrt{T}}}$$

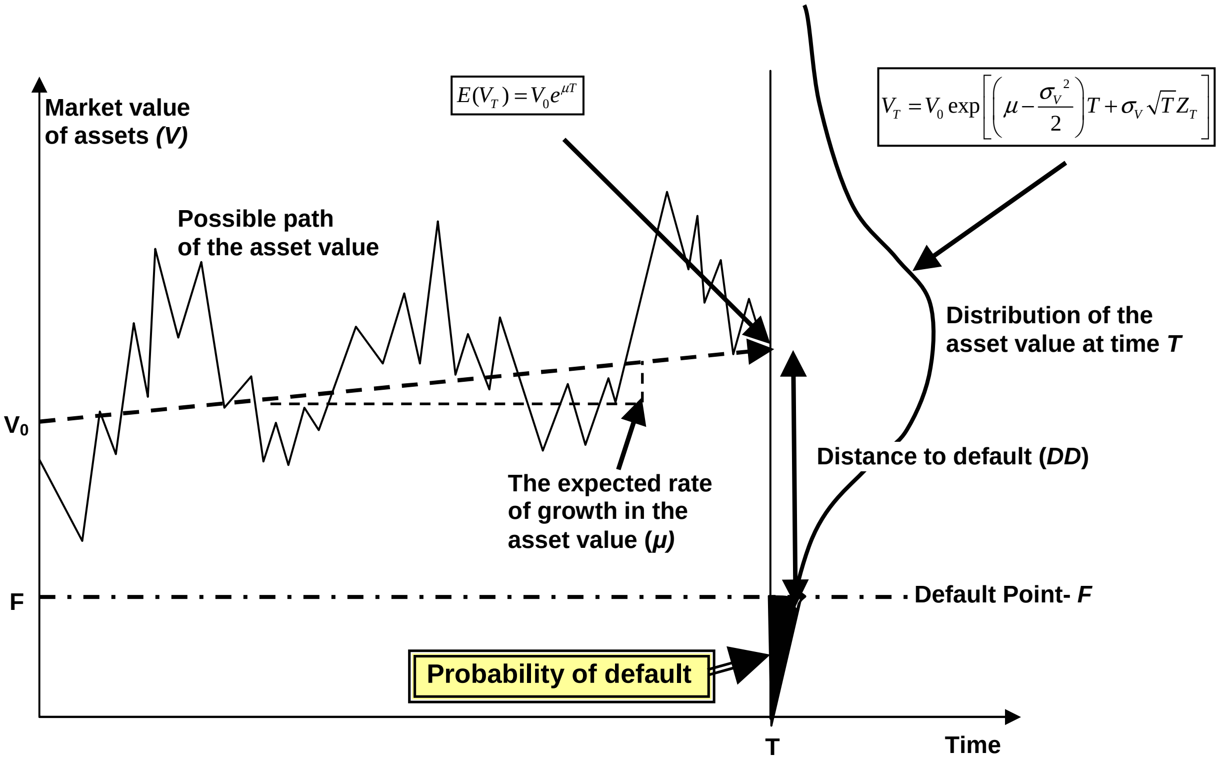
and therefore

(27) $DP = N(-DD)$

[60] Hillegeist et al. (2004)

The following picture summarizes the development of the value of the firm in relation to the distribution of the future asset value. The dark area is the probability of default as expressed in Equation (25).

Figure 3: *Distribution of the firm's asset value at maturity of the debt*



Source: Moody's KMV, Crouhy et al. (2000), Schmid (2004)

4.4. The case with dividends

One of the assumptions of the original Merton model is that there **aren't any dividends paid out**. For the sake of simplicity for derivation of the default probability, the previous sections of this chapter held this assumption. But this assumption can be easily relaxed and the Equation (10) is modified to reflect the stream of dividends paid by the firm to the equity holders.[61]

(28) $$\boxed{E = Ve^{-\delta T} N(d_1) - Fe^{-rT} N(d_2) + \left(1 - e^{-\delta T}\right)V}$$

where

[61] For the derivation of Equation (28), see e.g. Chartkou et al. (2006)

$$d_1 = \frac{\ln\left(\frac{V}{F}\right) + \left(r - \delta + \frac{\sigma_V{}^2}{2}\right)T}{\sigma_V \sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T} = \frac{\ln\left(\frac{V}{F}\right) + \left(r - \delta - \frac{\sigma_V{}^2}{2}\right)T}{\sigma_V \sqrt{T}}$$

where δ is the continuous dividend rate expressed in terms of V.

The term δ appears twice in Equation (28), where the term $Ve^{-\delta T}$ accounts for the reduction of the value of the assets due to the cash dividends that are paid out before the maturity. The term $\left(1-e^{-\delta T}\right)V$ reflects that these dividends are obtained by the equity holders and equals zero when $\delta = 0$.[62]

As a result of taking into account the dividend payouts, the formula for DP as presented in Equation (25) has to be modified as well.

$$\boxed{DP = N\left(-\frac{\ln\left(\frac{V}{F}\right) + \left(\mu - \delta - \frac{\sigma_V{}^2}{2}\right)T}{\sigma_V \sqrt{T}}\right)} \tag{29}$$

Since the inclusion of the dividend payout ratio δ makes the model more "realistic" the default probability as expressed in Equation (29) is the base for the rest of this paper.

4.5. Step-by-step calculation of the probability of default

In order to compute the theoretical probability of default for real-life companies and markets, several modeling choices and estimations have to be made. Most importantly, Equation (29) contains **some variables, which cannot be directly observable** (the market value of assets V, the asset volatility σ_V and the expected return on assets μ) and must be therefore estimated. The following eight steps are sufficient for calculating the probability of default in the asset-based approach.

Step 1- choosing a forecasting horizon T

In the credit risk literature and modeling, it is common to use a **one-year** (T=1) time horizon for debt maturity and subsequent estimation of default probability.[63]

[62] Hillegeist et al. (2004)

[63] As Kulkarni et al. (2005) argue, one year is perceived as being of sufficient length for a bank to raise additional capital on account of increase in portfolio credit risk (if any). The one-year convention may

Step 2- estimating σ_E

The volatility of equity, σ_E, can be most easily estimated from **historical stock returns** data. It is computed as an annualized standard deviation of daily returns in one year, with the returns being expressed using continuous compounding. Using the lognormal property of stock/equity values, volatility can be easily calculated as follows:[64]

- Download the equity prices over the time period (e.g. the last year)
- Compute $u(t) = \ln\left[\frac{S(t)}{S(t-1)}\right]$, where S(t) is the share price at day t
- Find the standard deviation of $u(t)$, which is in fact σ_E
- Annualize the daily volatility: $\left(\sigma_E \cdot \sqrt{days}\right)$, where *days* is the number of trading days in the year (often assumed to be 252).

Another possibility of estimating the volatility of equity, σ_E, would be to use the **implied volatility**. The implied volatility of equity can be extracted from the market prices of options on equity and is received as an answer to the following question: "What volatility must I use to get the correct market price of the option?" The implied volatility is the volatility of the underlying asset (the firm's equity value) which, when substituted into the Black-Scholes formula (as given in Equation (13)), gives theoretical option price equal to the market price of the option. For finding the implied volatility, a Newton-Raphson algorithm can be used.[65] Hence, the implied volatility can only be obtained for those companies, which have options written on their stock.

Step 3- determining E

The value of equity, E, is found simply as the number of shares outstanding times the last day's equity price.

Step 4- setting the Default Point F

have arisen largely because, until recently, default rates and rating transition matrices were most easily available at a one-year horizon, and such data are key inputs to conventional portfolio credit risk models.

[64] As proposed by e.g Gülçiçek and Sinan (2005)

[65] A nice discussion on the implied volatility and the Newton-Raphson algorithm can be found in Wilmott (1998) on pages 183-185

The Default Point is defined as the threshold, which, when crossed, triggers default.[66] In the case when firm's liabilities would consist only of a single, zero-coupon debt (i.e. the assumptions of the original Merton model), the Default Point would be the face value of this debt F. But because this form of financing is highly unlikely to exist, some other measure of Default Point has to be introduced. To be consistent with the theoretical Merton model from the previous sections, this estimated Default Point is also labeled F.

Some of the authors of credit risk literature[67] consider the Default Point **F** to be equal to the **book value of total liabilities.**[68]

Other authors, [69] however suggest that it is more rational estimate F as

$$F = STL + \frac{1}{2} LTL \tag{30}$$

where STL is the book value of the companies' short term liabilities (debt due in one year) and LTL is the book value of long-term debt. Both of these variables are easily obtainable from the annual reports of the companies.

Step 5- setting r

The risk-free rate of return, r , is usually set as the **yield on** a government security, **T-Bill**, with one year remaining to maturity. This rate is converted into continuously compounded rate for further analysis. Another option is to use inter-bank offered rates, such as LIBOR or PRIBOR.[70]

Step 6- computing the dividend rate δ

The dividend rate, δ, is expressed as the sum of the prior year's common and preferred dividends, divided by the approximate market value of assets. This market value of assets is approximated by the sum of the market value of equity and the book value of liabilities. Even

[66] In reality, the default point is also a random variable. In particular, firms will often adjust their liabilities as they near default. It is common to observe the liabilities of commercial and industrial firms increase as they near default while the liabilities of financial institutions often decrease as they approach default.
[67] E.g. Hillegeist et al. (2004)
[68] Here it's important to point out that total liabilities are defined in this paper as the solution to the equation: total assets=equity + total liabilities. This contradicts the Czech accounting standards, where total liabilities are referred to as the sum of all items on the "right-hand side" of the balance sheet, i.e. total assets=total liabilities.
[69] This proxy is based on the observations of Moody's KMV, which has found out from a sample of several hundred companies that firms default when the asset value reaches a level somewhere between the value of total liabilities and the value of short-term debt. Crosbie and Bohn (2003) show that the model is surprisingly robust to the precise level of the liabilities.
[70] Prague Interbank Offered Rates, available from the Czech National Bank

though this approximation of the market value of assets is generally very inaccurate (as discussed in the following Step 7), it is used here to compute δ, because δ is being used to estimate *V*.

Step 7- finding *V* and σ_V

The price of the call option on stock can be easily computed since all of the variables are directly observable on the market (strike price, time to maturity, underlying asset price (i.e. stock) and the risk-free rate) or can be easily estimated (volatility of the underlying asset). However, the case of the Merton's model brings some problems, because **the value of company's assets, V, and the asset volatility, σ_V, remain unknown and need to be inferred.**

The market value of the firm is simply the sum of the market value of equity plus the market value of debt. But while equity values can be easily observed on the stock markets, the market value of debt is usually unavailable. The reason is that, especially in continental Europe, the company's financing usually does not come from an issue of tradable bonds.

In some literature, the market value of firm's assets is proxied by the sum of market value of equity and book value of debt. But using book values instead of market values can generate highly misleading results.[71] Therefore, more sophisticated methods for the estimation of *V* have to be used.

In Equation (28), one relationship between the value of equity (E) and the value of firm's assets (V) and asset volatility (σ_V) is pointed out. In order to identify the two unknowns (i.e. V and σ_V) with two equations, the model invokes again the geometric Wiener process to model equity value

(31) $$dE = \left(\mu_E - \delta\right) Edt + \sigma_E EdW$$

where μ_E is the expected continuously compounded return on E, δ is the dividend rate, σ_E is the volatility of equity value and dW is a standard Wiener process (the random component of the equity's return). After applying the Ito's lemma,[72] the process for equity can be represented as:

[71] E.g. Wong and Li (2004) show theoretically that "using sum of market value of equity and book value of corporate liabilities as a proxy for the market value of corporate assets generates significant bias of overestimating the asset values."
[72] See Appendix A

(32) $$dE = \left(\frac{\partial E}{\partial t} + (\mu - \delta) V \frac{\partial E}{\partial V} + \frac{1}{2} \sigma_V{}^2 V^2 \frac{\partial^2 E}{\partial V^2} \right) dt + \frac{\partial E}{\partial V} \sigma_V V dW$$

The diffusion terms (i.e. variance) in the equity process in (31) and (32) are equal,[73] and therefore:

(33) $$\sigma_E E = \sigma_V V \left(e^{-\delta T} \right) \frac{\partial E}{\partial V} = \sigma_V \left(e^{-\delta T} \right) V N(d_1)$$

often rewritten as

(34) $$\boxed{\sigma_E = \sigma_V \left(e^{-\delta T} \right) \frac{V}{E} N(d_1)}$$

If there are no dividends paid out, this equation turns itself in the more common form:

(35) $$\sigma_E = \sigma_V \frac{V}{E} N(d_1)$$

One way to obtain the values of V and σ_V is to **solve the equations** (28) **and** (34) **simultaneously**:

(28) $$\boxed{E = V e^{-\delta T} N(d_1) - F e^{-rT} N(d_2) + \left(1 - e^{-\delta T} \right) V}$$

where $d_1 = \dfrac{\ln\left(\dfrac{V}{F}\right) + \left(r - \delta + \dfrac{\sigma_V{}^2}{2} \right) T}{\sigma_V \sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T} = \dfrac{\ln\left(\dfrac{V}{F}\right) + \left(r - \delta - \dfrac{\sigma_V{}^2}{2} \right) T}{\sigma_V \sqrt{T}}$

(34) $$\boxed{\sigma_E = \sigma_V \left(e^{-\delta T} \right) \frac{V}{E} N(d_1)}$$

These two equations complete the system of two simultaneous nonlinear equations with two unknowns, which can be solved relatively easily.[74]

[73] Kulkarni et al. (2005)

[74] Chartkou et al. (2006) or Kulkarni et al. (2005) for example use the Solver routine in Microsoft Excel to come to a numerical solution. For more details on the Solver routine, see e.g. Hull (2003) or the Merton Model spreadsheet on the CD that comes with Allen (2003).
Hillegeist et al. (2004) use a Newton search algorithm to obtain the pair of values that solves both equations, and this process converges usually within five iterations.

Some authors refuse the simple numerical solution as they claim that it can be misleading, since Equation (34) only holds instantaneously.[75] For example Crosbie and Bohn (2003) assert that "[i]n practice the market leverage moves around far too much for [Equation (34)] to provide reasonable results." **The model biases the probabilities in exactly the wrong direction.** For example, a quick decrease in the market leverage (for a firm trending upwards and whose share prices are growing quickly) will lead to the overestimation of asset volatility from Equation (34). This would imply higher probability of default while at the same time, the decrease in the market leverage would suggest improved credit quality. To resolve this problem, a rather complicated **iterative procedure** is implemented.[76]

First, an initial value of $\sigma_V = \sigma_E \left(\frac{E}{E+F} \right)$ is proposed and subsequently this value of σ_V and Equation (28)[77] are used to infer the market value of firm's assets for every day of the previous year. The implied log return on assets each day is calculated and these returns series are used to generate new estimates of σ_V, which is used for the next round of iteration. The iteration on σ_V is repeated in this manner until the values of σ_V from two consecutive iterations converge[78]. Once the converged value of σ_V is obtained, it is used to back out V throughout Equation (28).

Step 8- estimating the asset drift μ

Once the values of V are obtained, the market return on assets μ can be calculated based on the actual return on assets for the entire year. But the actual return on assets based on the values of V coming from Step 7 may be negative. This contradicts the financial theory where the expected returns cannot be negative and cannot be even lower than the risk-free rate. One way of dealing with this problem is to set the growth rate equal to the risk-free rate of return in the cases, where μ would be otherwise negative or lower than the riskless rate.[79] Thus, $\mu(t)$ is calculated as follows:

[75] Du and Suo (2003)
[76] This iterative process is for example described in Bharath and Shumway (2004). Vassalou and Xing (2004) use the same procedure, except that they use σ_E for the initial estimate of σ_V.
[77] Bharath and Shumway assume there are no dividends paid out and therefore they use here the Equation (10), rather than Equation (28).
[78] Bharath and Shumway (2004) claim that the convergence is usually obtained within few iterations (the absolute difference in adjacent σ_V s is set as being less than 10^{-3}).
[79] Hillegeist et al. (2004)

(36) $$\mu(t) = \max\left[\frac{V(t) + \text{Dividends} - V(t-1)}{V(t-1)}; r\right]$$

where the variable "Dividends" is the sum of the common and preferred dividends declared during the year.

Another possibility to compute the asset drift is to use a theoretical relationship between the expected return on assets μ and the expected return on equity μ_E. The relationship is:[80]

(37) $$\mu = \frac{\mu_E E - \left(\frac{\partial E}{\partial t}\right) - \frac{1}{2}\sigma_V{}^2 V^2 \left(\frac{\partial^2 E}{\partial V^2}\right)}{V\left(\frac{\partial E}{\partial V}\right)}$$

It is assumed that the expected return on equity μ_E can be estimated from the stock market data. For example, the popular Capital Asset Pricing Model (CAPM) can be used to find the values for the drift of equity μ_E.[81]

After completing all of these 8 steps, all of the inputs that are necessary to compute the probability of default as given in Equation (29) are finally obtained.

4.6. Other issues in the asset-based framework

4.6.1. The Greeks

Equation (9) puts together many variables known from the financial markets as the "Greek letters". Each Greek letter measures a different dimension to the risk in an option position. In the asset-based approach, where the equity is seen as the call option on the company's assets, the Greek letters represent the different risks to the equity holder.

Option (equity) delta $$\Delta_E = \frac{\partial E}{\partial V} = N(d_1) > 0$$

[80] Derived and discussed in Kulkarni et al. (2005)
[81] Crouhy et al. (2000)

The delta of a call option is defined as the rate of change of the option price with respect to the price of the underlying asset. In the structural framework, the equity value rises as the value of company's assets increases.

Option (equity) gamma $\Gamma_E = \dfrac{\partial^2 E}{\partial V^2} = \dfrac{N(d_1)}{V\sigma_V\sqrt{T}} > 0$

Gamma measures the sensitivity of the option delta to a small change in its underlying asset. It is the second partial derivative of equity value with respect to asset value. The equity gamma is larger than zero suggesting that with an increase in the firm's asset value, the equity delta increases as well.

Option (equity) theta $\Theta_E = \dfrac{\partial E}{\partial t} = \dfrac{VN(d_1)\sigma_V}{2\sqrt{T}} - rFe^{-rT}N(d_2) ? 0$

The theta of an option is the rate of change of the value of the option (i.e. the equity value) with respect to the passage of time. As the development of equity value in time is unpredictable, the equity theta can be both positive and negative.

With the knowledge of the Greek letters, Equation (9) can be rewritten as:

$$(38) \qquad rE = \Theta_E + rV\Delta_E + \frac{1}{2}\sigma_V{}^2 V^2 \Gamma_E$$

4.6.2. Risk-neutral probability of default

In many research papers on the structural credit risk model, including Merton's original paper, the assumption of risk-neutrality is applied.[82] In the case of the Merton model, the risk-neutrality concept implies that all **assets grow at the risk-free rate** r because all other factors influencing the growth rate of the company's assets are already reflected in the share prices. Therefore, the probability of default under the assumption of risk neutrality would be:

$$(39) \qquad DP = N(-DD) = N(-d_2) = N\left(-\frac{\ln\left(\frac{V}{F}\right) + \left(r - \frac{\sigma_V{}^2}{2}\right)T}{\sigma_V\sqrt{T}}\right)$$

The main advantage of using the risk-neutral probability of default is that (besides easier computation of DP) expected returns on equities are usually estimated with significant

[82] For further discussion on this topic, see e.g. Hillegeist et al. (2004) or Kulkarni et al. (2005)

error. For example, Kulkarni et al. (2005) argue that "[b]ecause risk neutral probabilities of default can be calculated without estimating the firm's expected return, they may be more accurately estimated than objective default probabilities."

However, the underlying asset is risky and therefore it does not actually drift at the riskless rate. The objective distribution should have a mean greater than the risk free rate (the drift is generally higher than the risk free rate of return). From properties of normal distribution it follows that the risk neutral assumption implies a higher default probability and therefore risk-neutral default probabilities serve as an upper bound to objective default probabilities.[83] Since the probability of bankruptcy depends upon the actual distribution of future assets (which is a function of μ), the **objective probability measure of default** as given in Equation (29) should be used as the measure of default probability rather than the risk-neutral DP from Equation (39).

4.6.3. The value of debt

The market value of a firm is the sum of market value of debt and market value of equity. Assuming that the firm's debt comprises only of one zero-coupon bond, the value of this bond is equal to the value of the firm less the value of equity (European call option), i.e.:

(40) $V = E + D$

(41) $D = V - E = V - \text{European call on the firm's assets}$

From the previous discussion and assuming, for the sake of simplicity, that there are no dividends being paid out, the following equality holds:

(42) $D = V - \left[VN(d_1) - Fe^{-rT} N(d_2) \right]$

(43) $D = VN(-d_1) + Fe^{-rT} N(d_2)$

Or alternatively, the put-call parity can be used. Under the put-call parity, the value of the firm's debt is equal to the value of a riskless discount bond less the value of the put option written on the firm, again with a strike price equal to the face value of debt *F* and a time to maturity of *T*. In other words, the value of debt can be seen as a portfolio comprising of money lent at the risk-free rate and a short put option.

(44) $D = Fe^{-rT} - \text{European put}$

[83] Deliandes and Geske (2003)

(45) $D = Fe^{-rT} - \left[-VN(-d_1) + Fe^{-rT} N(-d_2) \right]$

(46) $D = VN(-d_1) + Fe^{-rT} N(d_2)$

which is exactly the same result as in Equation (43).

Rather than referring to value or price of debt, it is usual in dealing with bonds to discuss them in terms of yields. The **yield-to-maturity**, *y*, of a corporate zero-coupon bond is (within the continuous time framework) the solution to:

(47) $D = Fe^{-yT}$

The yield-to-maturity for the firm's debt as of today (t=0) maturing at *T* is therefore

(48) $$y = -\frac{\ln\left(\frac{D}{F}\right)}{T}$$

4.6.4. Credit spreads

A credit spread (or credit margin) is the difference between the interest rate that a client has to pay for the granted loan and the risk-free rate. The Merton model can be used to estimate credit spread on debt, which can, in return be useful in evaluating the performance of the model. In order to obtain an explicit formula for the credit spread within the asset-based approach, it is necessary to define F' as the present value (discounted at the risk-free rate) of the debt *F* maturing at time *T*

(49) $F' = Fe^{-rT}$

and let Ψ be a measure of leverage, called Quasi-Debt ratio

(50) $$\Psi = \frac{Fe^{-rT}}{V} = \frac{F'}{V}$$

Then, the yield-to-maturity of the corporate debt can be obtained from:

(51) $D = Fe^{-yT} = F'e^{(r-y)T}$

Defining the credit spread, *CS*, as the difference between the yield-to-maturity and the risk-free rate:

(52) $CS = y - r$

and after substituting Equation (46) into Equation (51), a **simple formula for the credit spread** is attained:

$$F' e^{(r-y)T} = D$$

$$T(y-r) = \ln\left(\frac{D}{F'}\right)$$

$$(y-r) = \frac{1}{T}\ln\left(\frac{VN(-d_1) + Fe^{-rT}N(d_2)}{F'}\right)$$

:(53) $$\boxed{CS = -\frac{1}{T}\ln\left(N(d_2) + \frac{N(-d_1)}{\Psi}\right)}$$

From this equation, it can be seen that the credit spread is an increasing function of the Quasi-Debt ratio Ψ and of the volatility of the firm's assets σ_V. This is both intuitive and economically justifiable because the higher probability of default (which is also an increasing function of leverage and asset volatility), the higher should the credit spreads be. The larger credit spreads are a consequence of the borrower's request to be compensated for the potential losses that come from the higher probability of default of the loans.

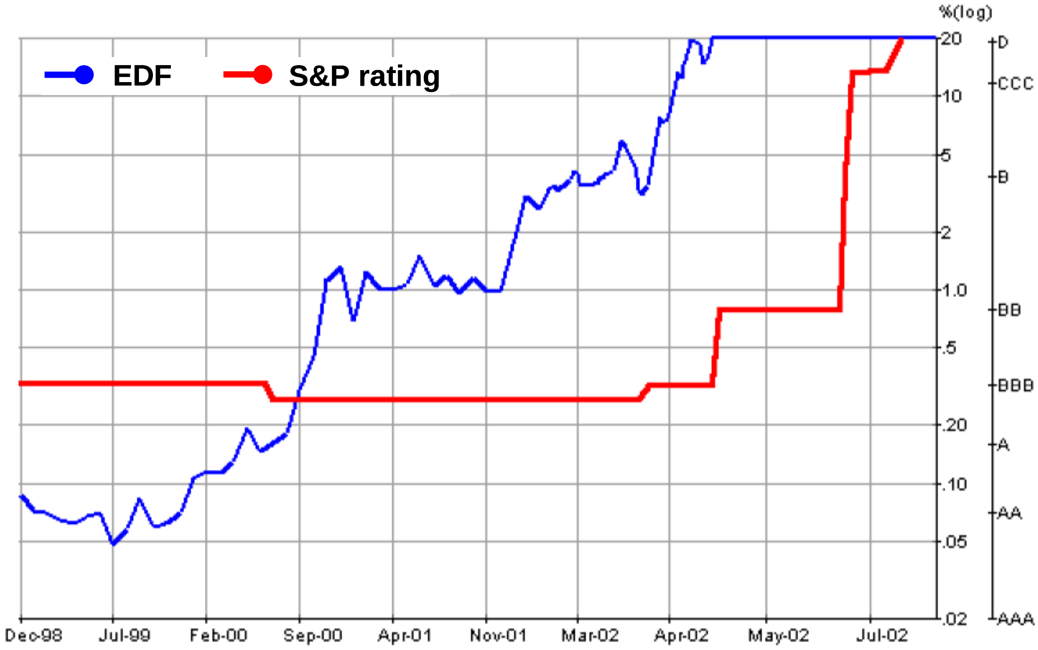
4.7. Advantages of the structural models

The basic idea of Merton's (structural) approach is to relate the default probability of a certain company to its asset value and volatility. This makes the Merton model very intuitive and the actual calculations are not prohibitively difficult.

The main advantage of the asset value-based approaches is that, by using the market prices of debt and equity, it is **inherently forward looking**, because these prices (on efficient markets) reflect the future prospects of the company. On the contrary, credit risk models based on financial statements are inherently backward looking since they are designed to measure past performance and may not be very informative about the future status of the firm. Unlike accounting-based models, the structural model **incorporates the measure of asset volatility, which is a crucial variable in bankruptcy prediction**.

The Merton model instantly reflects the actual credit risk of the firm, because the share prices change almost continuously. Therefore, the probability of default can be estimated at any point in time for any publicly traded company regardless of the time period and industry. It takes some time for the credit rating agencies to make changes to the credit ratings of companies. This advantage of the Merton model can be easily seen on the examples of the bankruptcies of Enron and WorldCom. Moody's KMV, whose asset-based model is built on the Merton framework (described in more detail later in Section 4.10.) plots in Figure 4 the probability of default, called the Expected Default Frequency, of WorldCom[84] compared to the rating of Standard & Poors.

Figure 4: *The Expected Default Frequency of WorldCom*



Source: Moody's KMV case studies

Clearly, using equity values to infer default probabilities allows the asset-based models to **reflect information faster than credit ratings**. However, when WorldCom's stock price was high, the EDF for WorldCom was actually significantly lower than the default probability predicted by the rating agencies.

In the asset-based framework, each issuer is specific and is characterized by its own asset returns distribution, its own capital structure and its own default probability. Therefore, the default probabilities obtained from the Merton model are unique numbers that are firm-specific and directly comparable. They can be viewed as a **"cardinal ranking"** of obligors relative to default risk, instead of the more conventional "ordinal ranking" proposed by rating agencies, which group companies of potentially differing credit risks into same rating

[84] In 1998, WorldCom's share price was $71.75 and the probability of default- EDF was 0.09%, equivalent to AA- rating. When WorldCom's stock price began to fall (to $14 in 2000 and to $2.75 on April 30, 2002), its distance to default immediately decreased. The ratings agencies didn't incorporate the warning signals of falling share prices into their ratings and it took them several months (until April 2002) to downgrade WorldCom's credit rating.

categories. To give a trivial non-economic example, it is certainly better to know the exact result of a competitor in a race, rather than knowing that he/she finished among the top ten.

The development of share prices of an individual company also **reflects the development of the market as a whole**.[85] For example, economic downturns or political instability are likely to imply higher probability of default of the companies active in this country. The model incorporates this fact in the sense that the decrease of share prices and increased volatility of market value (which are the likely consequences of an increase in the systematic risk) implies higher credit risk.

Moreover, unlike accounting-based models that are subject to different national accounting rules, the Merton model can be consistently applied across differing countries, and therefore is internationally comparable.

Finally, another fact that speaks for the assertion that the option-pricing theory perspective is very useful is the fact that many leading commercial credit risk models (e.g. Moody's KMV or CreditMetrics[86]) have been built on the foundations of the original Merton model.

4.8. Disadvantages and problems

The model in its basic form, as introduced by Merton (1974) and discussed in this paper, is considered to be **oversimplistic and based on too strong assumptions**.

The **assumption of efficient capital market**, in the sense that equity prices reflect all relevant publicly available information, is for the model crucial, since the share price is the key input of the model. Moreover, the predictive power of the model comes from the predictive power of shares. But several studies rebut the assertion of efficient capital markets.[87] Especially on the young markets, such as that of the Czech Republic, the **equity prices cannot be considered to be the perfect indicator of the real situation of the company**.[88] But this implies that if markets are not perfectly efficient, then conditioning on information not captured by the Merton model probably makes sense.[89] On the other hand, the

[85] The effect of a market downturn on the equity value of any particular firm can be estimated using the firm's equity beta. $\Delta E = \beta_E \Delta V_m$, where ΔV_m is the market change.

[86] This model based on the concept of VaR (Value at Risk) has been introduced in 1997 by J.P.Morgan in cooperation with Bank of America, KMV, Union Bank of Switzerland and others

[87] E.g. Sloan (1996) [cited by Hillegeist et al. (2004)] suggests that the market does not accurately reflect all of the information in the financial statements

[88] Pečená (2003)

[89] Bharath and Shumway (2004)

trading of big volumes of shares based on insider information may reflect important information about the real condition of the company and push the share prices closer to reality. Crosbie and Bohn (2003) argue that "it appears that it doesn't seem to take many economically motivated investors to move the equity price to reflect the risk of the firm."

The negligible number of listed companies and the relatively low trading volumes on the Prague Stock Exchange suggest further troubles with the use of the equity-based models. In continental Europe, compared especially with USA or UK, it is rather uncommon to use issue of shares to cover the financing of the companies and more traditional ways of financing are used (especially loans from banks or eventually bond issues).

One of the mentioned advantages is that the modeled probabilities of default reflect general movements on the stock markets. This can also be a big disadvantage of the model, since the equity-based models tend to be **very cyclical and are prone to overreaction due to market bubbles**.[90] At the portfolio level this overreaction can be problematic, because economic capital linked to default probabilities calculated on the basis of equity-based models will tend to be very volatile.

Another strict and strongly criticized assumption is that the company has only one outstanding zero-coupon bond. Also the impossibility of the company to default before the maturity of the debt (since there are no coupons to be repaid) and the assumption that the capital structure remains during the period static are very far from reality too. In particular, firms will often adjust their liabilities as they near default.[91]

Another problem of the Merton model is that if the default threshold is set greater than zero and if asset values are assumed to follow paths without jump processes, then the theoretical required spread over the risk-free rate can be driven as close to zero as desired by increasing the frequency with which observations of the asset value are taken.[92] Thus, **the model theoretically implies negligible (zero) credit spreads for short-term debt**, which contradicts reality.

Using the Brownian motion to model the asset value development, the use of cumulative normal distribution function to transform the distance-to-default into default

[90] Servigny (2004), pp.72
[91] It is common to observe the liabilities of commercial and industrial firms increase as they near default while the liabilities of financial institutions often decrease as they approach default. The difference is usually just a reflection of the liquidity in the firm's assets and thus their ability to adjust their leverage as they encounter difficulties.
[92] Allen (2003), pp. 341

probabilities and the assertion that in bankruptcy, a strict priority of claims is preserved[93] has also rightly received much critique.

4.9. Model improvements

The Merton model, as discussed in the previous parts of this Chapter is derived from the original Merton model outlined in his Merton (1974) paper. Ever since the publishing of the original Merton model, the structural asset-based approach has drawn much attention from both academic and commercial researchers. Consequently, during the last 30 years, many improvements to the classical Merton model have been made, most of which were aimed at relaxing the strict and unrealistic assumptions of the original model. Some of these improvements (such as the inclusion of dividends or the use of objective instead of risk-neutral default probabilities) have already been discussed in this paper.[94] Other important, but more advanced model extensions, are introduced in the following overview[95]:

- More realistic capital structures such as involvement of junior and senior debt, safety covenants and dividends e.g. Black and Cox (1976).
- Early bankruptcy (i.e. default outside time T) and liquidation costs introduced by Black and Cox (1976). Longstaff and Schwartz (1995) allow bankruptcy to occur at any random default time.[96]
- Duffie and Singleton (1994), Jarrow and Turnbull (1995), and Jarrow, Lando and Turnbull (1997) characterize bankruptcy as an exogenous process, e.g. a Markov process in the firm's credit ratings, which does not explicitly depend on the firm's assets and the priority rules for the various debt instruments.
- Inclusion of coupon payments, e.g. Geske (1977) or Kim, Ramaswamy and Sundaresan (1993).

[93] For example, Franks and Torous (1994) found that the strict absolute priority rule was violated in 78% of the bankruptcies they considered.
[94] However, several of the modeling choices presented in these sections (such as the iterative procedure for estimating the asset volatility) are a nontrivial extension of the ideas of the original Merton model. The KMV Corporation is especially responsible for these clever extensions and some authors, e.g. Bharath and Shumway (2004), choose to call the Merton model, as it is described here, the "KMV-Merton model".
[95] This overview is mostly based on Servigny (2004, pp. 68), Shimko (1999, pp.43-46 and 130-138), Shimko (2004, pp.93-95), Hanke (2003, pp. 92-103) and Crouhy et al. (2000)
[96] Beside that, Longstaff and Schwartz (1995) allow non-independence between credit risk and the interest rate, and they model recovery as a stochastic process.

- Stochastic interest rates, e.g. Hull and White (1993), or Shimko, Tejima, and Van Deventer (1993) who use the Vasicek (1977) stochastic interest rates model.
- Stochastic processes including jumps in the value of the firm, e.g. Zhou (1997) who models the value of the firm with the help of a Poisson process.
- Strategic bargaining between shareholders and debt holders, e.g. Anderson and Sundaresan (1996).
- The effect of incomplete accounting information in Duffie and Lando (2001).
- Incorporation of bankruptcy costs and taxes (which makes it possible to work with optimal capital structure) by Leland (1994).

Despite the theoretical limitations of the assumptions of the classical Merton model, **the paper has stood the test of time**. In the last 30 years, many extensions to the model, addressing the criticized issues, have been made to improve its performance, out of which a short selection is listed above. But the most self-contained and elaborated framework built on the original Merton platform has been carried out by the Moody's KMV company.

4.10. The Moody's KMV model

The Moody's KMV model (MKMV) is a popular commercial credit risk model used extensively in various parts of the world. In the heart of the MKMV model is the Vasicek-Kealhofer model, which is a generalized form of the Merton model.[97] Amongst the most important improvements is the inclusion of five different classes of liabilities (short-term, long-term, convertible, preferred equity, and common equity), letting the firm default at any time the value of assets crosses the Default Point, or estimating an implicit corporate-risk-free reference curve instead of using the treasury curve. Empirical setting of the Default Point or using a more complex procedure to solve for asset value and volatility[98] are among the other enhancements of the original model. This leads to obtaining the distance-to-default (DD) measure, which is within the MKMV framework defined as:

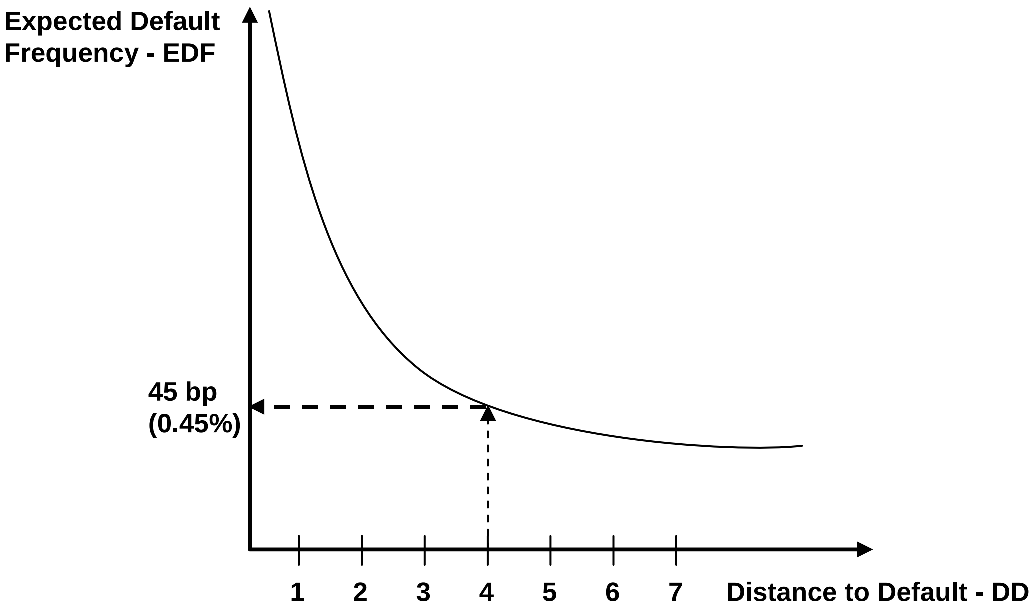
(54) $$DD_{MKMV} = \frac{V - F}{V \sigma_V}$$

[97] This unpublished model was proposed around 1984. See Vasicek (1984) for some of the improvements of the Vasicek-Kealhofer model to the original Merton model.

[98] For example the derived asset volatility from the iteration is combined in a Bayesian manner with country, industry and size averages to produce a more precise estimate of the firm's asset volatility.

But the main difference between the classical Merton model and the Moody's KMV model is that the original MM uses the cumulative normal distribution to convert distances-to-default into default probabilities. Moody's KMV rightly argues that the probability from the Normal distribution is too low and the credit risk isn't normal. The statistics books don't go beyond 3.49 standard deviations from default, whereas firms that are 4-6 standard deviations from default have defaulted.[99] Therefore, Moody's KMV uses its wide **historical company database** (since 1973) to estimate the empirical distribution of distances-to-default and it calculates default probabilities (in the MKMV terminology called the Expected Default Frequency- EDF) based on that distribution. The **distance-to-default is mapped into the Expected Default Frequency** for a given time horizon. An example[100] of such mapping is outlined in Figure 5.

Figure 5: *Mapping the distances-to-default into Expected Default Frequencies*



Source: Crouhy et al. (2000), Crosbie and Bohn (1999)

The resulting distribution of default rates has much wider tails than the Normal distribution. For example, a distance-to-default of 4 (4 standard deviations away from default) would from the Normal distribution mean essentially zero probability of default. However, Moody's

[99] Measuring & Managing Credit Risk: Understanding the EDF Credit Measure for Public Firms (2004)
[100] As shown in Crouhy et al. (2000), Crosbie and Bohn (1999). Crosbie and Bohn (2003) give the DD of 4 already a 1% probability of default.

KMV maps this distance-to-default of 4 to a default rate of around 45 bp (0.45%), which is roughly equivalent to S&P's BBB-/BB, which is hardly an investment grade bond.[101].

To make the EDF variables comparable with the more widespread credit ratings, Table 5 shows the correspondence between EDFs and the ratings of Standard & Poor's and Moody's.

Table 5: *EDFs and risk rating comparisons*

EDF (bp)	S&P	Moody's
2–4	⩾ AA	⩾ Aa2
4–10	AA/A	A1
10–19	A/BBB+	Baa1
19–40	BBB+/BBB−	Baa3
40–72	BBB−/BB	Ba1
72–101	BB/BB−	Ba3
101–143	BB−/B+	B1
143–202	B+/B	B2
202–345	B/B−	B2

Source: Crouhy et al. (2000)

According to the researchers employed by Moody's KMV,[102] the MKMV model outperforms the original Merton model significantly[103] and shows a very good predictive power.[104] But the possibility to use the MKMV model and the EDF measures in the Czech Republic is questionable. The distance-to-default should capture most of the relevant inter-country differences in default risk[105] (the different economic prospects for countries are already captured by the individual equity and asset valuation). But MKMV's empirical default distribution is built on publicly listed companies in the United States and, as a result, its translation to other countries is unsure. However, Moody's KMV claims that their experience with EDF values internationally has been very good and that over half of their customers operate outside of the US. Moreover, using a modification of the MKMV model, Moody's

[101] Investment grade bond is a corporate bond with a credit rating of BBB or above from Standard & Poor's, or Baa and above from Moody's.
[102] Cossin (2001, pp. 287) points out that "[t]here is no published systematic and scientific study that assesses the performance of EDFs compared to classical ratings. Practical studies tend to come from Moody's KMV itself and are rarely based on large samples or on valid econometric methods."
[103] E.g. Arora, Bohn and Zhu (2005). Sobehart, Keenan and Stein (2000) report that the Merton model performs almost as well as the Moody's model in the case of extremely poor quality firms. However, the Moody's KMV model clearly performs better beyond 10% of the population and is much better at discriminating defaults in the middle ranges of credit.
[104] As shown in e.g. Bohn (2000) and Agrawal, Arora, and Bohn (2004).
[105] Crosbie and Bohn (2003)

KMV is able to assess the credit risk of private, non-traded companies, which makes it potentially useful for assessing credit risk of bank's loan portfolio.[106]

But the reason why the MKMV model is not being applied in this paper is that, unfortunately, the Vasicek-Kealhofer model and many of the modeling choices of Moody's KMV are **proprietary information**, thus not being publicly available. Hence, for the scope of this paper, the publicly available Merton model, with several improvements made by Moody's KMV, is the basis.

4.11. Empirical testing of the Merton model

Over the past several years, many researchers have examined the contribution of the Merton model to the assessment of credit risk. Many of these studies utilize the credit spreads observed on financial markets to evaluate the model's performance. A brief overview of some of the most unambiguous results follows, roughly divided into evidence speaking for and against the asset-based model:

4.11.1. Evidence speaking for the use of the Merton model

- Sarig and Warga (1989) estimate the term structure of credit spreads and show it to be consistent with contingent claim model predictions.
- A study by Wei and Guo (1997) tests the models of Merton (1974) and Longstaff and Schwartz (1995) and finds the Merton model to be empirically superior.
- Lardic and Rouzeau (1999) implement the Merton model to bond market pricing issues and find that the model provides valuable and informative results about the fundamental credit quality of the firm. Their study confirms that the Merton's model is efficient for monitoring purposes but, however, quite inaccurate for trading and pricing needs, since the Merton's firm-specific spread does not explain all of the market spread.
- Cossin (2001) reviews the comparative statics of several more complicated asset-based models and concludes that the basic intuition of the Merton model seems to be useful for pricing risky debt.

[106] Crosbie and Bohn (1999) hint that when the information about the equity value of the firm is not available, peer comparisons are used and the asset value and asset volatility are estimated using financial statement data and industry and country comparables. The methodology may have changed recently as this remark does not appear in the later version of the paper- Crosbie and Bohn (2003).

- Gemmill (2002) shows that Merton's model works well in the particular case when zero-coupons are being used for funding. He employs zero-coupon corporate bonds data and concludes that model and market spreads are on average of similar magnitude. He draws a conclusion that market spreads are high (relative to model spreads) for bonds which have low risk and for bonds which are near to maturity.
- Hillegeist et al. (2004) use a relative information content test and find that structural default probability measures contain relatively more information than Altman's *Z-Score* and Ohlson's *O-Score*.
- Duffie and Wang (2004) show that Merton-implied default probabilities have statistically significant predictive power in a model of default probabilities over time, which can generate a term structure of default probabilities.
- Bohn, Arora and Korablev (2005), practitioners of Moody's KMV, argue that the MKMV model captures all of the information in traditional agency ratings and well known accounting variables.

4.11.2. Evidence speaking against the use of the Merton model

- Frank and Torous (1989) find that contingent-claim models yield theoretical credit spreads much lower than actual credit spreads.
- Similarly, Jones et al. (1984) use a sample of companies with relatively simple capital structures and find low theoretical spreads compared to actual spreads. They conclude that the Merton model is not an improvement over their naive (riskless) model for investment grade bonds.
- Mella-Barral and Perraudin (1997) show that the simple structural models (eg. Merton, Geske) forecast spreads which are smaller than market spreads, particularly for companies which have low leverage and low volatility, but the more complicated structural models which produce larger spreads (eg. Longstaff/Schwartz and Leland/Toft) also produce large errors. Another finding is that whether a model allows for stochastic interest rates or not does not make much difference.
- Stein (2000), Sobehart and Stein (2000) and Sobehart and Keenan (1999) argue that the basic Merton model can easily be improved upon. They provide some evidence that unmodified, Merton-type models are not, in fact, complete.

- Bohn (2000) surveys some of the main theoretical models of risky debt valuation that build on Merton (1974) and Black and Cox (1976) and finds empirical evidence that the actual credit spreads are higher than model spreads.
- Bharath and Shumway (2004) find evidence that the probability of default derived from the Merton's model is a marginally useful default forecaster, but it's not a sufficient statistic for default.
- Campbell, Hilscher and Szilagyi (2004) estimate hazard models that incorporate both probability of default (derived from the Merton model) and other variables for bankruptcy, finding that Merton model's probability of default seems to have relatively little forecasting power after conditioning on other variables.
- Arora, Bohn and Zhu (2005) assert that the basic Merton model is not good enough for practitioners and appropriate modifications to the framework make a difference.
- Du and Suo (2003) conclude that structural models hardly provide any significant additional capability when they are used for forecasting credit ratings.

The previous research results can be, in a very simplified manner, summarized as that the Merton's model is **efficient for monitoring purposes** and **gives an early warning signal about the nearing default of a company but is not a sufficient statistic for predicting default and is unsuitable for pricing and trading purposes**. Moreover, the Merton model estimates credit spreads that are significantly lower than those observed on the markets.

5. Merton model in the Czech Republic

5.1. Previous research

The **Merton model hasn't drawn so far much attention from researchers within the Czech Republic.**[107] The papers discussing the asset-based approach come prevailingly form the Czech National Bank (CNB) and are mostly aimed at comparing alternative credit risk models for the purpose of determining capital requirements for banks within Basel II.

[107] Here I mean the Merton model for individual companies, which derives the credit risk from the development of credit risk from market prices of bonds and/or equity. For example Jakubík (2006) applies a single-factor Merton-type model for macroeconomic credit risk modeling and stress testing, but this model derives the credit risk for the whole economy (i.e. financial stability of the Czech Republic) based on macroeconomic variables.

Even though some CNB authors[108] approve the use of Moody's KMV model for banks, which have a large share of their portfolio represented by corporate credits, this endorsement is caused by MKMV's potentially superior modeling techniques and especially their ability to estimate credit risk for privately-held companies. Nevertheless, **the possibility to use the Czech capital markets for credit risk modeling has been**, more or less, **rejected**. For example Jakubík (2006)[109] claims that **for the analysis of credit risk, models based on market prices of shares or bonds are not of much use, because the Czech capital market is not very well developed.** Pečená (2003) argues that the asset-based model cannot be used in the Czech Republic even for companies that are publicly traded. The reason is that it is **impossible to historically rely on the fact that the share prices reflect the real economic situation of the firm.**

But, as far as I'm aware, there hasn't been a single academic study that would try to actually implement the Merton model on the listed companies in the Czech Republic and compare the obtained values with other measures of credit risk.

5.2. The probabilities of default for Czech companies

5.2.1. The company selection process

As of beginning of January 2007, there are totally 32 companies listed on the Prague Stock Exchange (PSE). Out of these 32 companies, only **non-financial companies** have been selected, because the business area of financial institutions differs considerably and the model, such as setting of the Default Point, would have to be adjusted, generating thus incomparable results.[110] In order to obtain at least a short time series data for further comparison, only companies that had been **listed in** (or prior to) **2004** have been selected.[111] Moreover, since the key input of the Merton model is the equity volatility, companies, whose stock is not being **actively traded,** are rejected. A rather lenient criterion was set,[112] which leaves, nevertheless, only **15 companies for subsequent analysis**.

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[108] E.g. Derviz, Kadlčáková (2001) or Kadlčáková, Sůvová (2002)
[109] As quoted in CNB, Financial Stability Report (2005)
[110] Two companies, Erste Bank AG and Komerční Banka, a.s. have been excluded on behalf of this restriction.
[111] The companies CETV (Central European Media Enterprises Ltd.), ECM (ECM Real Estate Investments A.G.), Orco (Orco Property Group S.A.) and Pegas Nonwovens SA did not meet this condition.
[112] The traded volume of the companies' stock during 2006 had to be at least CZK 15,000 (approximately EUR 550) for more than one day in that year. The companies that were excluded are: Česká námořní plavba, Česká zbrojovka, Energoaqua, Jihomoravská plynárenská, Lázně Teplice

These selected non-financial companies that are actively traded on the Prague Stock Exchange are, together with the excluded companies, depicted in Appendix B. Despite the fact that the 15 companies met the criteria for further analysis**, big differences in the trading activity and equity volatility** persist. The Appendix C has an overview of the share price development for all of the companies since 2001. Several companies, such as Severomoravská plynárenská, a.s. or Středočeská plynárenská, a.s.[113], are not traded very frequently and as a result, the share prices change only rarely.

5.2.2. Counting the default probabilities

In order to compute the probability of default implied by the Merton model, DP_{MM}, the eight steps that were already described in Section 4.5 have to be taken. In accord with the convention, a 1 year horizon for default has been chosen. The whole period for observation of default probabilities comprises of the **last five years (2001-2006)**. The equity volatility is computed using the historical stock price data in Excel (see Attachment 1 "Shares&ve") as an annualized standard deviation of daily returns in one year (as described in Step 2). The market value of equity, E, is simply the number of shares outstanding (see Attachment 1 "Issue" for the values and changes in the total issued shares) times the share price of the last day of the year. The Default Point F is set equal to the book value of total liabilities. The annual 1Y PRIBOR rates have been chosen as the risk-free rate. The method used to infer the values of V and σ_V is to **solve simultaneously** Equations (28) and (34). The main reason for preferring this option, despite the justified critique, is the simplicity of this method and the easy tractability of the exact computations. These values are used to calculate the asset drift μ from the relationship in Equation (36). With all of these model inputs, the **default probabilities,** DP_{MM} **, theoretical credit spreads,** CS**, and the distances-to-default,** DD**, are finally computed.** The results of the Merton model for the years 2001-2006 are reported in Appendix E.

The actual computations are made in the statistical software SAS and the program "Merton_final.sas" (see Appendix D) and the required dataset "Merton.sas7bdat" are attached in Attachment 2.

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v Čechách, Léčebné lázně Jáchymov, Pražská plynárenská, Pražské služby, Slezan Frýdek-Místek and Západočeská plynárenská. Despite meeting this condition, RMS-Holding had been excluded as well because its share price hasn't changed throughout the whole year 2006.

[113] Středočeská plynárenská for example did not encounter any movement in share prices throughout the entire year 2003.

5.3. Comparison with other indicators of credit risk

In order to assess the results of the **Merton model** (the Merton-implied probabilities of default) it is useful to **compare them to other indicators of credit risk**. These other indicators are the **Atman's *Z*- and Ohlson's *O*- *Score*** as proposed in Chapter 2, together with their **updated versions**, $Z\text{-}Score^{U}$ and $O\text{-}Score^{U}$.[114]

The *Z*- and *O-Scores* are calculated using fiscal year-end data and the way they are constructed (as scoring functions), do not represent bankruptcy probabilities. Therefore, in order to comply with the probabilistic form of the DP_{MM} variable, these **scores** are turned **into probabilities** using the **logistic transformation** via logistic cumulative distribution function.[115]

$$DP = \frac{e^{Score}}{1+e^{Score}} \tag{55}$$

In this manner, the probabilities implied by the original *Z-Score,* $Z\text{-}Score^{U}$, *O-Score* and $O\text{-}Score^{U}$ are obtained and denoted as *Z-Prob,* $Z\text{-}Prob^{U}$, *O-Prob* and $O\text{-}Prob^{U}$ respectively. From the properties of logistic transformation, these values indicating the probability of default fall naturally between 0 and 1.

Another indicator of company solvency could be represented by **credit ratings from external rating agencies**. But unfortunately **very few ratings** have been assigned by external rating agencies to the 15 selected companies. Altogether, only several companies based in the Czech Republic have acquired from the rating agencies Standard & Poor's and Moody's a credit rating[116], and out of these rated companies, only two companies are analyzed in this paper.[117] Moreover, despite my expectations, the largest local rating agency, CRA Rating Agency[118] (CRA), has assigned for the Czech listed companies only two ratings as well.[119] The main reason for the insufficient number of ratings from these agencies is their relatively

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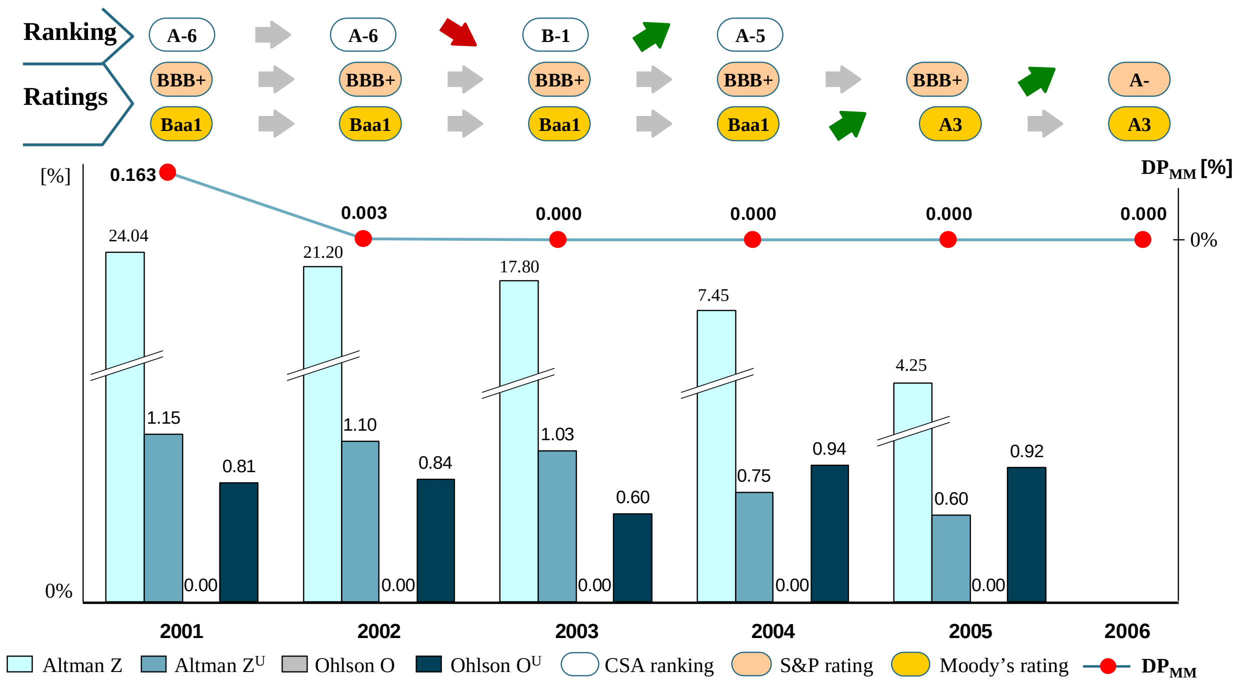
[114] The coefficients of the Altman's *Z-Score* had to be inverted so that the lower the *Z-Score*, the better-off is the company. This step is necessary for transforming the scores into default probabilities.
[115] It can be easily recognize that, regardless of the size of the "Scores" this transformation always produces values (the default probabilities) in the range of 0 to 1. For details on logistic transformation, refer to e.g. http://www.statsoft.com/textbook/glosl.html or Hillegeist et al. (2004).
[116] As of January 2007, Standard & Poor's has given a long-term rating to 21 Czech companies, whereas Moody's only to 8 companies.
[117] Out of the 15 analyzed companies, the only two that have an assigned rating from Standard & Poor's- are ČEZ, a.s. (A-) and Telefónica O2 Czech Republic,a.s. (BBB+). Moody's ratings for these companies are A2, respectively Baa1.
[118] In the Czech Republic, the CRA has assigned to date around 80 international ratings.
[119] Středočeská plynárenská, a.s. (Baa) and Spolek pro chemickou a hutní výrobu, a.s. (Ba).

high price and the fact that the base of small individual investors relying on the rating data to place their investments is very narrow.

The last source of information about credit risk that has been included in this analysis are the **Czech Sector Award rankings** already introduced in Section 2.5. These rankings for the years 2001-2004[120] are included in the comparison as well, because they allow simple comparison across companies and give an idea about the development of the company's financial indicators in time. The comparison of all of the analyzed measures of credit risk is depicted on the following diagrams.

ČEZ, a.s.

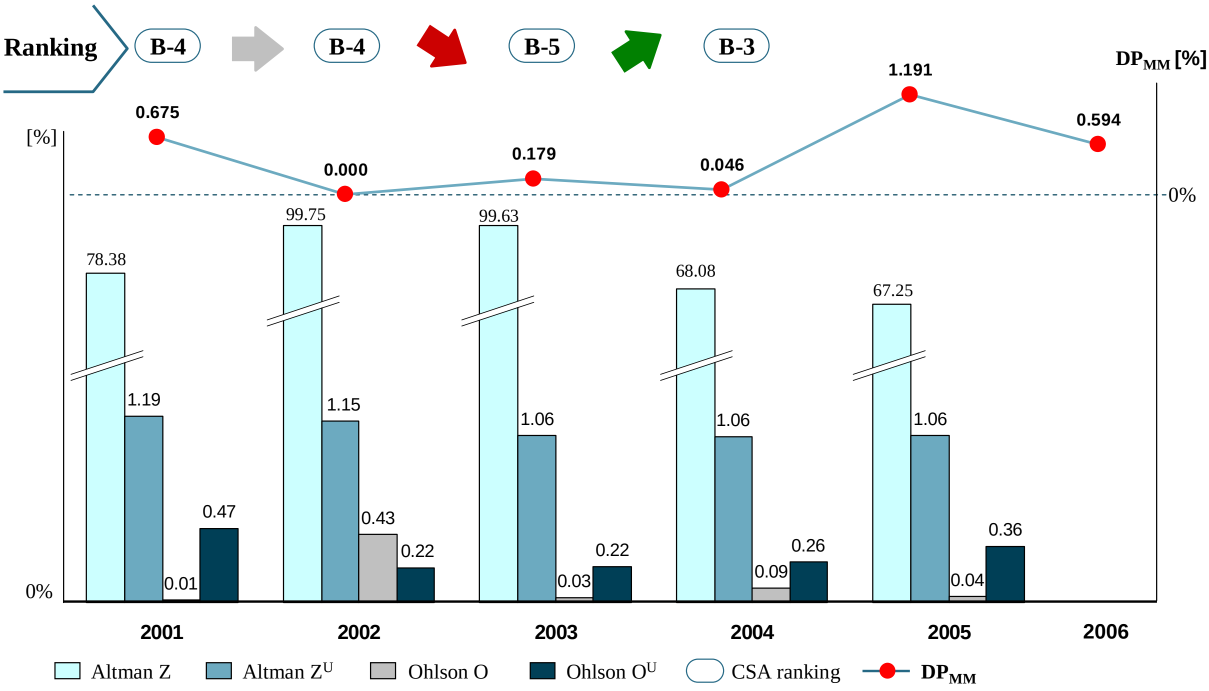
Probabilities of default [in %], CSA ranking and S&P and Moody's ratings



120 The rankings for the year 2005 haven't been published yet. Rankings haven't been awarded to Zentiva a.s. for the year 2004, as well as to Spolek pro chemickou a hutní výrobu, a.s. for the year 2002.

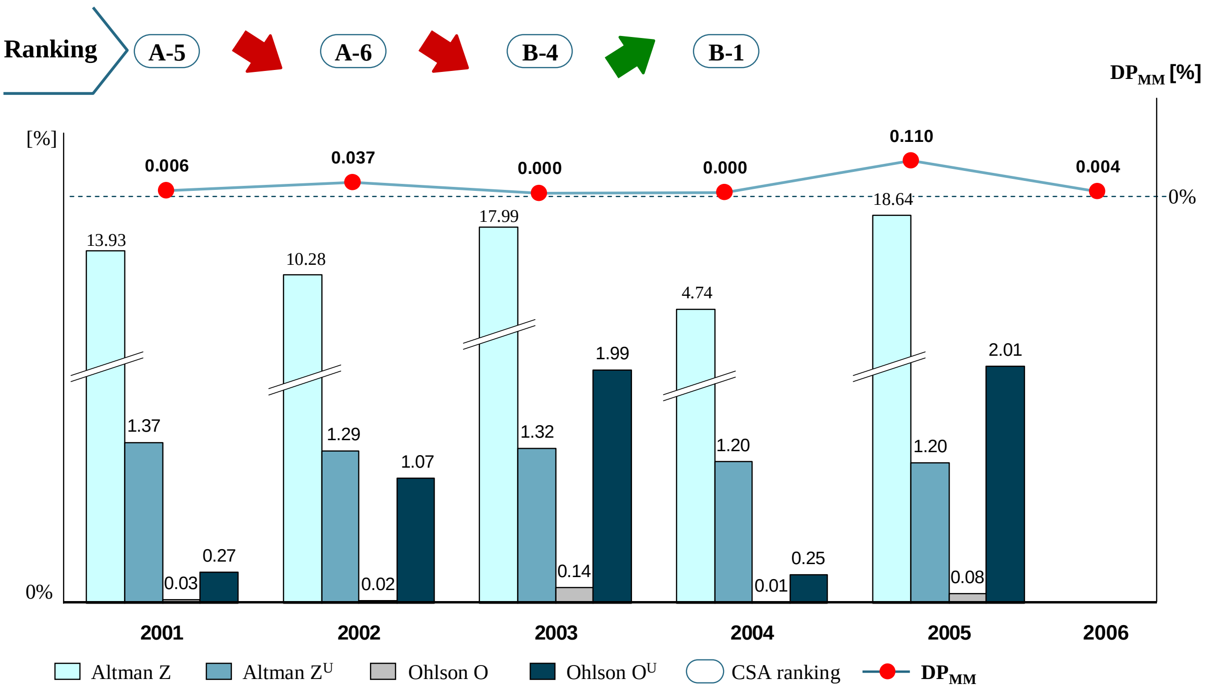
Jihočeské papírny Větřní, a.s.

Probabilities of default [in %] and Czech Sector Award ranking



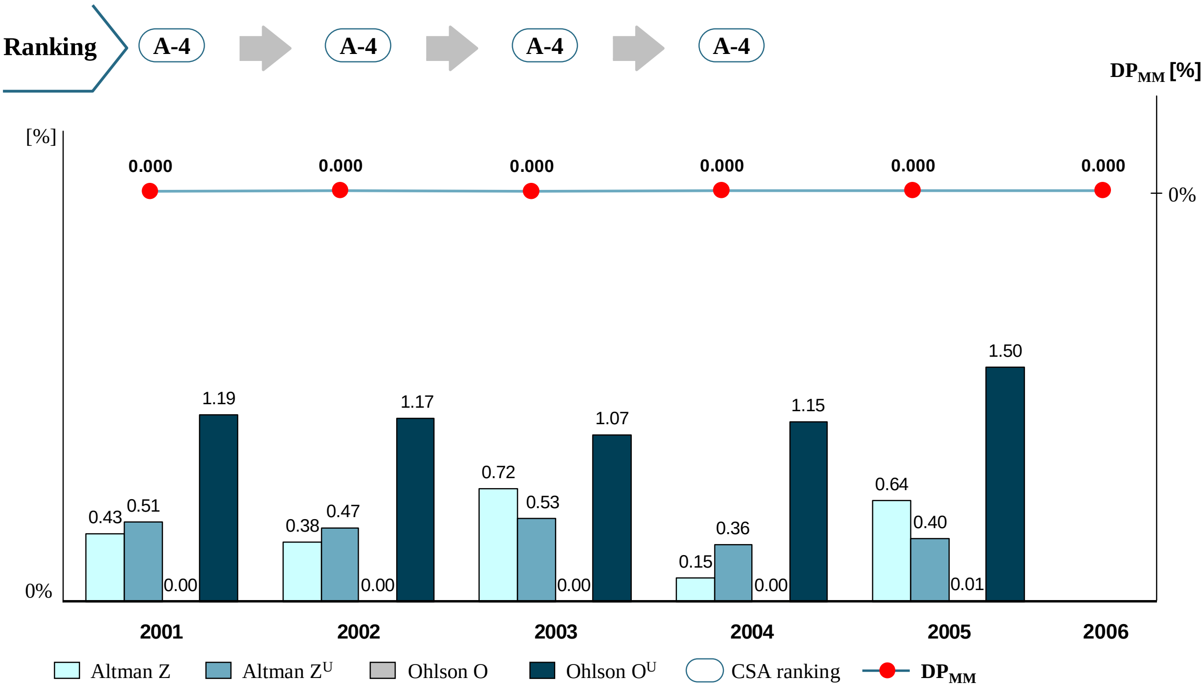
PARAMO, a.s.

Probabilities of default [in %] and Czech Sector Award ranking



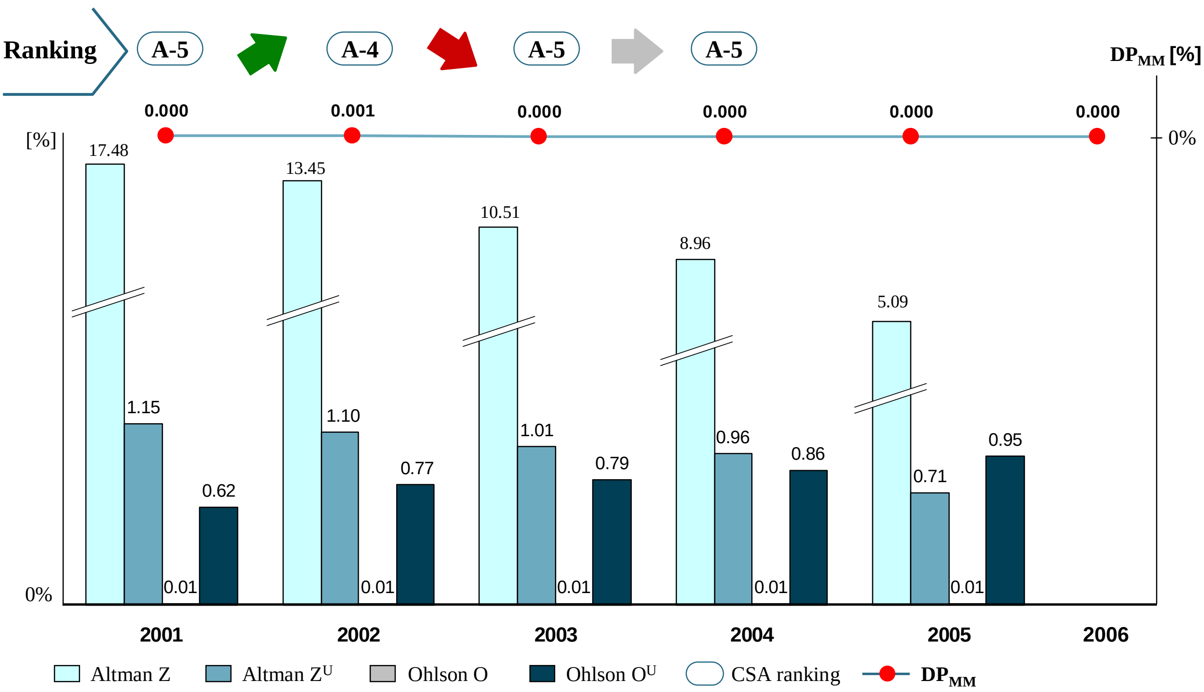
Philip Morris ČR a.s.

Probabilities of default [in %] and Czech Sector Award ranking



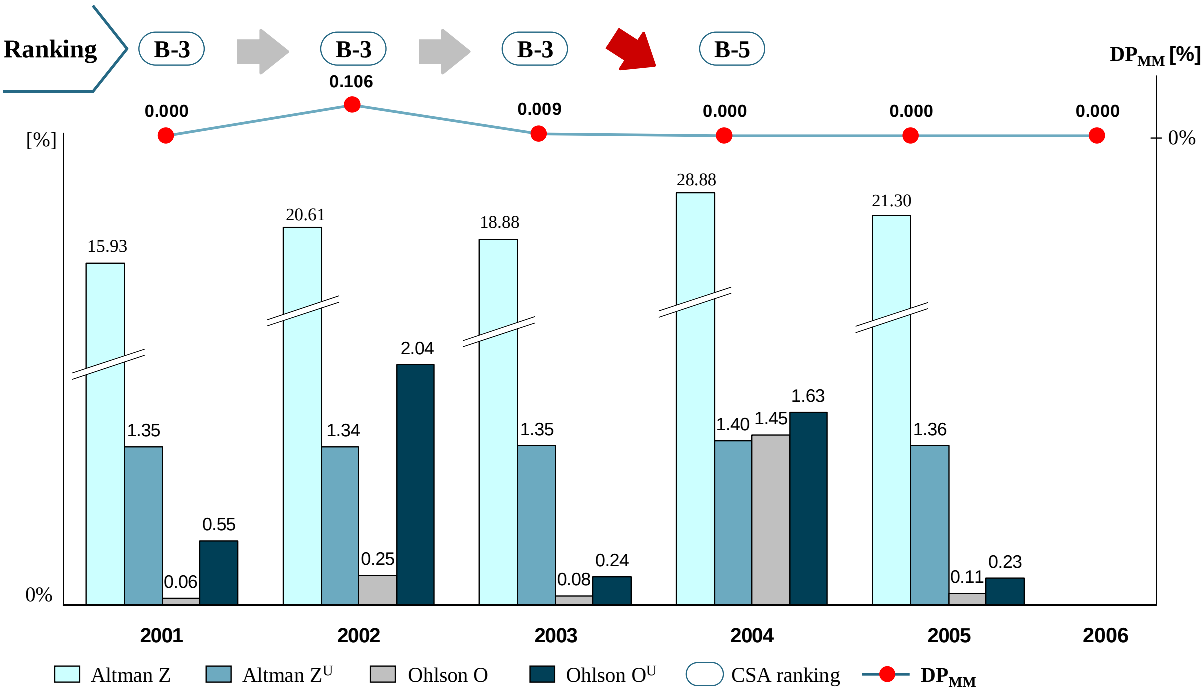
Pražská energetika, a.s.

Probabilities of default [in %] and Czech Sector Award ranking



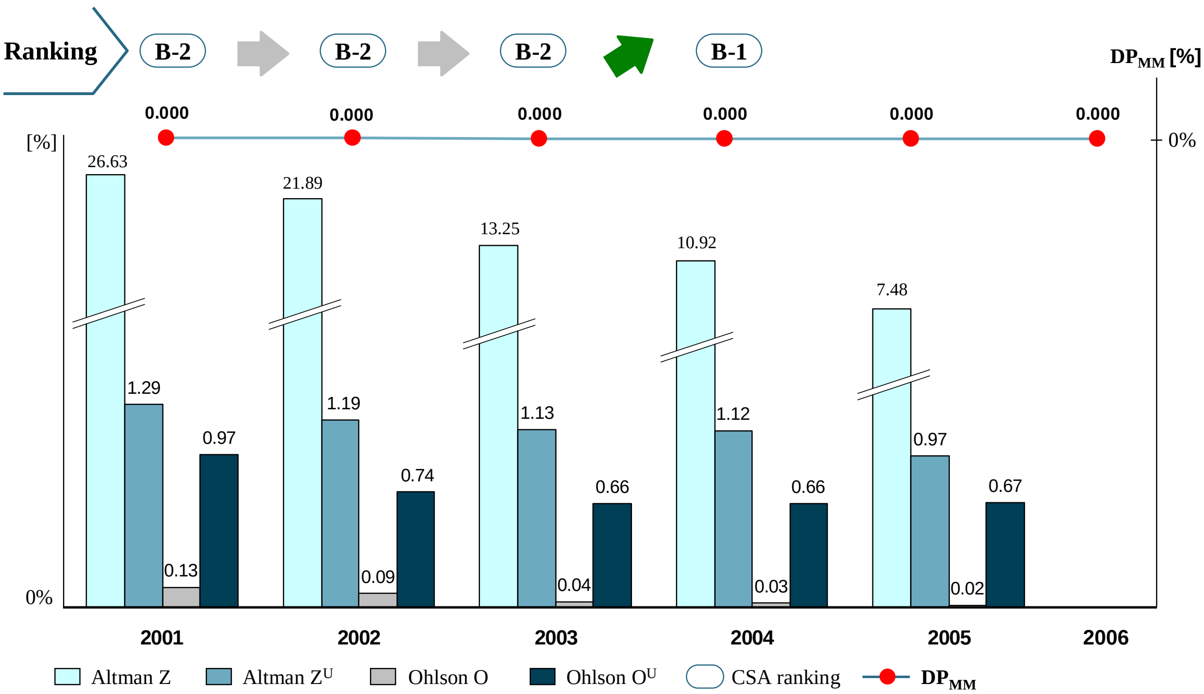
SETUZA a.s.

Probabilities of default [in %] and Czech Sector Award ranking



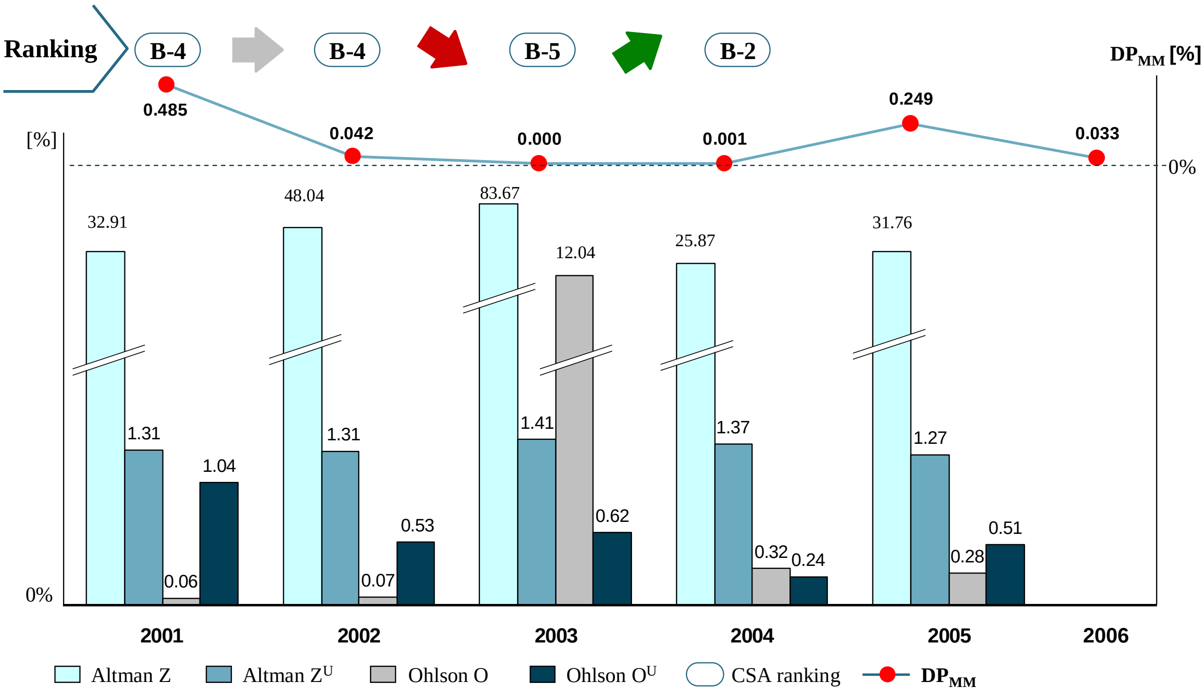
Severomoravská plynárenská, a.s.

Probabilities of default [in %] and Czech Sector Award ranking



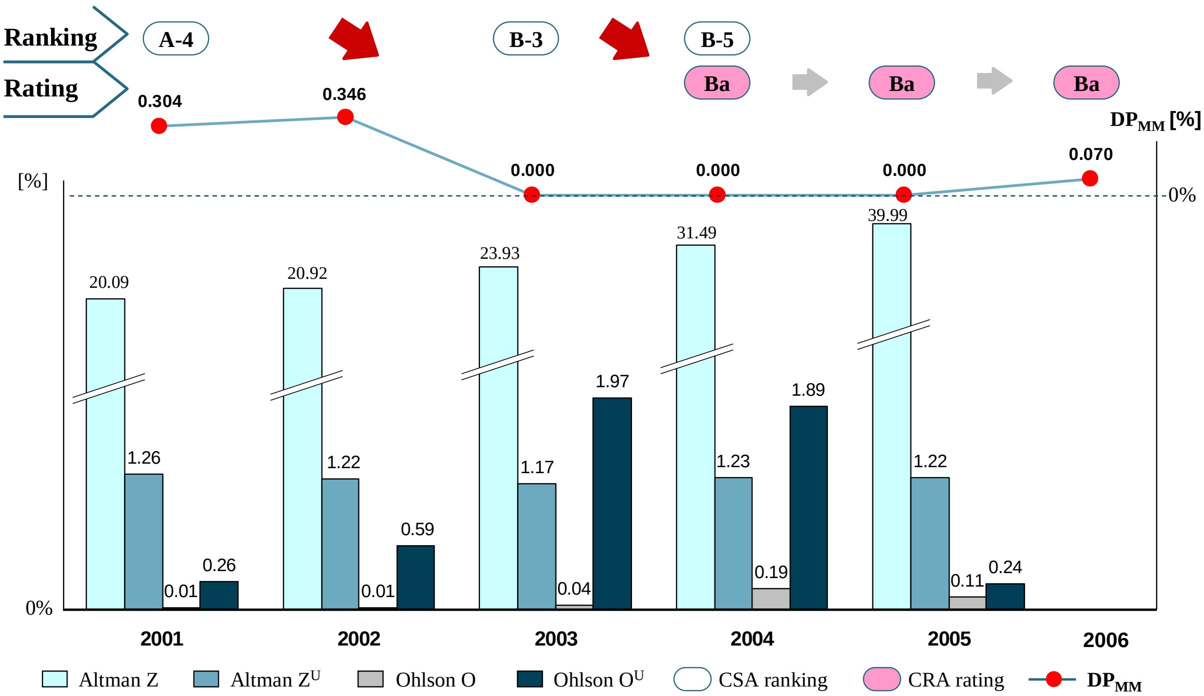
SPOLANA a.s.

Probabilities of default [in %] and Czech Sector Award ranking



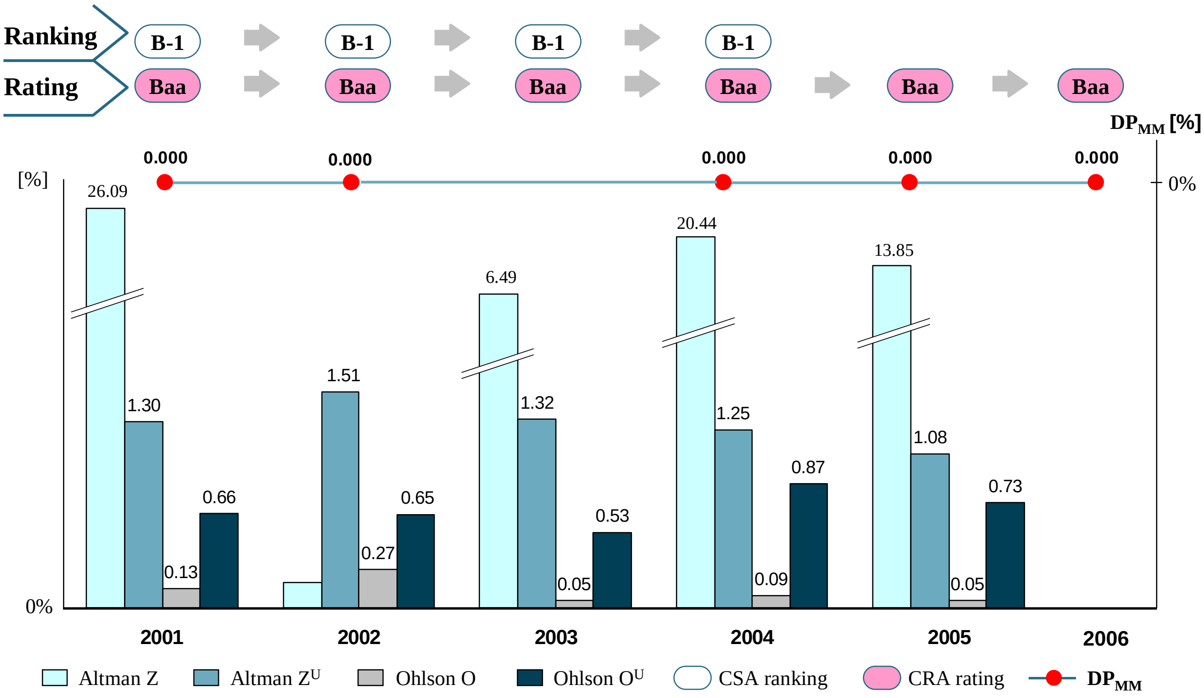
Spolek pro chemickou a hutní výrobu, a.s.

Probabilities of default [in %] and Czech Sector Award ranking and CRA international rating



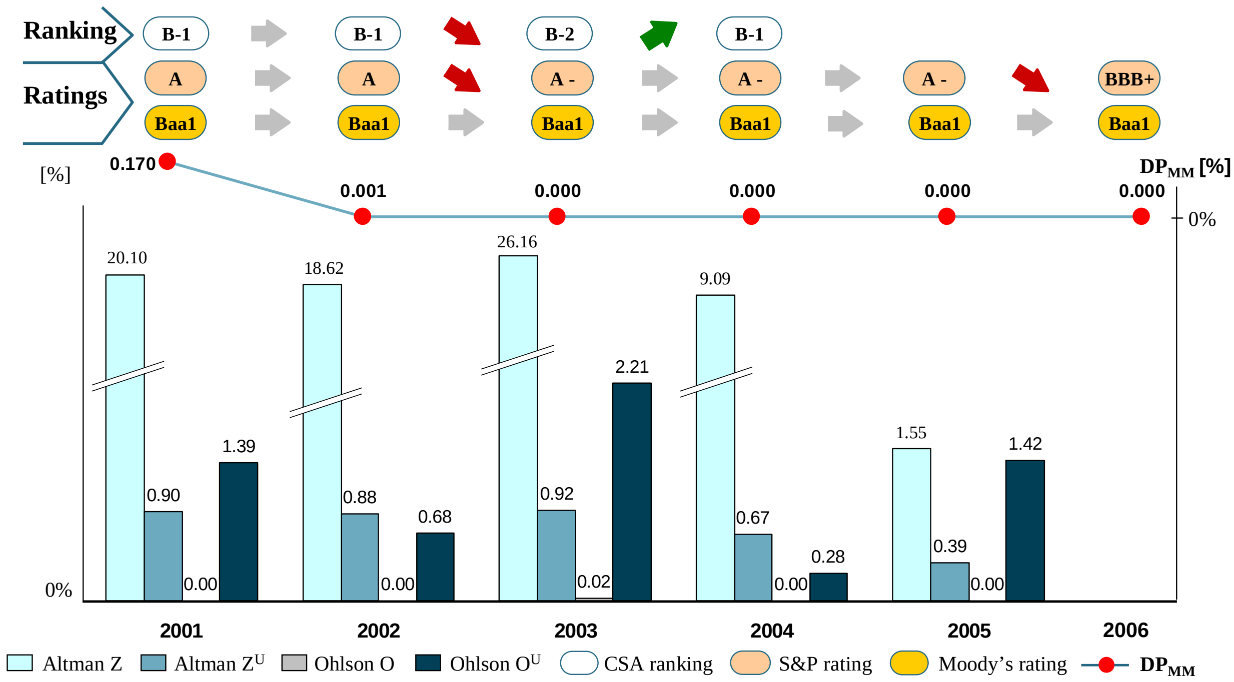
Středočeská plynárenská, a.s.

Probabilities of default [in %] and Czech Sector Award ranking and CRA international rating



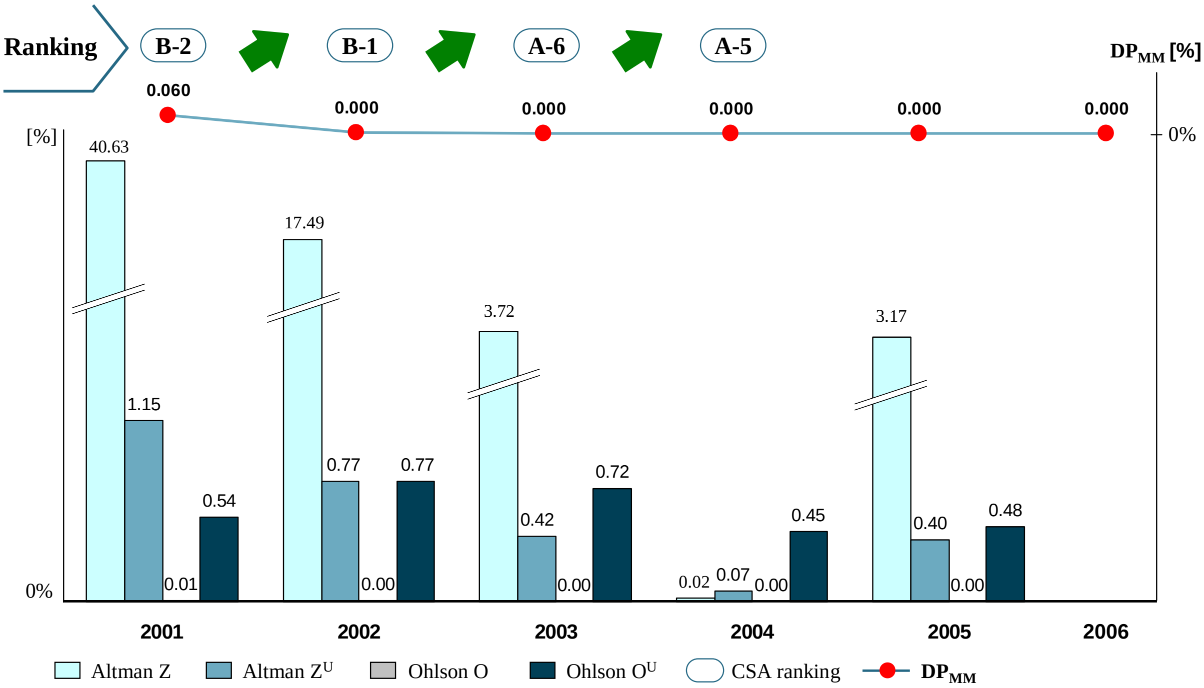
Telefónica O2 Czech Republic,a.s.

Probabilities of default [in %], CSA ranking and S&P and Moody's ratings



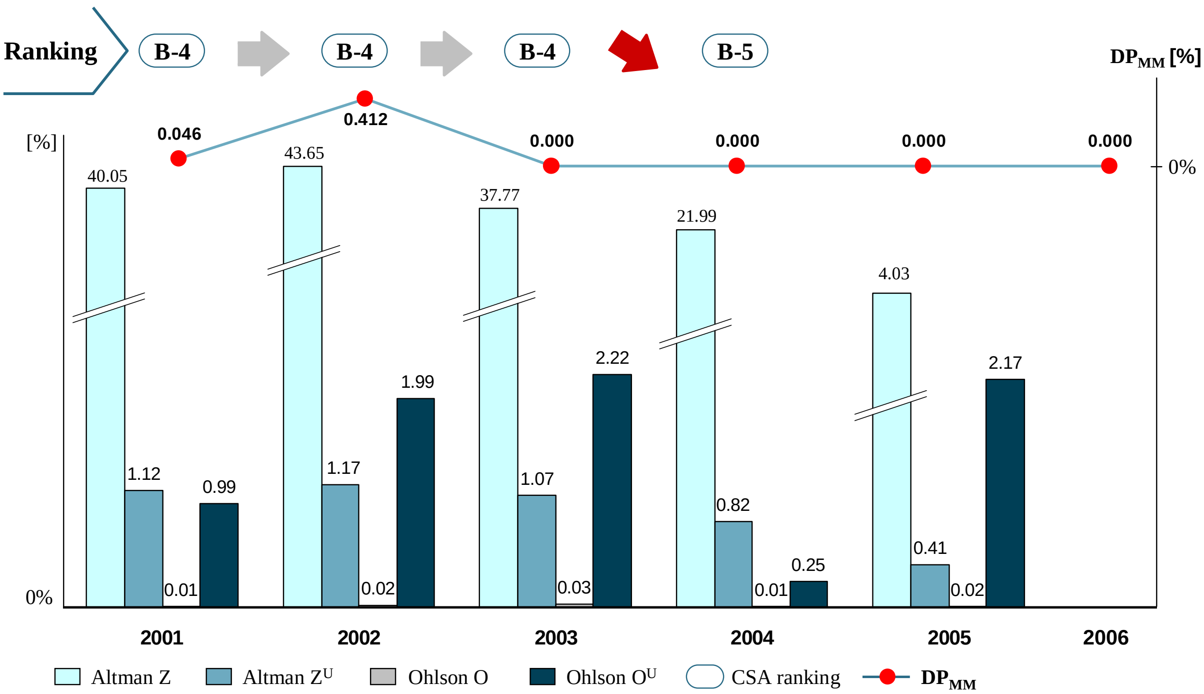
TOMA, a.s.

Probabilities of default [in %] and Czech Sector Award ranking



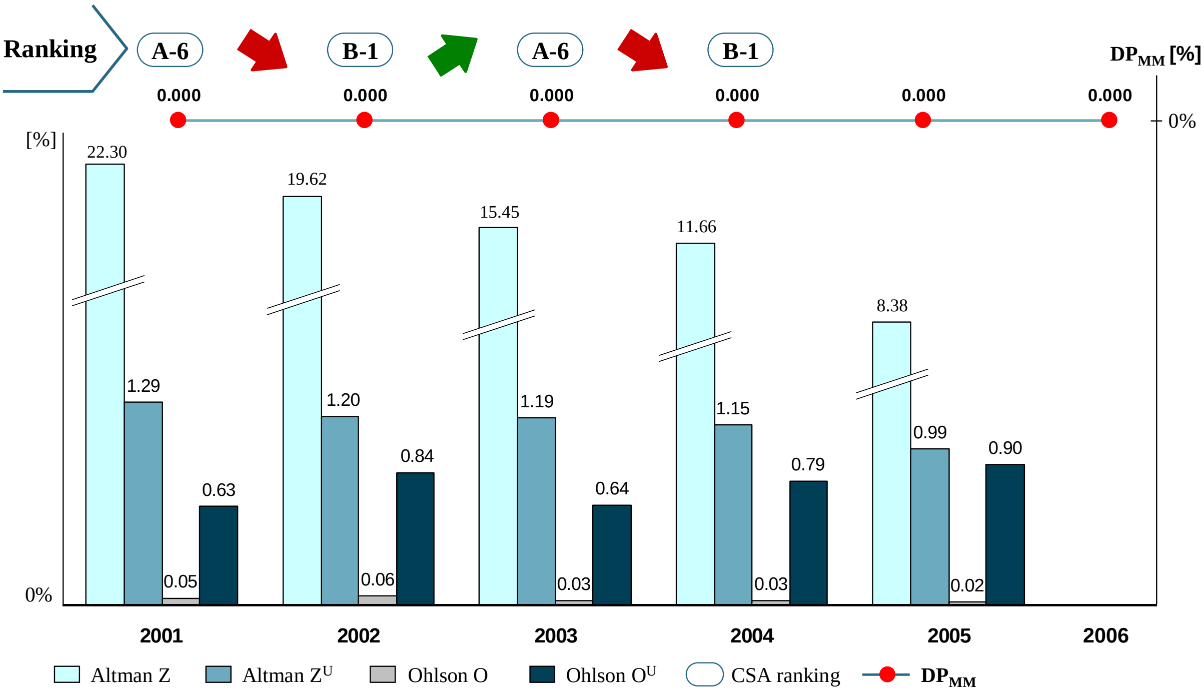
UNIPETROL, a.s.

Probabilities of default [in %] and Czech Sector Award ranking



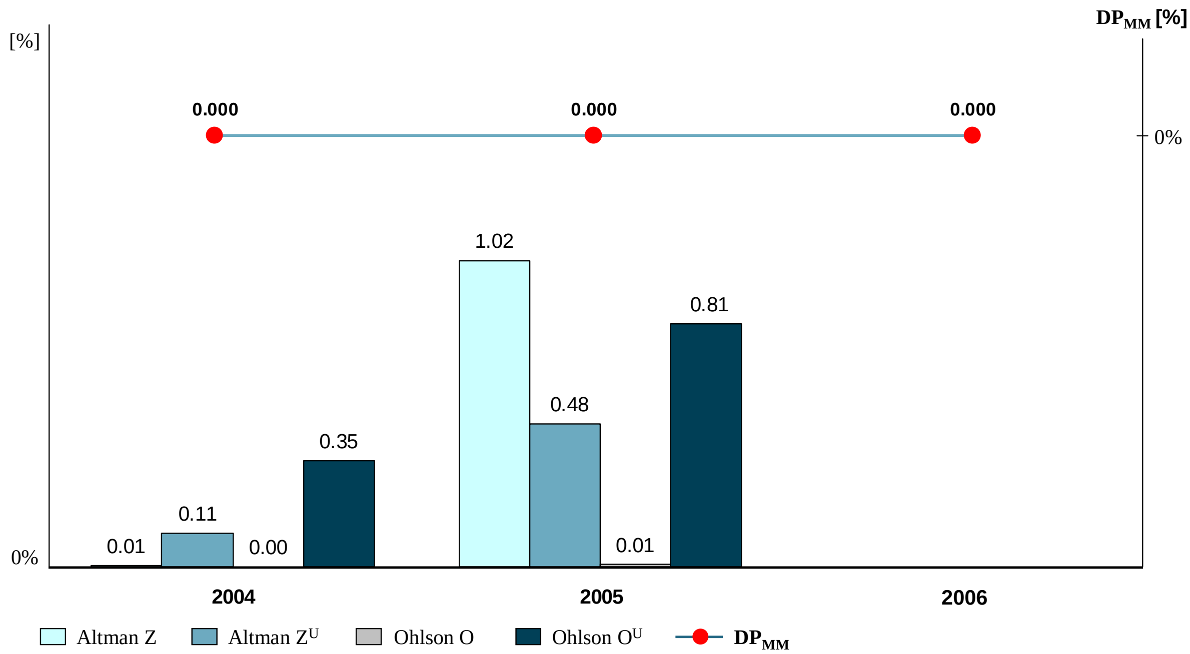
Východočeská plynárenská, a.s.

Probabilities of default [in %] and Czech Sector Award ranking



Zentiva a.s.

Probabilities of default [in %]



5.4. Comments on the results

5.4.1. The ability of the measures to incorporate new information

One way to evaluate the performance of the individual credit risk measures is to look at their **ability to incorporate new information about an adverse change in the financial health of a company**. The capabilities of these measures to identify a firm in financial distress can be most clearly demonstrated on the examples of Spolana, a.s. and Jihočeské papírny Větřní, a.s.

The floods that hit the Czech Republic in 2002 had a very big negative impact on **Spolana, a.s.** As a result of stopped production and high costs associated with the damaged facilities, Spolana has made a loss of CZK 0.5 billion in 2002. Moreover, in 2003 the company streamlined its production, which required the creation of a corrective item to assets in the amount of nearly two billion Czech crowns and led to a lost of CZK 2.6 billion. This loss was more than a half of the firm's total assets in the year 2003 (which decreased as well by almost CZK 3 billion). The CSA ranking given to Spolana was B-5, which is the second lowest ranking possible. The general manager of Spolana described the situation at the end of 2003 as "rising from the dead"[121] after the 2002 floods. The company survived this critical period and since 2004 started to be profitable.

The **original *Z*- and *O*- *Scores* reflected the financial distress** of the company as both of these measures predicted a higher default probability in the hard times for Spolana. Especially the otherwise very low Ohlson's *O-Score* appraised the huge loss of the year 2003 by a sharp increase from 0.07 to 12.04%. The improved financial situation since 2004 led to a decrease in these two scores. The *Z*- and *O*- *Scores* with updated coefficients also moderately increased in 2003 and decreased again in 2004.

Despite the evident troubles of Spolana, the **Merton model didn't reflect the adverse information** and the DP_{MM} was even decreasing during the years 2002 and 2003. The main reason is that despite the low-lying value of the company's shares, the equity volatility remained rather low and the firm's total liabilities stayed almost constant as well. Nevertheless, in the case of Spolana, the **Merton model failed to identify the serious worsening of the company's financial situation** and the implied default probability had been unreasonably low.

[121] Spolana, 2003 Annual Report

Interestingly, the predicted DP_{MM} increased in the year 2005 as a result of the sharp increase in the stock prices. This may be due to the **bias from the simultaneous inferring of the asset value and volatility** as described in Step 7 of Section 4.5. The quick growth in market share prices represents a quick decrease in the market leverage, which leads to an overestimation of asset volatility from Equation (34). This erroneously implies a higher probability of default while at the same time, the decrease in the market leverage would suggest improved credit.

Another example of the ability of the credit risk measures to reflect nearing default is that of **Jihočeské papírny Větřní, a.s.** This company had a highly unsuccessful year 2002 when it made a loss of almost CZK 1 billion and had to sell all of its minority capital interests and several subsidiaries to cover this loss and the losses from previous years. The net total assets fell to CZK 800 mil, which was lower than the absolute value of the total loss. The credit quality of the company had certainly been affected by the financial troubles and the company received a ranking of B-5.

The *Z-Score* was predicting for the years 2002 and 2003 a default with almost a 100% certainty and the *O-Score* increased from 0.01 to 0.43%. Nevertheless the default probability implied by the *O-Score* and all of the other measures of default risk (besides *Z-Score*) remained unreasonably low.

Despite the obvious plunge in share prices in the middle of 2003, the Merton model predicted a relatively low 0.18% default probability for this year as well. In 2005 (when the situation of the company was already relatively stabilized), the shares were highly volatile. Despite the fact that the shares were trading around the value approximately three times higher than the share price in the second half of 2003, the DP_{MM} reached its maximum for the whole sample- a 1.2% default probability. This would suggest that the **Merton-implied default probabilities are more sensitive to share price volatility than to the market value of equity**.

5.4.2. Comparison with credit ratings

In the case of ČEZ a.s. and Telefónica 02 Czech Republic, a.s.[122], some comparison with the ratings of external rating agencies is possible. According to Table 5, the probability of default for a BBB+ (Baa1) rated company moves between 10-40 (10-19) basis points. For

[122] Formerly Český Telecom, a.s.

the year 2001, this corresponds precisely to the values obtained by the Merton model for ČEZ a.s. In other years, the Merton model implies even lower probabilities, mostly as a result of the growing equity value of ČEZ. It is important to point out that a BBB+ rating still represents a very high investment grade as the Czech Republic never had a rating higher than A+.[123] The Merton-implied default probabilities for Telefónica 02 are almost identical as those of ČEZ. The ratings from the rating agencies are also very favorable and suggest a very low investment risk. The two companies that have ratings assigned by the CRA Rating Agency, Spolek pro chemickou a hutní výrobu, a.s. and Středočeská plynárenská, a.s., are also rated as low-risk companies, which corresponds to the low values of DP_{MM}.

Overall, the only conclusion that can be drawn for the Merton model from the comparison with credit ratings is that the **Merton model correctly estimates very low default probabilities** for these blue-chip companies. On the other hand, the probabilities implied by the original *Z-Score* and the updated *O-Score* seem to be very high for these highly-rated companies.

5.4.3. The quality of the accounting-based measures

The probabilities of default derived from the Altman's ***Z-Score*** have been **very high** for many of the firm-year observations. The reason can be that scoring functions for Altman's ***Z-Score*** and Ohlson's *O-Score* were estimated on the data from the U.S. companies. But there are major differences in the form of financing between the United States and Europe. In Europe, the ratio of debt to equity is traditionally higher.[124] But at the same time, this ratio significantly influences the resulting *Z-Score*. This has the consequence that several companies are evaluated by the *Z-Score* as having high probability of bankruptcy, whereas in reality they are considered to be well established and safe. To give an example, the probability of ČEZ and Telefónica 02 defaulting between the years 2001 and 2003 has been assessed by the *Z-Score* to be above 15%, which is unreasonably high. Looking aside from the generally high default probabilities, the **Z-Score seems to be able to reflect adverse changes in companies' financial situation**, such as in the cases of Spolana and Jihočeské papírny Větřní.

The situation for the **original *O-Scores*** is similar except that the implied default probabilities are for most of the companies on the contrary very low (below 0.01%).

[123] As of January 2007, the S&P rating for the Czech Republic is A-. Current rating from Moody's is A1.
[124] Pečená (2003)

Nevertheless, they seem to be able to **correctly identify firms in financial distress** as well. This is caused by their relatively high sensitivity to the profitability of the company, such as in the case of Spolana in 2003.

The analysis of results for the ***Z-* and *O- Scores* with updated coefficients** gives ambivalent results. They generally tend to be very stable across companies and in time. This is mostly caused by their relatively low coefficients of the explanatory variables entering the scoring functions in comparison to the constant term. Even though they correctly incorporate some of the information from the financial statements, the **fluctuations in the implied probabilities are too low** to call these measures good approximations of reality.

5.4.3. The quality of the Merton model

The computed **Merton-implied default probabilities** were for all of the companies **very low**. This partially corresponds to the results of several empirical studies.[125] For example, Hillegeist et al. (2004) on a very large sample of US companies find that the median estimate for DP_{MM} is for non-bankrupt companies essentially zero (0.01%). Nevertheless, the mean estimate of DP_{MM} for these companies is already 5.61% and for the bankrupt companies in the sample, the mean rate was 24.76%. Therefore a zero DP_{MM} for Spolana, a.s. and Jihočeské papírny, a.s. in 2003 (2002), cannot be justified as both of these companies were in these years in serious troubles. For most of the other companies, the low Merton-implied default probabilities can represent the fact that the **companies listed on PSE** are considered to be of **high quality** and creditworthiness. Moreover, the companies with the highest liquidity (i.e. members of SPAD)[126] are perceived as the Czech blue-chip companies. **Another reason for the low Merton-implied default probabilities can be the use of Normal distribution to map the distances-to-default into probabilities, because the** probabilities from the Normal distribution are too low**.**[127] For example, a DD of 4 (similar to that of Spolana in 2004) is mapped by the Moody's KMV model into a default probability of 1%.[128] The corresponding DP_{MM} based on the Normal distribution is essentially zero.

[125] E.g. Bohn (2000), Mella-Barral and Perraudin (1997), Frank and Torous (1989), and Jones et al. (1984) find theoretical credit spreads (a function of the default probabilities) lower than the actual credit spreads.
[126] These companies are ČEZ, Philip Morris, Telefónica 02, Unipetrol, and Zentiva. SPAD (System for Support of the Share and Bond Markets) trading is based on the activities of market makers who are responsible for providing enough liquidity on the market.
[127] Measuring & Managing Credit Risk: Understanding the EDF Credit Measure for Public Firms (2004)
[128] Crosbie and Bohn (2003)

Despite these justifications for the low default probabilities, in my opinion the **Merton model does not sufficiently incorporate the information about the worsening situation of the companies**, such as in the case of Spolana in 2003. One of the reasons can be that the key assumption about the market efficiency may be violated. The Merton model relies on the ability of financial markets to properly incorporate information from the firms' financial statements and reflect the underlying credit risk of the company. This assumption for the Czech Republic has already been rejected by previous studies (see Section 5.1). Moreover, the last six years can be defined as time of "stock market enthusiasm", which can be illustrated by the impressive growth rates of PX, the PSE index- (see Appendix C). All of the analyzed companies (except Jihočeské papírny) ended the year 2006 with share price values significantly higher than those of 2001. In such time of investors' "euphoria", the share prices are more of a reflection of general climate on the markets than an appraisal of the "real" situation of the company. But worse yet, in the cases when the stock market correctly reflected financial distress, such as a in the cases of Spolana, a.s. and Jihočeské papírny Větřní, a.s., and the share prices fell considerably, the **Merton model failed to incorporate this information from the financial market** and predict higher probabilities of default.

One of the reasons may be that simultaneous inferring of the asset value and volatility already discussed earlier in this paper cause a **bias of the model in both directions**. Therefore, a sharp decrease in share prices leads to an underestimation of default probability and the sharp increase results in an overestimation of DP_{MM}. The "downward" bias can be deduced for example from the low implied probabilities in the year 2002 for Spolana or the year 2003 for Jihočeské papírny. The "upward" bias can be witnessed in the periods of high growth in share prices for e.g. Spolana, Jihočeské papírny or PARAMO in the year 2005.

Moreover, the original Merton model is especially good at predicting defaults for extremely low quality firms[129] but the companies listed on the Prague Stock Exchange are considered to be blue-chip companies.

Based on the observations of the Merton-implied default probabilities and their comparison with the other measures of credit risk and "reality", the **Merton model cannot be seen as a good measure of credit risk for the analyzed companies**. There can be many reasons for the poor performance of the model ranging from the violated assumption of

[129] Sobehart, Keenan and Stein (2000)

market efficiency to the wrong modeling choices. But as many reasons as there might be, none of them however defends the use of the DP_{MM} as a good approximation of credit risk.

6. Model quality evaluation

In the previous chapter, the method for assessment of quality of the respective measures of credit risk was largely based on interpretation of the diagrams and their confrontation with the financial statements of the analysis. Such analysis makes it possible to comment on the performance of the individual credit risk indicators and make suppositions about their quality. But some questions like: **“Which of the measures of credit risk is the best approximation of the real situation of the company? How good is this approximation?”** can arise. To give an answer to these questions, some **test of model quality** has to be carried out.

In order to carry out an empirical study testing the performance of a credit risk model, some **ground for model quality evaluation is necessary**. In the context of the Merton model, such ground can be represented by observed credit spreads on the markets, large samples of companies (with a certain part of this sample being a population of bankrupt firms), and credit ratings from external rating agencies.

6.1. Tests of model quality based on credit spreads

It has often been the case that empirical tests of the Merton model have used the Merton-implied **theoretical credit spreads** (as introduced in Section 4.6.4) and compared these spreads with the actual spreads observed on the markets.[130] Nevertheless, such test of the performance of the Merton model **does not allow the comparison with other measures of credit risk**, such as the accounting-based variables, as these measures don’t give explicit formulas for credit spreads. Moreover, using credit spreads as a measure of default probability has many shortcomings. The most important one is that the credit spreads are a result of many other factors besides the purely economic ones.[131]

[130] For a selection of empirical tests utilizing the credit spreads, refer to the Section 4.11. of this paper.
[131] E.g. a bank is trying to attract a new client, which may result in lower credit spread.

6.2. Tests of model quality based on large samples of bankrupt and non-bankrupt companies

Such ground for model quality evaluation that allows comparison of alternative methods/models could ideally be represented by a **sample of bankrupt and non-bankrupt companies**. In this context the **model quality is determined by its ability to correctly identify failed and non-failed firms.** Most of the empirical studies assessing the ability of the Merton model to forecast default have been conducted on these large samples of bankrupt and non-bankrupt companies.[132] One option of how to use this ground for model quality evaluation is to use prediction-oriented tests to distinguish between alternative statistical models and/or different groups of explanatory variables.[133] Because the dependent variable is binominal in nature (the company defaults or it doesn't) a single-period logit[134] approach is usually taken, where only one observation per company is used.[135] Another possibility is to use multi-period relative information content tests to compare the amount of bankruptcy-related information contained in each of the compared bankruptcy measures.[136]

Both of these methods have been in the last years continuously refined and can be seen as the best possibility to assess the models' ability to estimate the probability of default. However, for evaluation of the Merton model, these tests **require large databases of publicly traded companies** with a part of the population consisting of bankrupt firms. Such datasets can be mostly found in the Anglo-Saxon countries and access to these databases is not free of cost.

6.3. Tests of model quality based on credit ratings

An alternative ground for model quality analysis can be represented by **credit ratings**. Thus, the **ability to correctly predict corporate credit rating defines the model quality**. The tests of quality of the Merton model (or other models of default probability) that are based on credit ratings from the external rating agencies assume that **credit rating is the best**

[132] For example Hillegeist et al. (2004) use a sample of 78,100 firm-years including 756 initial bankruptcies.
[133] These tests often involve determining a cutoff value that is used to classify which firms are expected to remain solvent and which are expected to declare bankruptcy within a particular (typically one-year) time horizon. Prediction accuracy is assessed by comparing the total Type I and II error rates for each alternative specification, and the model with the lowest total error rate is deemed the best. [Hillegeist et al. (2004)]
[134] See Section 6.3.2 of this paper for the discussion on logit and logistic transformation.
[135] Shumway (2001)
[136] E.g. Hillegeist et al. (2004) use a discrete hazard model to assess how well each bankruptcy probability measure explains the actual probability of bankruptcy.

available proxy for the probability of default. The criteria employed by rating agencies for determining specific ratings are in these tests considered to be the most comprehensive because they involve both quantitative and qualitative factors.

In order to understand the tests of model quality based on credit ratings, it is necessary to take a small detour and look at the rating models.

6.3.1. Rating models

Derived from the traditional models of credit risk described in Chapter 2, the rating models have represented a new framework in assessing the credit quality of public firms and were based on the pioneering work of McKelvey and Zavoina (1975).[137] The rating models are a standardized approach to assigning a credit rating based on a predetermined information set. The credit rating, as a discrete variable, is assessed through ordered dependent variable models, such as logit or probit. These models allow an ordered specification for the credit quality of firms where default can be regarded as a special case of credit rating.

The rating models lack the qualitative approach of the credit ratings assigned by international rating agencies (as introduced in Chapter 3). As a result, these rating agencies have been skeptical about whether models using publicly available information can replicate the professional rating process. Nevertheless, the **methodology of the ratings models,** where the credit rating, as a discrete variable, is assessed through ordered dependent variable models, can be used for assessment of explanatory power of the individual input variables.

6.3.2. The ordered logit regression[138]

The **ordinary logit model** has the following form

$$(56) \qquad p_i = \frac{e^{\alpha + X_i\beta}}{1+e^{\alpha + X_i\beta}}$$

where p_i is the actual probability of bankruptcy of i-th firm, α is a constant, X_i is a vector of (continuous) explanatory variables, and β is the coefficient vector. This model can be easily linearized using logistic transformation into

[137] Notable contributions in explaining and predicting credit ratings with ordered dependent variable models are from Cantor and Packer (1996), Blume, Lim and Mackinlay (1998) and Pottier and Sommer (1999)

[138] Before going into details about the logit and ordered logit, the interested reader should refer to the used-sources list of web pages dedicated to the issue of logit and logistic transformation (especially the comprehensible pages of STATISTICA software http://www.statsoft.com/textbook/glosl.html).

(57) $$\operatorname{logit}(p_i) = \ln\left(\frac{p_i}{1-p_i}\right) = \alpha + X_i\beta$$

In the case of credit ratings, there are more categories (the ratings), in which the dependent variable can fall and therefore an **ordered logit regression should be used**.[139] The ordered logit model depends upon the idea of the cumulative logit, which in turn relies on the idea of the **cumulative probability**. The cumulative probability C_j can be thought of as the probability that the rating of a company, *y*, is in the *j* th or lower category (i.e. it has a rating of j or a lower rating):

(58) $$C_j = \Pr(y \le j) = \sum_{k=1}^{j} \Pr(y = k)$$

The logistic transformation (see Equation (57)) turns this cumulative probability into the **cumulative logit** expressed as a **linear function of independent variables**

(59) $$\boxed{\operatorname{logit}(C_j) = \ln\left(\frac{C_j}{1-C_j}\right) = \alpha_j + \beta_1 x_1 + \beta_2 x_2 ... + \beta_n x_n}$$

where *j* denotes the given category of the dependent variable (the rating), α_j is the intercept for this rating category, and $x_{1,2,...n}$ are the independent variables that enter the model with their respective betas β.

If $p_{1,2,...k}$ are the probabilities that the response variable will fall in the *j*-th category ($j = 1, 2, ...k$) or lower, the cumulative ordered logit can be expressed as:

(60) $$\begin{aligned}
&\operatorname{logit}(p_1) \equiv \ln\left(\frac{p_1}{1-p_1}\right) = \alpha_1 + \beta_1 x_1 + \beta_2 x_2 ... + \beta_n x_n \\
&\operatorname{logit}(p_1 + p_2) \equiv \ln\left(\frac{p_1 + p_2}{1-p_1-p_2}\right) = \alpha_2 + \beta_1 x_1 + \beta_2 x_2 ... + \beta_n x_n \\
&\qquad \vdots \\
&\operatorname{logit}(p_1 + p_2 + ... + p_k) \equiv \ln\left(\frac{p_1 + p_2 ... + p_k}{1-p_1-p_2 ... - p_k}\right) = \alpha_k + \beta_1 x_1 + \beta_2 x_2 ... + \beta_n x_n \\
&\text{and} \quad p_1 + p_2 + ... + p_k = 1
\end{aligned}$$

139 For discussion on this topic, see e.g. Hao (2006) or the elaborate web pages of the Columbia University in New York (http://www.columbia.edu/~ag2319/teaching/G4075_Outline/node27.html)

There is a different intercept α_j for each level of the cumulative logit, but $\beta_{i=1,2\ldots n}$ remain constant. Each α_j indicates the logit of the odds of being equal to or less than category *j* when all independent variables are zero. Thus, these intercepts will increase over *j*.[140] The β determines, how a one-unit increase in the independent variable increases the log-odds of being lower than category *j*.

It is important to point out that the above mentioned model is the **single-period ordered logit model**, which only includes one firm-year observation for each company. In order to model time-varying changes in the underlying risk of bankruptcy, a **multi-period ordered logit**[141], written here in the vector form where *t* denotes time period (i.e. year of observation)**:**

$$\text{(61)} \qquad \operatorname{logit}\left(C_{i,t}\right) = \alpha(t) + \beta x_{i,t}$$

would have to be used.

6.4. The test of the Merton model in the Czech Republic

In order to compare the results of the Merton model with those of the traditional accounting based measures (the *Z*- and *O- Scores*) of companies in the Czech Republic, the **tests based on large samples of bankrupt and non-bankrupt companies cannot be used**. The reason is that until now, **not a single firm among the few publicly listed companies on PSE has defaulted**.

For the purpose of testing model quality based on the **credit ratings**, a considerable amount of companies with an assigned external rating has to be obtained. **The total number of fifteen analyzed companies** would already be **too small** to obtain any statistically reliable results. Moreover, there is a major problem with the **insufficient number of ratings** assigned by external rating agencies to the 15 selected companies. Standard & Poor's and Moody's assigned ratings only to ČEZ, a.s. and Telefónica O2 Czech Republic, a.s. and the CRA Rating Agency rated only two companies as well (Středočeská plynárenská, a.s. and Spolek pro chemickou a hutní výrobu, a.s.).

[140] These intercepts are sometimes referred to as cutpoints, or cut-off points as they indicate the boundaries of the dependent variable (the rating categories).
[141] See e.g. Hao (2006) for an implementation of the multi-period ordered logit.

Therefore I can conclude that **in the Czech Republic, there is no ground for the evaluation of the Merton model, which would allow a statistically reliable comparison with other** (accounting-based) **measures of credit risk.**

6.4.1. Rankings as the ground for model quality

Nevertheless, I would like to **illustrate the theoretical concepts of tests based on credit ratings** (described in Section 6.3.2) on a **practical example**. In these tests, the credit rating, as a discrete variable, is assessed through ordered dependent variable model to determine the explanatory power of the individual input variables. Here, the computed measures of credit risk from Chapter 5 (DP_{MM}, the *Z-Score*, $Z\text{-}Score^U$, *O-Score* and $O\text{-}Score^U$) enter the model as the independent variables and their ability to correctly explain the response variable is assessed. Since the external credit ratings as the dependent (response) variables are missing, some other discrete (ordinal) variable of credit risk has to be used instead. The only such available source of information about solvency for all of the analyzed companies are the **Czech Sector Award rankings** already introduced in Section 2.5 and in Chapter 5. These rankings are available for the analyzed companies between the years 2001-2004[142] (see Attachment 1 "CSA rankings" for the list of these rankings).

But rankings are, unlike credit ratings, purely quantitative methods based on financial indicators, and the **ability of the CSA rankings to correctly represent the "real" default probabilities is very limited**. Because of the **very small sample size** and the **use of the simple rankings instead of credit ratings**, running an **ordered logit regression on this dataset cannot generate any reliable results with statistical power**. Nevertheless, the **methodology**, which should be seen as the main focus of this section, **is equivalent to using credit ratings** as the ground for model quality evaluation.

In this actual test, **the tested hypothesis is whether the probability of default estimated from the Merton model (**DP_{MM}**) will be able to explain the company rankings better than the accounting-based measures.**

[142] The rankings for the year 2005 haven't been published yet.

6.4.2. The ordered logit regression for the Czech companies

The independent variables in the form of probabilities are not consistent with the ordered logit model. Therefore, DP_{MM} should be the **transformed into a score** using the **inverse logistic function** (i.e. the opposite process to that described in Equation (55)), so that

$$(62) \qquad MMscore = \ln\left(\frac{DP_{MM}}{1-DP_{MM}}\right)$$

As DP_{MM} approaches zero (one), *MMscore* approaches negative (positive) infinity. In order to exclude extremely large values from the regression, the scores are "trimmed" so that the minimum (maximum) value of *MMscore* is -13.8155 (+13.8155), which is equivalent to DP_{MM} being 0.000001 (0.999999).

The single-period **ordered logit model from** Equation (59) **takes** (after substituting for the explanatory variables, transforming the DP_{MM} into *MMscore* and taking into account the total number of ranking categories[143]), the **following form**:

$$\begin{aligned}
\operatorname{logit}(C_1) &\equiv \alpha_1 + \beta_1(MMscore) + \beta_2(\text{Z-Score}) + \beta_3(\text{Z-Score}^{\text{U}}) + \beta_4(\text{O-Score}) + \beta_5(\text{O-Score}^{\text{U}}) \\
\operatorname{logit}(C_2) &\equiv \alpha_2 + \beta_1(MMscore) + \beta_2(\text{Z-Score}) + \beta_3(\text{Z-Score}^{\text{U}}) + \beta_4(\text{O-Score}) + \beta_5(\text{O-Score}^{\text{U}}) \\
&\dots \\
&\dots \\
\operatorname{logit}(C_8) &\equiv \alpha_8 + \beta_1(MMscore) + \beta_2(\text{Z-Score}) + \beta_3(\text{Z-Score}^{\text{U}}) + \beta_4(\text{O-Score}) + \beta_5(\text{O-Score}^{\text{U}})
\end{aligned}$$

Each α_j indicates the logit of the odds of being equal to or less than category *j* when all independent variables are zero. Thus, these intercepts (i.e. the cut-off points between the different ranks) will increase over *j*.

The β determines how a one-unit increase in the independent variable increases the log-odds of being higher than category *j*. The economic intuition suggests that with an increase in the probability of default (increase in the *MM-*, *Z-* and *O- Scores*), the ranking should decrease (i.e. the cumulative probability that the ranking will be lower or equal to *j* should increase) and therefore positive signs before the respective betas can be expected.

[143] The rankings in the sample only range from B-5 (minimum=1) to A-4 (maximum=8), which is only 8 categories. Besides the year 2003, not all of the rankings were present in the sample for the other years and therefore, the actual number of response variables in the model ranges between 6 and 8.

The four single-period ordered logit regressions for the years 2001-2004 were again computed by the statistical software SAS. The SAS program for the ordered logit regression is in Appendix F. The SAS file “Logit.sas” as well as the permanent datafile “Logit.sas7bdat” are again attached (Attachment 2) for the interested reader to obtain the same results. The output window has been saved as “Logit_Output.doc” and is included in the Attachment 2 as well.

6.4.3. Results of the ordered logit regression with all variables

According to various statistical tests[144], the ordered logit model, as expected, displays **very poor performance**. For example, the Wald and Score Chi-Square tests for global null hypothesis (suggesting that all of the coefficients are zero) could not be rejected at the 5% level for any of the four years.[145] For the year 2004, SAS reports that a maximum likelihood estimate of coefficients does not even exist. Based on the Wald Chi-Square statistics and their respective p-values, the absolute majority of coefficient was evaluated as insignificant.

Judging by the p-values, the variables that have generally been evaluated as the best **are the original *Z*- and *O*- *Score***. Moreover, the sign for these two scores has been in all four years positive, which complies with the intuition and the increasing intercepts over ordered values (the rankings). The relatively “best” performance of these traditional, accounting-based methods is most likely caused by the construction of the rankings. The ranking is, just like the Altman’s *Z* and Ohlson’s *O*, a scoring function computed from the financial statements of the company.[146]

What may be a little surprising is the **poor performance** of these two scores with the **updated coefficients**. This may be caused by the fact that some of the new coefficients, which were estimated on US data, are against the economic intuition.[147]

On average, the ordered logit test based on rankings found the **Merton model** to be the **worst measure of credit risk assessment**. The *MMscore* hasn’t been significant in any of the years at the 15% level and for two years, the sign of the coefficients has been negative, which would imply that with an increase in the default probability, the probability of getting a

[144] For an annotated SAS output of an ordered logit regression and explanations of the various tests and statistical issues, refer to the web pages of Columbia University:
http://www.ats.ucla.edu/stat/sas/output/sas_ologit_output.htm
[145] Except the year 2003, when this hypothesis based on the Score test could be rejected.
[146] However, the *Z-Score* also incorporates a market-based measure- the market value of equity.
[147] For example, the updated *Z-Score* implies that an increase in the Sales/Total Assets ratio leads to an increase in the default probability.

higher ranking is increasing. As mentioned before, the results of the regression analysis are not reliable and no sound conclusions can be made on their ground. . However, the actual results suggest that **the Merton model is not a better prediction of the company rankings than the accounting-based measures**.

6.4.4. Ordered logit regression without original *Z-Score*

I carried out a **correlation analysis** to see the explanatory variables are correlated with each other (see Appendix G). The highest correlations are between the variables O and Z^U, Z and Z^U, and between Z and *O-Scores*. These high positive correlations across the accounting-based measures could have been expected as they often utilize the same, or related, financial ratios. Surprisingly high was the positive **correlation between the Merton model and the original *Z-Score***. Based on the nonsensical high probabilities of default generated by the original *Z-Score* and the high correlations of this score with other measures of credit risk (including the *MMscore*), I carried out another round of **ordered logit regressions without the original *Z-Score***.

The SAS file "Logit_noZ.sas" and the dataset "Logit.sas7bdat" can be found in the Attachment 2. The results (the SAS output window) are saved as "Logit_Output_noZ.doc."

By excluding the original *Z-Score* from the regression, the performance of the model was even worse than with the *Z-Score*. The exception was the year 2003, when the global null hypothesis had been rejected at the 5% level by both Wald and Score Chi-Square tests.[148] But generally, the biggest change caused by the exclusion of the *Z-Score* was that the **significance of the *MMscore* increased substantially** for the years 2001-2003[149] and the *MMscore* has actually been evaluated as statistically significant at the 7%, 5% resp. 3% level. For these three years, the coefficient of *MMscore* also had the correct positive sign.

The results of the correlation analysis and the two regressions suggest that the **Merton model and Altman's *Z-Score* contain some related information**. Moreover, unreported results of an ordered logit regression without the *MMscore* also resulted in an overall worsening of the model's explanatory power and at the same time the significance of the *Z-Score* increased considerably. Nevertheless, when both of the variables are present, the **ability of the *Z-Score* to explain company rankings significantly outperforms that of the *MMscore***. The other accounting-based scores seem to have higher explanatory power than the

[148] In this year, all variables, except the Z^U, came out as significant on the 10% level.
[149] The regression for the year 2004 has generally the worst results and in both regressions, the *MMscore* was insignificant.

Merton model as well. This is probably caused by the fact that these scores use the same source of information (i.e. the companies' financial statements) as the ranking method. But the fact that the Merton model does not correctly reflect the financial situation of the company, which determines the company rankings, can have two possible explanations. It could be potentially caused by the inability of the financial markets to incorporate this information from financial statements into the share prices. The other possibility is that the Merton model in the simple form as proposed in this paper is unable to extract such information from the share price development.

However, as mentioned before a few times, the regression analyses that have been carried out are due to the **extremely small sample** size and the substitution of credit ratings with **simple rankings not a full-fledged assessment of quality of the various models.** The results should be used only for guidance purposes and **no definite conclusions can be drawn from these test**.

7. Conclusion

In the previous parts of this paper, several methods for assessment of credit risk have been introduced. The main focus was on the Merton model, which deduces the probabilities of default of individual companies mainly from the equity value and volatility. The model's strong assumptions, the improvements to the model as well as the major advantages and drawbacks of the asset-based approach have been discussed theoretically. Nevertheless, the main purpose of this paper has been to explore the applicability of the Merton model in the Czech Republic, which is a relatively small country with a relatively young and not very liquid capital market. The main question was whether this model will be able to predict "reasonable" default probabilities for the 15 most actively traded non-financial companies listed on the Prague Stock Exchange. In order to give an answer to this question, the computed default probabilities were confronted with other measures of credit risk. These measures were the traditional, accounting-based indicators of credit risk- the Altman's *Z*- and Ohlson's *O -Scores* (with both original and updated coefficients), the available credit ratings from external rating agencies and Czech Sector Award rankings. .

The analysis of the results was mainly based on their "common sense" interpretation, especially based on the observation of the diagrams and the comparison with the "real" situation of the companies (represented by annual reports and financial statements) and with the share price developments. Based on this “rule of thumb” analysis I found the accounting-based measures outperform the Merton model in the ability to correctly identify companies in financial distress. Moreover, in several cases the Merton model failed to correctly include substantial movements of share prices into the default probabilities of the companies concerned. On the other hand, the Merton model correctly predicted very low probabilities of bankruptcy for the high-quality, blue-chip companies.

In order to confirm or reject these suppositions, the possibility of statistical testing of the Merton model has been explored. I come to the conclusion that there is no ground for the evaluation of the Merton model, which would allow a statistically reliable comparison with other (accounting-based) measures of credit risk. Nevertheless, in order to illustrate some of the theoretical concepts of rating models, I carry out an ordered logit regression. In the ordered logit models, the measures of credit risk are the explanatory variables and their ability to correctly reflect the external credit rating represents their quality. The extremely low sample size and the use of simple and purely quantitative rankings instead of the missing credit ratings unfortunately impede the possibility to use the results of the ordered logit regression to draw any sound results. Nevertheless, the results of this “quantitative” analysis match the conclusions drawn in the previous, “common sense” assessment of the different credit risk indicators. The Merton model is unable to predict the company rankings and its explanatory power is inferior to that of the accounting-based variables.

The original Merton model in the form described in this paper seems to be unsuitable for predicting default for the publicly traded Czech companies. One of the most probable reasons is that the Czech capital market violates the assumptions of an efficient market, which has already been the result of some previous studies addressing this issue. I find as well that the ability of the share prices to efficiently reflect financial situation and the credit quality of the listed companies, is very doubtful.

The bottom line of this work is that the Merton model can potentially be used as a source of information about the underlying credit risk for the Czech publicly traded companies but these probabilities are an insufficient measure of the underlying risk and some other measures of credit risk should be used instead of or in addition to the Merton model.

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  - http://en.wikipedia.org/wiki/Ito%27s_Lemma
  - http://www.mathserv.okanagan.bc.ca/math/math414/walk/Ito.htm

- The web pages dedicated to logit and ordered logit regression:
  - http://www.columbia.edu/~ag2319/teaching/G4075_Outline/node27.html
  - http://staff.washington.edu/~glynn/olr.pdf
  - http://www.indiana.edu/~statmath/stat/all/cat/2b1.html
  - http://www.statsoft.com/textbook/glosl.html
  - http://en.wikipedia.org/wiki/Logit
  - http://www.ats.ucla.edu/STAT/sas/output/sas_ologit_output.htm

Some of the share prices for Czech listed companies were provided by **Patria Finance**, a.s. (www.patria.cz) free of cost. Others were downloaded from public sources, especially from the servers www.akcie.cz and www.pse.cz.

The company financial statements, as well as the information about dividends and number of outstanding shares, were obtained from the database **Magnus**. Missing and most up-to-date information was obtained directly from the companies' web pages and annual reports.

The software used for the computation of Merton-implied probabilities of default and the ordered logit regression is **SAS, version 8.2**.

Microsoft Excel was used to count the accounting-based variables and to process the input data for SAS.

The Czech Sector Awards rankings were provided by **Čekia** (www.cekia.cz) in cooperation with **Moody's Central Europe** (formerly **CRA Rating Agency**, www.moodys.cz).

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Diploma thesis

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Academic year: **2006/2007**

The applicability of Merton's credit risk model to the Czech market

This diploma work will examine the problem of credit risk and the approach of Merton's model to its measurement. Credit risk can be defined as the possibility that a contractual party shall not meet its obligations. All lending institutions, especially banks, face such risk and use varying instruments to quantify and manage credit risk. Moreover, according to national regulations, banks are obliged to keep sufficient capital to cover the potential loss from default of debtors. At the moment, several different approaches to credit risk measurement exist and are used by financial institutions.

The Merton's model is unique since it uses, unlike other credit risk models, market value of shares to estimate the probability of default. The basic notion behind this model is that financial statement analysis is inherently backward looking, whereas market prices are by their nature forward looking. The model depends solely on value of liabilities, stock value and volatility, which makes its use cost and time efficient. But since the basic variable of the Merton's model is the market price of equity, some assumptions about effectiveness of the markets have to be made. The model then employs the options theory to calculate the probability of default.

The object of this diploma work is to determine, whether the development of share prices on the Prague Stock Exchange can be used to predict the probability of default of selected listed companies using the basic Merton's model. The obtained results will then be confronted with other, more conventional methods of credit risk measurement such as rating from rating agencies.

Outline of the diploma work:

- Defining credit risk
- Reasons for development of credit risk models (New Basel Capital Accord)
- Brief overview of currently used credit risk models
- Merton’s model
    - Theoretical approach
    - Testing Merton’s model on the Czech market (Prague Stock Exchange)
- Conclusion

Basic literature:

- Server www.defaultrisk.com
- Papers from the Institute of Economic Studies of the Charles University in Prague
- Materials from Moody’s KMV Company, for example:
    - Modeling Default Risk, January 1999
    - Reduced Form vs. Structural Models of Credit Risk: A Case Study of Three Models, February 2005
- Research papers and publications of the Basel Committee on Banking Supervision, Bank for International Settlements
- Research papers and publications of Česká národní banka

Appendix A

Itō's lemma

In mathematics, Itō's lemma is used in stochastic calculus to find the differential of a function of a particular type of stochastic process. Essentially, Ito's Lemma provides a derivative chain rule for stochastic functions; i.e. if $f = f(x,t)$ where x is some stochastic function, what is the derivative df / dt? The lemma is widely employed in mathematical finance.

Statement of the lemma

Let $x(t)$ be an Itō (or generalized Wiener) process. That is let

$$dx(t) = a(x,t)dt + b(x,t)dW_t$$

where W_t is a Wiener process, and let $f(x,t)$ be a function with continuous second derivatives.

Then $f(x(t),t)$ is also an Itō process, and

$$df(x(t),t) = \left(a(x,t)\frac{\partial f}{\partial x} + \frac{\partial f}{\partial t} + \frac{1}{2} b(x,t)^2 \frac{\partial^2 f}{\partial x^2} \right) dt + b(x,t)\frac{\partial f}{\partial x} dW_t \quad \blacksquare$$

Informal proof

A formal proof of the lemma requires defining the stochastic integral, which is an advanced concept in between functional analysis and probability theory and for its complexity is not done here.[150]

Expanding $f(x,t)$ in a Taylor series in x and t we have

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial t} dt + \frac{1}{2}\frac{\partial^2 f}{\partial x^2} dx^2 + ...$$

and substituting $adt + bdW$ for dx gives

$$df = \frac{\partial f}{\partial x}\left(adt + bdW \right) + \frac{\partial f}{\partial t} dt + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}\left(a^2 dt^2 + 2abdtdW + b^2 dW^2 \right) + ...$$

[150] For the formal proof, see e.g. Cossin (2001).

In the limit as dt tends to 0, the dt^2 and $dtdW$ terms disappear but the dW^2 term tends to dt. The latter can be shown if we prove that

$dW^2 \to E\left(dW^2\right)$, since $E\left(dW^2\right) = dt$

The proof of this statistical property is however beyond the scope of this appendix.

Deleting the dt^2 and $dtdW$ terms, substituting dt for dW^2, and collecting the dt and dW terms, we obtain

$$df = \left(a\frac{\partial f}{\partial x} + \frac{\partial f}{\partial t} + \frac{1}{2}b^2\frac{\partial^2 f}{\partial x^2} \right)dt + b\frac{\partial f}{\partial x}dW$$

as required. ■

Appendix B

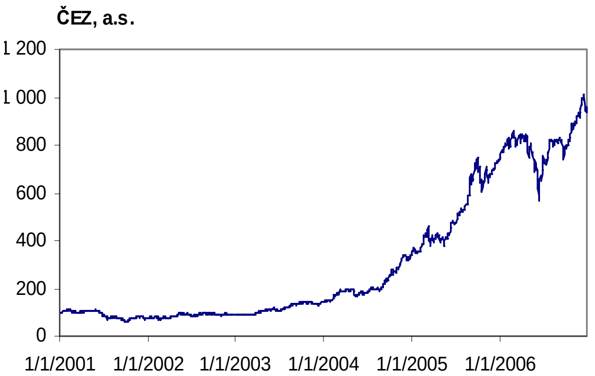
Companies listed on PSE				
Issuer	**ISIN**	**Market**	**Trading group**	**Reason for elimination**
CETV (Central European Media Enterprises Ltd.)	BMG200452024	Main market	3	Issue after 2004
Česká námořní plavba	CZ0008413556	Official free market	1	Insufficient trading volumes
Česká zbrojovka	CS0005029156	Secondary market	1	Insufficient trading volumes
ČEZ, a.s.	CZ0005112300	Main market	3	X
ECM (ECM Real Estate Investments A.G.)	LU0259919230	Main market	3	Issue after 2004
Energoaqua	CS0008419750	Official free market	1	Insufficient trading volumes
Erste Bank AG	AT0000652011	Main market	3	Financial institution
Jihočeské papírny, a.s., Větřní	CZ0005005850	Official free market	1	X
Jihomoravská plynárenská	CZ0005078956	Secondary market	1	Insufficient trading volumes
Komerční Banka, a.s.	CZ0008019106	Main market	3	Financial institution
Lázně Teplice v Čechách	CS0008422853	Official free market	1	Insufficient trading volumes
Léčebné lázně Jáchymov	CS0008446753	Official free market	1	Insufficient trading volumes
Orco (Orco Property Group S.A.)	LU0122624777	Main market	3	Issue after 2004
PARAMO, a.s.	CZ0005091355	Official free market	1	X
Pegas Nonwovens SA	LU0275164910	Main market	3	Issue after 2004
Philip Morris ČR a.s.	CS0008418869	Official free market	3	X
Pražská energetika, a.s.	CZ0005078154	Secondary market	1	X
Pražská plynárenská	CZ0005084350	Secondary market	1	Insufficient trading volumes
Pražské služby	CZ0009055158	Official free market	1	Insufficient trading volumes
RMS-Holding	CS0008416251	Secondary market	1	Constant share price
SETUZA a.s.	CZ0008460052	Secondary market	1	X
Slezan Frýdek-Místek	CZ0005018259	Official free market	1	Insufficient trading volumes
Severomoravská plynárenská, a.s.	CZ0005084459	Secondary market	1	X
SPOLANA a.s.	CS0008424958	Secondary market	1	X
Spolek pro chemickou a hutní výrobu, a.s.	CZ0005092858	Official free market	1	X
Středočeská plynárenská, a.s.	CZ0005078659	Secondary market	1	X
Telefónica O2 Czech Republic,a.s.	CZ0009093209	Main market	3	X
TOMA, a.s.	CZ0005088559	Official free market	1	X
UNIPETROL, a.s.	CZ0009091500	Main market	3	X
Východočeská plynárenská, a.s.	CZ0005092551	Secondary market	1	X
Západočeská plynárenská	CZ0005078758	Secondary market	1	Insufficient trading volumes
Zentiva a.s.	NL0000405173	Main market	3	X

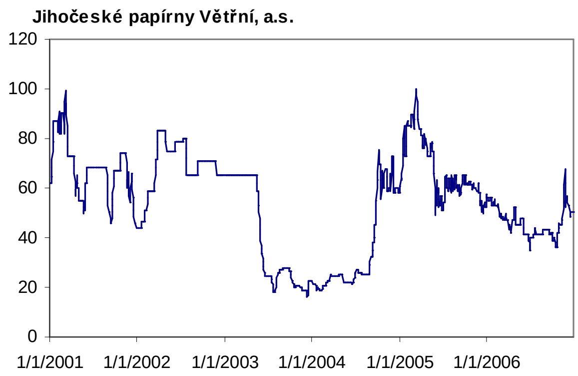
Note: For definitions of the market types and trading groups see www.pse.cz
Telefónica O2 Czech Republic,a.s. is the successor of Český Telecom, a.s. after the acquisition by Telefónica in 2005

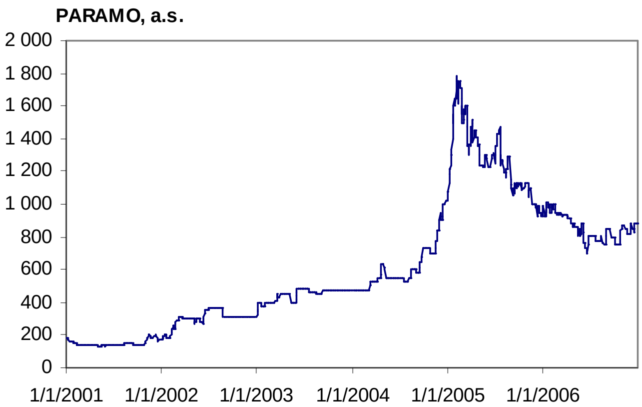
These companies have been selected for further analysis

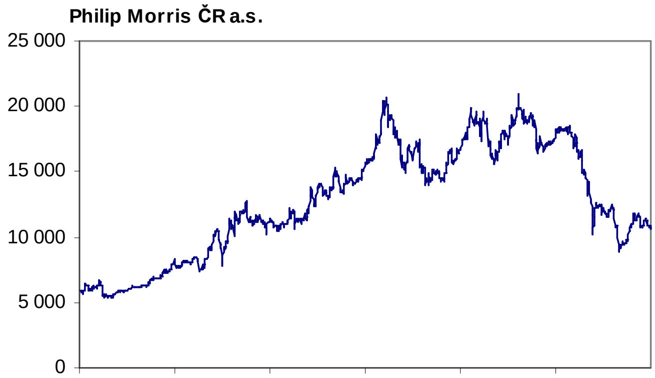
Appendix C

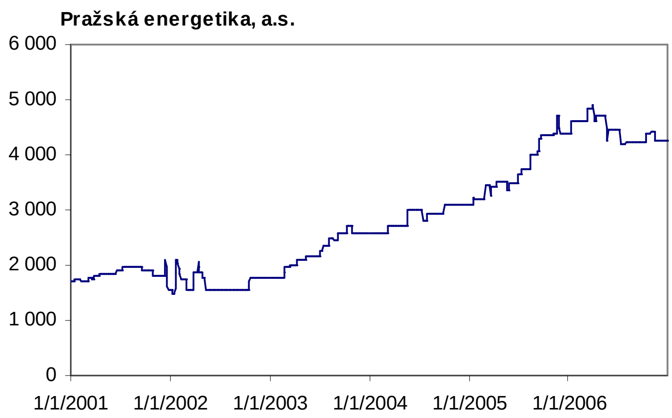
Development of share prices for the 15 analyzed companies in 2001-2006

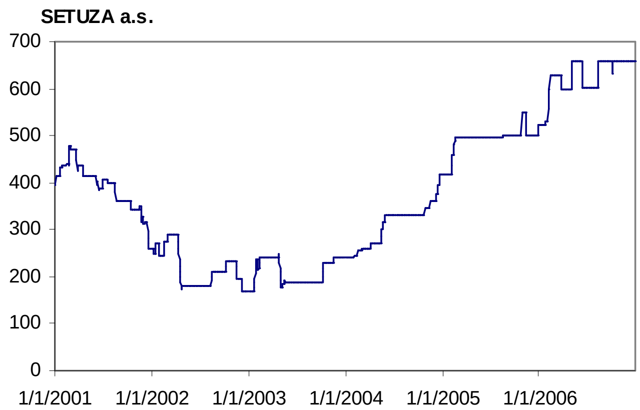


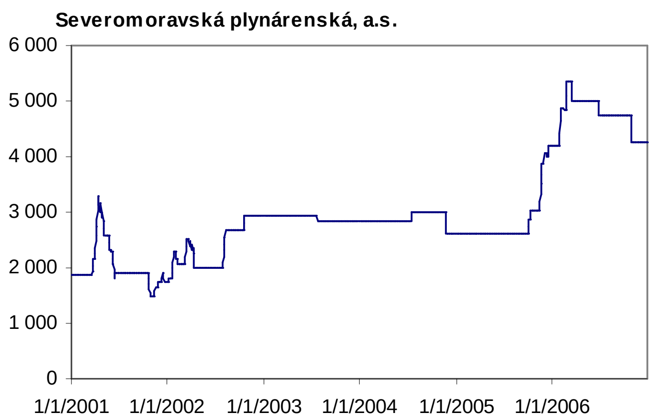


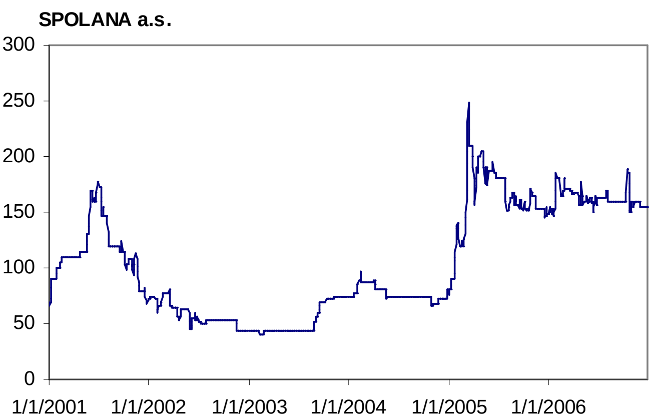


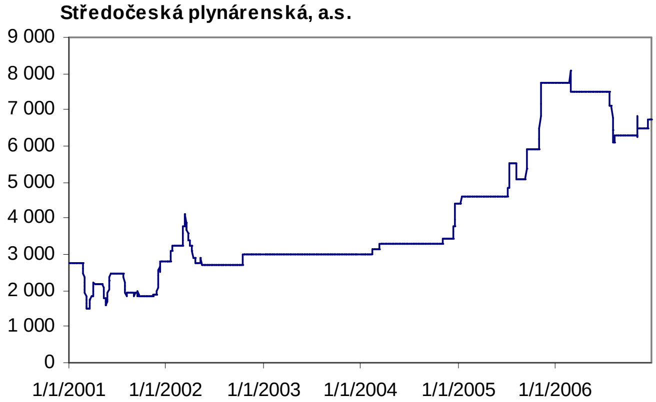


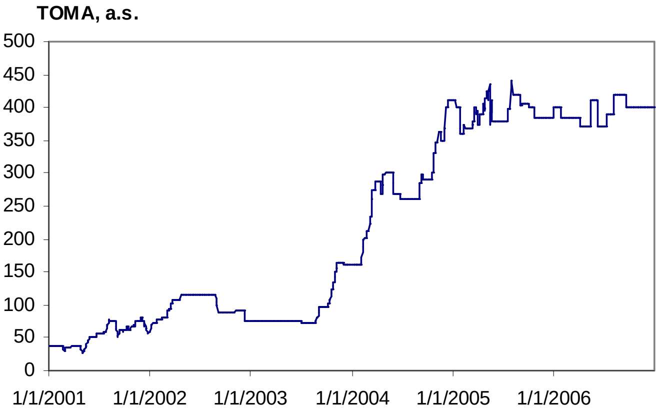


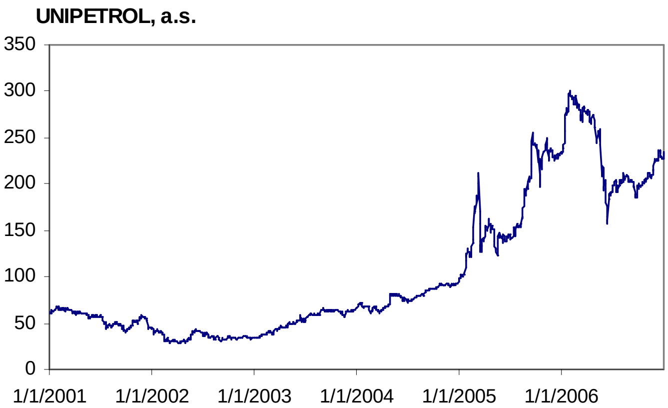


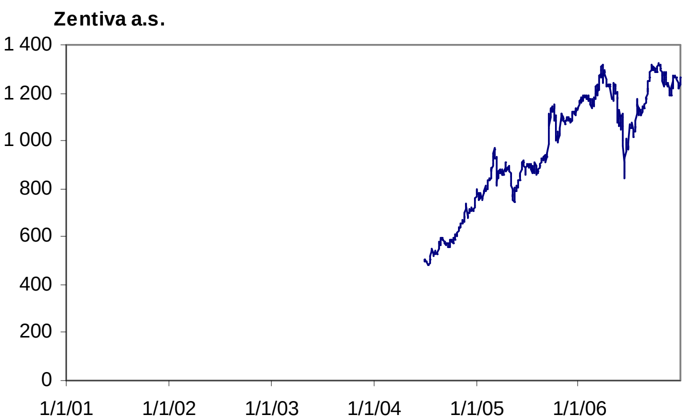


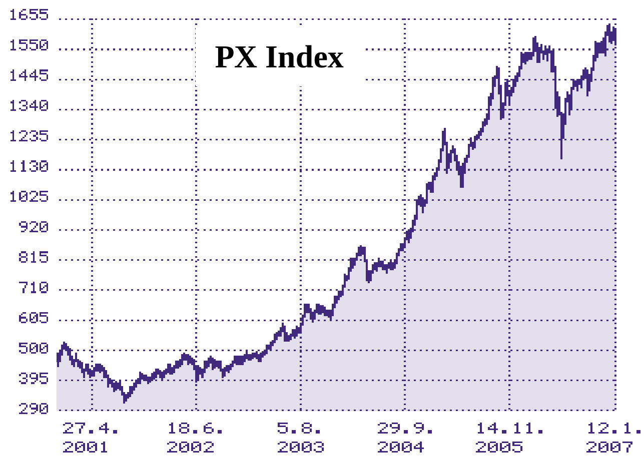












[151] Note: Zentiva a.s. has been listed in June 2004.

Appendix D

The SAS code for computing default probabilities

```
/*'Annual probabilities of default for 15 companies listed on PSE'*/


/*Choose the directory with the dataset ssd.Merton*/

LIBNAME ssd 'C:\SAS\';

data temp;
set ssd.Merton;
    divrate = (dividends)/(e + f) ;
    if divrate < 0 then divrate =. ;
    if divrate >1 then divrate =. ;
      f=f/10000000;
      e=e/10000000;
      dividends=dividends/10000000;
      v = f + e;
      va = ve*e / (f+e);
      t= 1.0;

*/ Simultaneously estimates the asset values "v" and volatility of firms'
assets "va" /;

proc model data= temp MAXERRORS=1 noprint ;
    by comp year ;
    bounds 0 < v  va;
    eq.call = v*exp(-divrate*t)*probnorm(((log(v / f)
    + t*(r - divrate + va ** 2 / 2))) / (va * sqrt(t)))
    - f * exp(-r * t) * probnorm(((log(v / f)
    + t*(r -divrate - va**2/2))) / (va * sqrt(t))) + (1-exp(-divrate*t))-e;

     eq.hedge = (va * v / e)* (exp(-divrate*t)) * probnorm(((log(v / f)
    + t * (r - divrate + va**2 / 2))) / (va * sqrt(t)))-ve ;
    solve v va
    / out = ssd.Results  maxiter =50 maxsubit =20 ;
    id name comp year r dividends divrate f e ve v va ;
run ;

proc sort data=ssd.Results;
      by comp year;
run ;


*/Computes the asset drift "mu"/;

data ssd.Results;
set ssd.Results;
    complag = lag(comp) ;
    yearlag = lag(year) ;
    tempvar = lag(v) ;
    if (comp = complag and year = yearlag + 1) then vlag = tempvar ;
      else vlag=v;
    mu = (v + dividends - vlag) / v ;
    if mu < r then mu = r ;
    if mu >1 then mu =1 ;
    if vlag = . then mu = . ;
```

```
*/Computes the probability of default "DPmm"/;

    if _errors_ = 0 then mmtemp1 = ((log(v / f)
    + t * ( mu - divrate - va**2 / 2))) / (va * sqrt(t));
    if _errors_ = 0 then dpmm = 1-probnorm(mmtemp1) ;


*/Computes and trims "MMscore"/;

     mmtemp2 = dpmm / (1 - dpmm) ;
     mmscore = log(mmtemp2) ;
     if dpmm < 0.000001 then mmscore = -13.8155;
     if dpmm > 0.999999 then mmscore =  13.8155 ;
     if dpmm = . then mmscore = .;
run;


*/Computes distance-to-default "DD", credit spread "cs" and loss given
default "LGD"/;
data ssd.Results ;
set ssd.Results;

DD=(log(v / f)+ t * ( mu - divrate - va**2 / 2)) / (va * sqrt(t));

cs=-(1/t)*log (probnorm((log(v / f)+ t*(mu - divrate - va**2 / 2)) / (va *
     sqrt(t)))+ v*probnorm(-(log(v / f)+ t*(mu - divrate + va**2 / 2)) /
     (va * sqrt(t)))/(f * exp(-r * t)));


     drop  _type_  _mode_  _errors_  tempvar mmtemp1 mmtemp2 complag t
            yearlag vlag;

     if year=2000 then delete;
run;
```

Appendix E

Company name	Year	DPmm	MMscore	DD	Credit spread
ČEZ, a.s.	2001	0.001632076	-6.416268909	2.941701	5.28035E-05
	2002	2.98987E-05	-10.41766557	4.013609	7.90849E-07
	2003	0	-13.8155	9.073269	-0
	2004	0	-13.8155	8.306124	-0
	2005	1.66533E-15	-13.8155	7.87616	6.66134E-16
	2006	3.44169E-15	-13.8155	7.787589	6.66134E-16
Jihočeské papírny Větřní, a.s.	2001	0.006753434	-4.990927838	2.470118	0.000300537
	2002	7.01922E-07	-13.8155	4.824458	1.61164E-08
	2003	0.001790875	-6.323258531	2.912825	4.77067E-05
	2004	0.000457665	-7.688916034	3.315336	0.000119199
	2005	0.011911662	-4.418254182	2.259967	0.000903232
	2006	0.005937832	-5.120455603	2.515818	0.000355152
PARAMO, a.s.	2001	6.21493E-05	-9.685909332	3.83749	4.18559E-07
	2002	0.000367746	-7.907749828	3.375995	6.76149E-06
	2003	0	-13.8155	9.402036	-0
	2004	1.85233E-07	-13.8155	5.083539	1.38862E-08
	2005	0.001103281	-6.808363362	3.060923	0.000123419
	2006	4.34156E-05	-10.0446478	3.924718	2.9559E-06
Philip Morris ČR a.s.	2001	0	-13.8155	10.9132	-0
	2002	5.07372E-13	-13.8155	7.128481	9.9809E-14
	2003	0	-13.8155	9.276598	-0
	2004	6.32827E-14	-13.8155	7.409622	-2.22045E-16
	2005	7.77156E-16	-13.8155	7.972875	-0
	2006	6.14362E-10	-13.8155	6.07645	-4.91986E-11
Pražská energetika, a.s.	2001	1.98308E-12	-13.8155	6.938381	-5.55112E-14
	2002	1.13028E-05	-11.39045182	4.237467	3.17621E-07
	2003	0	-13.8155	13.23327	-0
	2004	0	-13.8155	11.87118	-0
	2005	0	-13.8155	10.62815	-0
	2006	0	-13.8155	12.62248	-0
SETUZA a.s.	2001	2.41013E-09	-13.8155	5.853261	7.39349E-11
	2002	0.001061576	-6.846938332	3.072442	5.64569E-06
	2003	8.56871E-05	-9.364722928	3.757858	4.98798E-07
	2004	8.88178E-16	-13.8155	7.961414	1.11022E-16
	2005	1.10522E-11	-13.8155	6.691401	5.22915E-14
	2006	1.33958E-08	-13.8155	5.561206	1.20945E-10
Severomoravská plynárenská, a.s.	2001	1.31724E-06	-13.53997045	4.697436	1.18684E-07
	2002	4.616E-08	-13.8155	5.341226	4.6767E-09
	2003	0	-13.8155	39.35274	-0
	2004	0	-13.8155	9.655617	-0
	2005	0	-13.8155	13.28749	-0
	2006	0	-13.8155	10.35998	-0
SPOLANA a.s.	2001	0.004853547	-5.32318012	2.586092	9.76392E-05
	2002	0.000424352	-7.764522804	3.336402	1.72545E-05
	2003	1.02575E-08	-13.8155	5.607602	4.95083E-11
	2004	1.25078E-05	-11.28914888	4.21466	1.24365E-07
	2005	0.002486535	-5.994375332	2.808773	0.000313345
	2006	0.000331714	-8.010905059	3.404263	8.64047E-06
Spolek pro chemickou a hutní výrobu, a.s.	2001	0.003042908	-5.791894077	2.743122	9.57797E-05
	2002	0.00345656	-5.664018744	2.701001	0.000119344
	2003	4.12899E-08	-13.8155	5.361399	4.54128E-09
	2004	5.9952E-14	-13.8155	7.41698	1.79856E-14
	2005	5.778E-07	-13.8155	4.863102	2.03663E-07
	2006	0.000695528	-7.270143595	3.1965	3.0625E-05
Středočeská plynárenská, a.s.	2001	3.08695E-06	-12.68832524	4.520345	4.57713E-07
	2002	3.45879E-12	-13.8155	6.859346	3.24518E-13
	2003				
	2004	1.11022E-16	-13.8155	8.22548	-0
	2005	4.36873E-13	-13.8155	7.149065	4.04121E-14
	2006	2.22045E-16	-13.8155	8.131156	-0
Telefónica O2 Czech Republic,a.s.	2001	0.001695619	-6.378010579	2.929852	0.000149481
	2002	8.123E-06	-11.72080316	4.311079	4.38761E-07
	2003	3.1411E-06	-12.67093556	4.516662	3.45085E-07
	2004	8.21565E-15	-13.8155	7.675489	1.11022E-16
	2005	0	-13.8155	14.50474	-0
	2006	0	-13.8155	8.978872	-0
TOMA, a.s.	2001	0.000604869	-7.409894296	3.236574	5.08613E-05
	2002	1.09851E-10	-13.8155	6.346896	3.08031E-12
	2003	0	-13.8155	13.32625	-0
	2004	0	-13.8155	11.90172	-0
	2005	0	-13.8155	8.411338	-0
	2006	0	-13.8155	12.65533	-0
UNIPETROL, a.s.	2001	0.000461112	-7.681408398	3.313238	1.7585E-05
	2002	0.00412461	-5.486650645	2.641695	0.000168696
	2003	1.44551E-12	-13.8155	6.982929	2.82441E-13
	2004	5.77538E-13	-13.8155	7.110653	1.22125E-14
	2005	6.84003E-07	-13.8155	4.82961	2.85512E-07
	2006	5.69544E-13	-13.8155	7.112579	2.24265E-14
Východočeská plynárenská, a.s.	2001	3.61933E-14	-13.8155	7.483659	4.21885E-15
	2002	1.75584E-07	-13.8155	5.093686	2.56646E-08
	2003	0	-13.8155	9.11228	-0
	2004	0	-13.8155	19.22611	-0
	2005	0	-13.8155	12.49849	-0
	2006	0	-13.8155	16.33661	-0
Zentiva a.s.	2004	0	-13.8155	15.89448	-0
	2005	0	-13.8155	8.626581	-0
	2006	0	-13.8155	8.384504	-0

Note: Observations are missing for Středočeská Plynárenská in 2003 because of the zero equity volatility

Appendix F

SAS code for ordered logit regression

```
*Ordered logit regression for 15 companies;

LIBNAME ssd 'C:\SAS\';
/*'Choose the directory with the dataset ssd.logit'*/;


*Ordered logit regression in 2001;
data logit2001 ;
set ssd.logit;
if year ne 2001 then delete;

proc logistic data=logit2001;
model rank = Z ZU O OU MM;
run;


*Ordered logit regression in 2002;
data logit2002 ;
set ssd.logit;
if year ne 2002 then delete;

proc logistic data=logit2002;
model rank = Z ZU O OU MM;
run;


*Ordered logit regression in 2003;
data logit2003;
set ssd.logit;
if year ne 2003 then delete;

proc logistic data=logit2003;
model rank = Z ZU O OU MM;
run;


*Ordered logit regression in 2004;
data logit2004;
set ssd.logit;
if year ne 2004 then delete;

proc logistic data=logit2004;
model rank = Z ZU O OU MM;
run;
```

Appendix G

Correlations between variables

2001					
2001	*Z*	*ZU*	*O*	*OU*	*MM*
Z	1				
ZU	0.727231	1			
O	0.585674	0.879331	1		
OU	-0.229681	-0.5067	-0.239079	1	
MM	0.523281	0.132684	-0.137902	-0.100218	1

2002					
2002	*Z*	*ZU*	*O*	*OU*	*MM*
Z	1				
ZU	0.239555	1			
O	0.4049	0.839792	1		
OU	-0.493936	-0.080043	-0.268686	1	
MM	0.126757	0.376125	0.120437	0.571159	1

2003					
2003	*Z*	*ZU*	*O*	*OU*	*MM*
Z	1				
ZU	0.489253	1			
O	0.511522	0.698866	1		
OU	-0.390602	-0.091109	-0.064904	1	
MM	0.698035	0.175288	0.064184	-0.668859	1

2004					
2004	*Z*	*ZU*	*O*	*OU*	*MM*
Z	1				
ZU	0.890359	1			
O	0.72161	0.671874	1		
OU	-0.061492	0.083146	0.18135	1	
MM	0.428663	0.168881	0.319376	-0.446738	1

Note: Because of the missing values, the observations for Spolek chemické a hutní výroby in 2002 and for Středočeská plynárenská 2003 have been ommited from the correlation analysis