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DIPLOMOVÁ PRÁCE

Modely kreditního rizika a jejich vztah k ekonomickému cyklu

Vypracoval: Ing. Ing. Petr Jakubík, Ph.D.
Vedoucí: PhDr. Petr Teplý
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Charles University in Prague

Faculty of Social Sciences

Institute of Economic Studies

DIPLOMA THESIS

Credit Risk Models and Their Relationship with Economic Cycle

Author: Ing. Ing. Petr Jakubík, Ph.D.
Consultant: PhDr. Petr Teplý
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Prohlášení

Prohlašuji, že jsem diplomovou práci vypracoval samostatně a použil pouze uvedené prameny a literaturu.

Hereby I declare that I compiled this thesis independently, using only the listed literature and resources.

Prague, 11 May 2006

Petr Jakubík

Abstrakt

Význam kreditních rizikových modelů vzrůstá se zaváděním nové basilejske direktivy známé pod názvem Basel II. Cílem této studie je modelování míry defautu. Tato práce sleduje dva základní možné přístupy k makroekonomickému modelování kreditního rizika. Nejprve jsou zkoumány empirické modely, poté je použit koncept latentních faktorových modelů jdoucích v ideologické rovině mertonovských modelů. Oba tyto přístupy vychází z modelování individuální pravděpodobnosti defaultu. V první části této studie byla použita data finské ekonomiky za období 1988 až 2003, nejvýnamnějšími časovými řadami byly bankroty a počty firem pro agregovanou ekonomiku i pro specifická odvětví. Nejprve byl odhadnut jednoduchý lineární regresní model, který posloužil k výběru později uvažovaných makroekonomických indikátorů. V rámci dynamických empirických modelů byl použit lineární vektorový autoregresní model. Bylo zkoumáno jak významně ovlivňují makroekonomické indikátory míru defaultu v celé ekonomice a jednotlivých ekonomických odvětví. Nicméně narozdíl od latentních faktorových modelů, empirické modely nemohou poskytovat mikroekonomické zdůvodnění. Pro odhad modelovaného vztahu byl nakonec použit jednofaktorový latentní model ačkoli byl zkoumán i multifaktorový. V případě odvětvových modelů byly koeficienty odhadnuty na základě desagregovaných dat. Tyto odhady mohou přispět k pochopení vztahu mezi kreditním rizikem a makroekonomickými indikátory. Dosažené závěry byly použity v druhé části této práce, která využívá získaných poznatků k odhadu makroekonomického modelu kreditního rizika agregované české ekonomiky pro potřeby zátěžového testování bankovního sektoru Českou národní bankou. Tento přístup umožňuje modelovat dopady nejrůznějších makroekonomických šoků na kvalitu úvěrového portfolia a následně v kombinaci se zátěžovým aparátem na kapitál celé bankovní soustavy.

Klíčová slova: bankovnictví, úvěrové riziko, latentní factorový model, míra defaultu

JEL klasifikace: G21, G28, G33

Abstract

The significance of credit risk models has increased with the introduction of new Basel accord known as Basel II. The aim of this study is default rate modeling. This thesis follows the two possible approaches of a macro credit risk modeling. First, empirical models are investigated. Second, a latent factor model based on Merton's idea is introduced. Both of these models are derived from individual default probability models. We employed data over the time period from 1988 to 2003 of the Finnish economy in the first part of this thesis. Time series of bankruptcy and firm's numbers were used. Aggregate data for whole economy as well as industry specific data were available. First, linear vector autoregressive models was used in case of dynamic empirical model. We examined how significant macroeconomic indicators determined the default rate in the whole economy and in the industry specific sector. However these models cannot provide microeconomic foundation as latent factor models. We employed a one-factor model in our estimation although, multi-factor models were also considered. A one-factor model was estimated using disaggregated industrial data. This estimation can help understand relation between credit risk and macroeconomic indicators. Obtained results were used in the second part of this thesis. The macroeconomic credit risk model of the Czech aggregate economy was estimated for purpose of stress testing in the Czech National Bank. The impact of different macroeconomic shocks on credit portfolio quality and change in capital adequacy ratio of banking sector can be provided by this approach together with stress test.

Key words: banking, credit risk, latent factor model, default rate

JEL classification numbers: G21, G28, G33

Credit Risk Models and Their Relationship with Economic Cycle

Petr Jakubík[1]

This thesis originated from research done at the Bank of Finland Research Department and Economic Research Department of the Czech National Bank. However views expressed are those of the authors and do not necessarily reflect the views of any of the above institutions.

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[1]Institute of Economic Studies of the Charles University in Prague and the Czech National Bank. Contact: Petr.Jakubik@cnb.cz
The findings, interpretations and conclusions expressed in this thesis are entirely those of the author and do not represent the view of any of the above institutions.

Contents

List of Tables

List of Figures

Introduction

Credit risk is one of the most important areas of risk management. Research in credit risk has rapidly increased during last decade. Credit risk plays an important role mainly for bank institutions. They try to develop their own credit risk models in order to increase bank profit. A new wave of interest originated with the introduction of the new Basel accord known as Basel II.

Three approaches can be distinguished. The first - traditional models – are based on comparing client specific information. The objective of these models is a good prediction of future client quality. The default probability is obtained from empirical information. These models are widely used for business clients and this approach is also very popular for transitional economies with insufficient capital markets. Models based on option pricing ("Structural models") represent the second possible approach. They are based on financial pricing theory. Here, the value of a firm is modeled as an option price. The firm default is specified in relation to firm value and leverage. The third approach is summarized in so called reduced form models. These models use market bond price as input, and from this information they try to derive default probability and recovery rate. The aim of all approaches is an estimation of firm default probability and loss given default. Together with estimation of exposure at default and effective maturity these credit risk components can be used for determining the capital requirement - Internal Ratings-Based Approach (IRB).

One question which has become important is the relationship between credit risk models and business cycle. Research on this relationship has increased mainly during last few years. Targets of these studies are credit risk models taking into account the macroeconomic environment. Some researches are focused on developing a macro model for credit risk estimation. In general these types of models try to estimate the default rate from macro data. These models are used for stress testing. This testing is emphasized by the new Basel accord. Bank with IRB models must use stress testing in the assessment of capital adequacy. Stress testing must involve identifying possible events or future changes in economic conditions that could have negative effect on the bank capital requirements (Basel Committee on Banking Supervision 2004). Macro models are also a very useful tool for central banks for research and management of banking system financial stability. Through the application of these models central bank can estimate impact of introducing changing monetary policy or expected or unexpected macroeconomic shocks.

Two basic approach in default probability modeling can be distinguished. Banks can base borrower's assessments on the current economic condition. Default probability is then conditioned on the point in the cycle. When risk assessments take into account possible change in macroeconomic climate, then forward looking ratings can be derived. The second approach becomes important due to the possibility of implementing different type of cyclical policy. Macroeconomic models can help with understanding influence of macroeconomic change on the default events.

This paper contributes to contemporary research by comparing two basic approaches in macroeconomic default prediction. First, empirical models are introduced. Second, latent factor models based on Merton's idea are investigated.[2] Our study is connected to previous research which was done in Bank of Finland (Virolainen 2004). It extends the previous analysis of Finnish default data by introducing latent systematic risk factors. We tried to offer an alternative to the previous study, where an empirical approach to modeling was employed. However very similar macroeconomic indicators were used. Factors models can be a better way of default rate modeling, because they provide microeconomic foundation.

We focus on developing macro models for default rate prediction in this paper. The target of this paper was investigation of the possible approach of default rate macro modeling in literature and the selection of a model for the Finnish and Czech economy. There are several reasons for being interested in the relationship between business cycle fluctuations and default. First, financial regulators need to have a good understanding of the potential downside credit risk in loan and corporate bond portfolios. They therefore need to be able to estimate the potential cyclical variability of default rates. Second, management and regulators will want to have some idea of the likely rate of default in the immediate future. Macroeconomic indices are informative indicators of future default rates, requiring the direct modeling of these relationship. Third, as encouraged by the Basel committee, banks need to be able to develop stress tests of their portfolio performance in business cycle downsturns and these tests should be interpretable in terms of the magnitude of some underlying macroeconomic shock. This study can help in all these tasks. A latent factor model is a natural and popular way of to estimate potential downside credit risks. This is why the latent factor model is the basis of Pillar 1 of the new Basel accord (Gordy 2003). But relatively little work has been done on estimating the crucial parameter, representing correlation with systematic factor. Combining a latent factor model with macroeconomic indicators provides a natural test of the

[2] Merton's models are based on the option price model, which estimates value of the firm as a price of put option. For the first time this idea was introduced by Merton (1974).

specification of the macro-relationship. If the macro indicators are indeed informative predictors then the share of fluctuations explained by the latent factor will be relatively small. The latent factor represents the unexplained component of the macro-model. We found that latent factor remains important even with the inclusion of macro indicators. Therefore both simulation and forecasting should include allowance for latent factors as well as observed macroeconomic indicators.

This paper is structured as follows. Chapter 1 introduces related studies. Chapter 2 contains all considered data in this study. Bankruptcy data as proxy of defaults and macroeconomic indicators are described. Chapter 3 presents used macroeconomic credit risk models. The dynamic models are discussed within the framework of empirical models. Linear dynamic vector autoregressive models and their vector error correction forms were used for investigation of mutual relationship between default rate and some macroeconomic indicators. Lastly, a more sophisticated nonlinear one-factor model is used for default rate modeling. This model is derived from idea of return assets modeling by systematic factor and idiosyncratic shocks. A multi-factor model is also suggested, but due to the complicated numerical solution, only one-factor models are estimated for the Finnish economy. Chapter 4 describes results of latent factor model for the Finnish economy. All relationships are investigated for the aggregate economy and also for five sector specific industry (agriculture, manufacturing, construction, trade, transport). Chapter 5 presents estimated macro credit risk model for the Czech economy. This model is used for the financial stability purpose in the Czech National Bank. Last chapter concludes and discusses possible further research issues.

Chapter 1

Related Studies

Some studies focus on business cycle effects on portfolio credit risk; others research procyclicality of credit risk measurement, or research relationships between financial crises and credit risk models. Four basic components are defined in the new Basel accord according Internal Ratings - Based Approach (Basel Committee on Banking Supervision 2004). There are default probability, loss given default, exposure at default and effective maturity. In discussions about relationship between business cycle and credit risk models the most important is default probability and loss given default. Some papers solve problem of correlation between default probability and loss given default. In general default probability changes over time depending on the macroeconomic environment. Some models use constant value of loss given default, but this also changes over time in practice. Many studies demonstrate this fact. The basic issue of relationship between credit risk models and the economic cycle is estimation of default probability as a function depending on time. Default probability is usually modeled by default rate. This indicator is defined as ratio between credits in default and total granted credits. This type of data on aggregate level of economy is sometimes very difficult to get. In this case some approximation must be used. These models use aggregate variables to explain default rate. Macro indicators are very often accounted. Such models are able to model impact of macroeconomic shock on credit industry.

This paper is related to literature on the influence of the macroeconomic environment on credit risk models. Few papers focus on the issue of the mutual relationship between economic cycle and credit risk. Those studies can be divided into two groups. The first group use company specific information and try to research the influence of the macroeconomic environment to individual risk. Other studies use only aggregate data and investigate the default rate in relation to macroeconomic indicators. In this paper only aggregate information is used and therefore it is in the second group of papers.

In the context of New Basel Accord, there are studies investigating cyclical effects in credit risk models. They try to model influence of cyclical policy on the bank capital requirement. You can find this issue in (Catarineu-Rabell, Jackson, Tsomocos 2003). They discuss the influence of different implementation of rating system to the bank capital requirement. They conclude that when banks assess a borrower's probability of default the assessment can be based on current economic condition or can take into account the effect on the borrower of possible adverse change in the economic climate. They show that even this approach could lead to a 15% increase in bank capital requirement in recession. Their result indicates that banks will not choose a more stable approach. Given completely freedom banks would choose a countercyclical approach reducing ratings in recession and if regulators prevent this, banks will adopt a procyclical approach. Lowe (2002) examined whether credit risk is low or high in economic booms. He described how macroeconomic consideration are incorporated into credit risk models and the risk measurement approach that underlies New Basel Capital Accord. Finally he researched influence of these measurement approaches on the macroeconomy. A survey of the literature on cyclical effects on default probability, loss given default and exposure at default can be found in (Allen, Saunders 2003). They noticed that although systematic risk factors have been incorporated into both academic and proprietary models for default probability, the same is not true for loss given default and exposure at default. Moreover systematic correlation effects between default probability and loss given default, default probability and exposure at default, and loss given default and exposure at default have been ignored in the literature.

There are studies used latent factor models for investigation business cycle effects on portfolio credit risk. These models are based on Merton model. Cipollini, Missaglia (2005) attempt to integrate market risk with credit risk. The estimation and identification of the common shock underlying the business cycle was obtained by fitting a dynamic factor model to a large number of macroeconomic credit drivers. They noticed relationship between default probability and recovery. Their empirical results show that, ignoring the main feature of recoveries, as stochastic and dependent on default, can imply serious under provision of minimum capital requirements. Rösch (2003) estimated one-factor model for German economy. He used data of bankruptcies for estimation of default probability and correlation between firm normalized return assets. This model is estimated for whole German economy and also for 16 industry specific sectors. The one-factor model is also employed in (Rösch 2005). Two rating philosophies are distinguished: through the cycle versus point in the time. Data from Standard & Poor's were used. It was shown that Point in Time Ratings will exhibit much lower correlation derived from nonlinear one-factor model, and default probability forecast should be more precise. As a consequence Value-at-Risk quantiles of default distribution should be lower than those generated by through the cycle ratings. This fact may affect bank punishment in time of economic stress if the implied reduction of asset correlation is not accounted in case of using point in time ratings. Hamerle, Liebig, Scheule (2004) used also static factor model, but they consider the effect of different assumptions about the error distribution function. The empirical analysis were based on a large data set of German firms provided by Deutsche Bundesbank. They used logistic distribution function in contrast to (Rösch 2003) or (Rösch 2005), where normal distribution function is used. They found that the inclusion of variables which are correlated with the business cycle improves the forecasts of default probabilities. Céspedes, Martín (2002) studies two-factor model for credit risk. They compared this model with one-factor model employed in Basel II. Lucas, Klaassen (2003) used simple mapping to cast discrete state regime switching models for credit risk into a continuous state factor model structure. They studied the implied default probabilities and asset correlations of the regime switching approach. They found that correlations implied by the model are low, and may appear too low given typical estimates of assets correlation in literature. They showed that assets and default correlation appear to be higher in recession than in expansion. Tasche (2005) investigated multi-factor extension of the asymptotic single risk factor model and derive exact formulae for the risk contributions to value-at-risk and expected shortfall. He introduced a new concept for diversification index as an application of the risk contribution formulae. He illustrated this concept by an example calculated with two-factor model. The results that there can be a substantial reduction of risk contribution by diversification effects is indicated. A three-factor structural model is developed for example in (Hui, Lo, Huang 2003). Pesaran, Schuermann (2003) used the idea of a simple Merton-type credit model for modeling credit risk as a function of correlated equity returns of the obligor companies. These equities are linked to correlated macroeconomic variable using an approach similar to the Arbitrage Pricing Theory. They estimated global macroeconomic model for generating a conditional loss distribution using stochastic simulation. They analyze the impact of a shock to set of specific macroeconomic variables on that loss distribution. Koopman, Lucas (2004) used multivariate unobserved components framework to separate credit and business cycle. They used this model for describing the dynamic behavioral of credit risk factors in their relation to real economy. They used data of real GDP, credit spreads and business failure for US economy. They distinguished two types of cycles in the data corresponding to periods of around 6 and 11-16 years, respectively. Cyclical co-movements between GDP and business failures mainly arise at the longer frequency. They empirically showed positive relationship of spreads and business failure rates and negative of GDP.

Some papers try to develop simple macroeconomic model of default rates predictions. These empirical models are derived from traditional models used for prediction of individual risk. Few papers focus on the developing macroeconomic model of default rates. Virolainen (2004) estimated this kind of model for Finnish economy. He used this model for stress testing and tried to investigate the influence of these shocks to the excepted and unexpected loss. His model is based on logistic regression.[1] Pesola (2001) published a study of the role of macroeconomic shocks in banking crises. This study also used data of Finnish economy.

[1]The logistic regression model corresponds to linear regression applied after logit transformation of the explained variable.
The logit trannsformation of the explained variable y is defined as $\ln \frac{y}{1-y}$. In the case of credit risk models this expression transforms original values from the interval $[0,1]$ to $(-\infty,\infty)$.

Chapter 2

Data Description

We used monthly data of the Finnish Economy for all calculation. Bankruptcy data and some macroeconomic indicators were employed.

2.1 Bankruptcy Data

The numbers of companies in default were the most important time series in our analysis. Default was defined the same way as in (Virolainen 2004). Defined default takes place when bankruptcy proceeding is instituted against firm for the first time. We considered that this definition is more strict than common applied, but it is still good approximation and data of bankruptcies are available for the Finnish economy. Event of default is commonly defined as payment delinquency with some minimum amount. 12-month default probability is usually employed in credit risk assessments. Generally M-month default at time t is defined when event of default is happen at time interval $(t, t+M]$ and subject is not in default at time $t-1$. Given definition corresponds to new event of default. This indicator is monitored by financial institutions as well as by central authorities. In this paper all calculations are based on monthly data. Monthly time series of firm's bankruptcies were available from 1/1988 to 5/2005. Time series of firm numbers are available on yearly basis form 1988 to 2003. Numbers of Firms data were disaggregated from annual data.[1] We computed 1M-default rates as ratio of number of bankruptcies at time t and number of firms at time $t-1$. As a result of this calculation time series of observed default rate approximation from 2/1988 to 1/2004 was available. Figure 2.1 shows 1M observed default rate in the Finnish economy. We computed industry-specific default rates as well as aggregate default rates for the whole economy. Data of active companies' numbers and bankruptcies data were available for the following five

[1] Number of firms were disaggregated from annual data with EKTA (Bank of Finland software)

industries: agriculture (AGR), manufacturing (MAN), construction (CON), trade, accommodation and restaurants (TRD), transport and communication (TRN) together with aggregate data for the whole economy. The same segmentation as in (Virolainen 2004) was used in this paper. The industry-specific default rates seem to convergence in the end of observed data, but there is significant distinguish in recession time. Increasing of default rates during recession was important for MAN, CON and TRD. Development of default rate for AGR and TRN was not significantly changed in recession time. Problem of observed default rates data is change in bankruptcy law, which was implemented from 1/1993.[2]

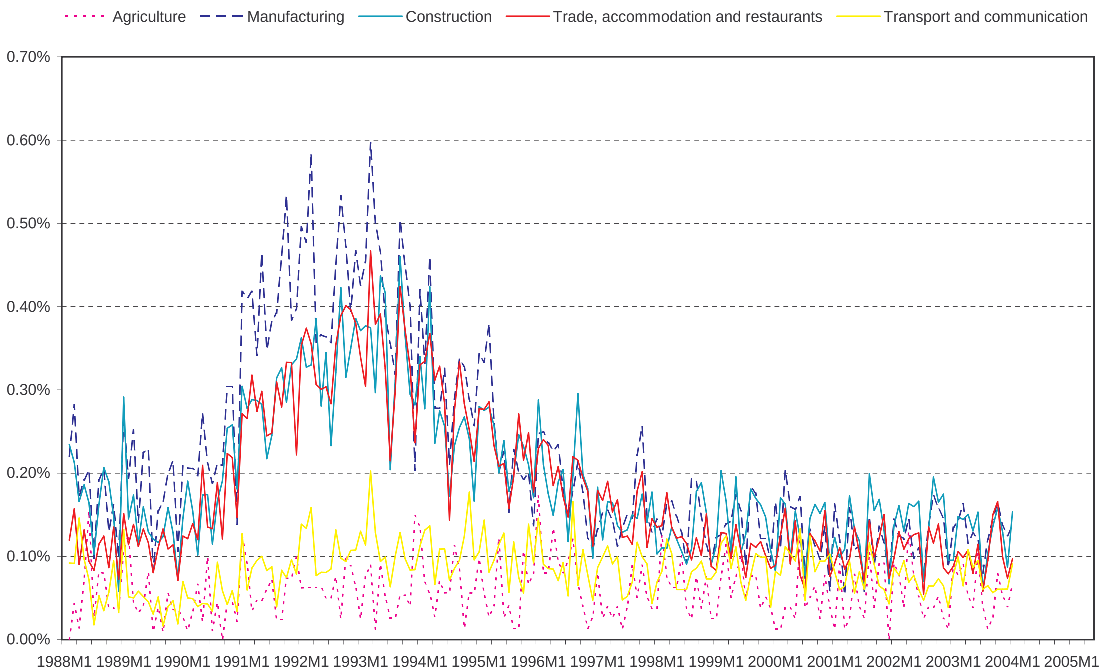

Figure 2.1: Monthly industry-specific default rates in the Finnish economy

2.2 Considered Macroeconomic Indicators

A lot of macroeconomic indicators as determinants of corporate default rates are usually considered. The most frequently determinants mention in studies are GDP

[2]The law was changed to facilitate restructuring instead of formal bankruptcy proceedings and so it may have reduced the number of bankruptcies. The change in the law was effected in february 1993 (Virolainen 2004)

and interest rates. In case of GDP, first difference of real GDP or difference from real GDP trend computed by Hodrick-Prescott filter[3] can be used.

$$\mathtt{GDPdif} = \frac{\mathtt{GDP} - \mathtt{GDP_{HP}}}{\mathtt{GDP}}, \tag{2.1}$$

where $\mathtt{GDP}$ is real GDP and $\mathtt{GDP_{HP}}$ is calculated by Hodric-Prescott filter. GDP data are available as quarterly. Monthly GDP data were obtained by disaggregation.[4] We considered 1M, 3M and 12M HELIBOR, form 1999 we took EURIBOR into account. Nominal and real interest rates were investigated. Real interest rates were calculated as

$$r = \frac{1+R}{1+\rho} - 1, \tag{2.2}$$

where r is real interest rate, R is nominal interest rate and ρ inflation during appropriate time period. Inflation was expressed by CPI and PPI indexes.[5] Nominal US/EURO exchange rate was used.[6] Finnish markka was considered before introducing of euro in Finland.

Loans to corporations and entrepreneurs were available for the time period 1989-1992 as annual time series and for the time period 1993-2004 as quarterly time series. We constructed debt indicator as ratio between outstanding loans to corporations and entrepreneurs and value added of the specific industry (GDP in case of aggregate economy was used). Formally,

$$\mathtt{DEBT} = \frac{\mathtt{LOANS}}{\mathtt{GDP_i}}, \tag{2.3}$$

where $\mathtt{LOANS}$ represents outstanding loans to corporations and entrepreneurs and $\mathtt{GDP_i}$ represents value added in the sector $\mathtt{i}$. It was available from 1/1990 after

[3]The Hodrick-Prescott filter is smoothing method that is widely used among macroeconomists to obtain a smooth estimate of the long-term trend component of series. The method was first used in the working paper (circulated in the early 1980's and published in 1997) by Hodrick and Prescott to analyze postwar U.S. business cycle.
Technicallly, the Hodrick-Prescott (HP) filter is a two-sided linear filter that computes the smoothed series s of y by minimizing the variance of y around s, subject to a penalty that constrains the second difference of s. That is , the HP filter chooses s to minimalize

$$\sum_{t=1}^{T}(y_t - s_t)^2 + \lambda \sum_{t=2}^{T-1}((s_{t+1} - s_t) - (s_t - s_{t-1}))^2$$

The penalty parameter λ controls the smoothness of the series σ.

[4]GDP was disaggregated from quarterly data with EKTA (Bank of Finland software)

[5]We used actual annual inflation rate. Ideally, expected inflation rate should be used, but data about inflation expectations were not available.

[6]Real effective exchange rate might be better, but only nominal exchange rate was available.

disaggregation to monthly data.[7]

In our analysis monthly growth rate of monetary aggregates M1 and M2 were considered. Furthermore, we accounted monthly data of unemployment rate, consumer confidence index or state budget as percentage of GDP.

[7] New loans to business is the another possible approach of debt indicator construction, but this data were not available for appropriate time period. However total outstanding loans can be important for explanation of default rate in the economy.

Chapter 3

Macroeconomic Credit Risk Models

The aim of this paper is a find a suite of macroeconomic models for default rate prediction and investigation of the relationship between macroeconomic indicators and default rate by these models (Jakubík 2006). In general we want to estimate function

$$d_{t_1} = f(I_{t_2}), \tag{3.1}$$

where d_{t_1} is default rate at time t_1 and $f(I_{t_2})$ is some function of macroeconomic indicators at time $t_2 \leq t_1$. The relationship between default rate and macroeconomic indicators can be modeled by this function.

These types of models are usually related to individual risk models, which is possible to express by the following general equation.

$$p_{t_1} = f(X_{t_2}), \tag{3.2}$$

where p_{t_1} is individual default probability at time t_1 and X_{t_2} are some indicators of client quality related to financial statement in the case of traditional model, firm value and leverage in the case of structural models or bond price in the case of reduced model. Macroeconomic indicators are part of this inputs for all types of these models. Originally macroeconomic factors were not considered, but in recent years a lot of papers research the influence of macroeconomic environment on the credit risk model. This issue is became important in nowadays.

Some empirical macroeconomic model maybe found in the literature. These models are based on the same idea as the traditional model. They try to find the empirical observed relationship between default rate and some macroeconomic indicators. This relationship is usually modeled very simple by linear, probit or logit models. Static or dynamic approaches are applied for modeling. Vector autoregressive models (VAR) are often used in the case of dynamic model. These models are able to modeled mutual relationship of times series even in case of time series nonstationarity. Vector autoregressive model can be applied for nonstationarity time series if cointegration exists. Vector error correction model (VEC) is able to distinguish long-run and short-run dependence. VEC model is only a reformulation of VAR model.

The other different approach is derived from Merton model (structural model). This model is employed in the Basel II framework for risk weight calibration. The model is based on modeling of assets return. Default event is defined as fall of borrowers return assets under some threshold. This models is originally used for estimation of individual risk, but in the last time was this idea extended to default rates estimation.

3.1 Dynamic Model

Empirical models try to estimate the empirical relationship between default rate and some macroeconomic indicators. Exact microeconomic substantiation is not important in this case. They explain default rate by some simple function, which is estimated on observed data. Linear, probit or logit models are usually use. A simple static approach can be used, but dynamic models are better in case of the mutual relationship investigation. In case of traditional dynamic models, investigation of the used time series stationarity is essential. Vector autoregressive models (VAR) can be used. Their reformulation into form of vector error correction model is able to separate long-term and short-term dependence. VAR models are generalized form of simple autoregressive process for n variables. These models are able to investigate mutual relationship between variables which are assumed random and simultaneously independent. The maximum length of time lag is known and assumed be the same for all consider variable.

Linear l dimensional autoregressive process of order p VAR(p) is defined by equation (3.3) .

$$Y_t = c + A_1 Y_{t-1} + \cdots + A_p Y_{t-p} + \epsilon_t, \tag{3.3}$$

where c is l dimensional vector of constants, $A_1, \ldots, A_p$ are $l \times l$ dimensional matrix of parameters,(ϵ_t) is l-dimensional gaussian white noise process.

VEC(p) model can be get by VAR(p) reformulation.

$$\triangle Y_t = c + \Pi Y_{t-1} + \sum_{i=1}^{p-1} \Theta_i \triangle Y_{t-i} + \epsilon_t, \tag{3.4}$$

where

$$\Pi = \sum_{i=1}^{p} A_i - I,$$

$$\Theta_i = - \sum_{j=i+1}^{p} A_j.$$

Long-term relationship is expressed by non-differentiate processes and short-term relationship by differentiate (stationary) processess.

We have started to investigate relationship between credit default rate and macroeconomic indicators by linear vector autoregressive models. However, our target has not been to detect exact relationships between the variables, but only the directions of influence. Exact relationship has been estimated by a more advanced approach derived from Merton's idea.

First, stationarity of time series were examined by Dickey-Fuller tests (see appendix table 5.3). Different stationarity orders of default rates time series for agriculture and the others economic sectors are observed. Time series of default rate in Agriculture is integrated order zero while default rate in whole economy is integrated order one and also default rate in manufacturing, trade, construction and transport is I(1).[1] Default rates in agriculture and transport seem be very similar. However, they have different order of stationarity (see figure 2.1). Time series of default rates in construction, manufacturing and trade have very similar character. Non-stationary times series can be used in VAR models only when they are cointegrated.

GDP and interest rates are often mentioned in studies, therefore we investigated relationship between corporate default rates, GDP and interests rates in case of dynamic model. Mutual relationship can be modeled by VAR or VEC model. We used the first difference of real GDP and difference from real GDP trend. 1M, 3M and 12M nominal and real interest rates were investigated. The order of stationarity is reported in appendix (see table 5.4).

Long-term and Short-term mutual relationship can be separated by VEC model. Long-term relationships are represented by matrix Π in (3.4). Non-stationary time

[1]According to the economic theory, default rate should be stationary in the long-term horizon. However in the 1990s we can observe a significant decreasing trend in many countries. Credit portfolio improvement can be caused by risk management techniques progress.

series can be used for this type of model when they are cointegrated. We investigated cointegration of default rates, interest rates and GDP by Johansen's test (Bierens 2004). Our tests showed cointegration of defaut rate, interest rates and GDP. Time series of GDP and interest rates are also cointegrated. It is important for agriculrure, where time series of default rate is already stationary. These results show, that original time series of default rates, GDP and interest rates can be used in VAR or VEC model.

Table 3.1 show the results of VAR(2) models estimation.

Model	R^2		
df,dGDP,R1M$_{CPI}$	0.800894	0.985970	0.959093
df$_{AGR}$,GDPdif,R12M$_{CPI}$	0.060148	0.998125	0.974966
df$_{CON}$,dGDP,R12M$_{CPI}$	0.654523	0.985846	0.974692
df$_{MAN}$,dGDP,R1M$_{CPI}$	0.321865	0.903338	0.135174
df$_{TRD}$,dGDP,R3M$_{PPI}$	0.280505	0.902341	0.094386
df$_{TRN}$,dGDP,R1M$_{PPI}$	0.442391	0.887201	0.035169

Table 3.1: VAR(2) models

The poor performance of the VAR(2) model in estimation of the default rate in agriculture is caused by different behavioral of default rates agriculture time series. Agriculture is probably more independent of the cycle of the whole economy. GDP should be to replace by industry specific value edited for improvement of VAR(2) models for industry specific sector. VAR(2) models of mutual relationship between default rates, GDP and interest rates were selected as models with the highest coefficient of determination for default rate. Two options for GDP were considered - difference of the real GDP from long-term trend and the first difference of the real GDP time series. Nominal and real interest rates were considered in case of interest rates. 1 month, 3 months and 12 months interest rates were examined. Consumer price index (CPI) and production price index (PPI) were used for real interest rate calculation. Cointegration relationships for selected models are introduced in the table 3.2.

Johansens' cointegration tests showed one cointegration relationship for selected models. Similar results were obtained for aggregate economy, construction and manufacturing. In this cases default rates are proportional to interests rates and non proportional to GDP. Values of cointegration vector are very close. In case of trade value of cointegration vector demonstrates a proportional relationship to interest rates as well as GDP. However a very low value of cointegration coefficient for GDP reveals an insignificant relationship between default rates and GDP for this sector of

Model	df	GDP	r
$\mathtt{df,dGDP,R1M_{CPI}}$	1.00000	0.000000274	-0.000113
$\mathtt{df_{CON},dGDP,R12M_{CPI}}$	1.00000	0.000000854	-0.000155
$\mathtt{df_{MAN},dGDP,R1M_{CPI}}$	1.00000	0.000000809	-0.000199
$\mathtt{df_{TRD},dGDP,R3M_{PPI}}$	1.00000	-0.0000000775	-0.000219
$\mathtt{df_{TRN},dGDP,R1M_{PPI}}$	1.00000	0.000000195	0.0000108

Table 3.2: Cointegration relationships between default rates, GDP and interest rates

economy. The coefficient of the interest rate is very similar to that of the aggregate economy, construction and manufacturing, but its value is a little higher. In case of transport, results show nonpropotional relationship GDP and interest rates with default rate. Coefficient of relationship with GDP is very similar to aggregate economy, construction and manufacturing, but the low value of interest rate coefficient demonstrates its insignificance. In case of agriculture, time series of default rates is already stationary.

Due to lower performance of VAR(2) for specific sectors, monthly time series of value added for AGR, CON, MAN, TRD, TRN were used. First, we examined stationarity of values added time series. The results of Dicky-Fuller tests are presented in appendix (see table 5.5).

All examined time series of value added were I(1) except agriculture. Time series of value added in agriculture is already stationary and it seems there is no cyclical behavioral in the sector of agriculture. In case of agriculture, stationarity of difference between value added and long term trend was also examined, but result was the same as for the first difference of this time series. VAR(2) models with replacing of GDP by value added did not improve the performance of considered VAR(2) models, except agriculture (viz table 3.3).

Model	$\mathtt{R}^2$		
$\mathtt{df_{AGR},dGDP_{AGR},R12M_{CPI}}$	0.483921	0.706177	0.202589

Table 3.3: VAR(2) model with value added for agriculture

However this kind of models are able to investigate mutual relationship between macroeconomic indicators, they are not very good for aggregate default rate estimation due to nonlinearity. Further, we focused to Merton type models.

3.2 One-factor Model

One of the variant of latent factor model is described by following equations. This model can be used for aggregate data which we had available for the Finnish economy. Application of this model to the German economy may be found in (Rösch 2003) or (Hamerle, Liebig, Scheule 2004) . This model is employed by Basel II accord. Following model appears in many papers, for example in (Rösch 2005), (Céspedes, Martín 2002), (Cipollini, Missaglia 2005) or (Lucas, Klaassen 2003).

The basic idea is based on Merton model. A normal distribution process is assumed for firm logarithmic return of assets. Discrete normalized logarithmic return process satisfies following equation for every company in the economy.

$$R_{it} = \sqrt{\rho} F_t + \sqrt{1-\rho} U_{it} \tag{3.5}$$

R denotes normalized logarithmic return of assets for each firm i at time t. F represents normalized logarithmic return in the economy independent on firm at time t. This return is assumed standard normal random distributed. It can be explained as the macroeconomic specific part of return. U denotes firm specific return. Standard normal random distribution is assumed. All random variables are assumed serially independent.

$$F_t \tilde{} N(0,1)$$
$$U_{it} \tilde{} N(0,1)$$

Coefficient ρ expresses the correlation between the normalized assets returns of any two borrowers.

$$E(R_{it}) = 0 \tag{3.6}$$

$$Var(R_{it}) = E(R_{it}^2) - E(R_{it})^2 = E(\rho F_t^2 + (1-\rho) U_{it}^2 + 2\sqrt{\rho}\sqrt{1-\rho} F_t U_{it}) = 1 \tag{3.7}$$

According the accepted assumption, return of assets for each firm i at time t is standard normal random distributed (3.6)(3.7). The basic idea of this model is derived from Merton model. Default event is assumed when return of assets decrease under some threshold. Formally,

$$P(Y_{it} = 1) = P(R_{it} < T), \tag{3.8}$$

where Y denotes random variable with the two potential state.

$$Y_{it} = \begin{cases} 1 & \text{borrower } i \text{ defaults at time } t \\ 0 & \text{else} \end{cases} \tag{3.9}$$

T can be assumed as constant or variable depends on time. In the second case change of this threshold is considered with changing in macroeconomic environment at time. Different macroeconomic indicators can be considered. Formally

$$T = \beta_0 + \sum_{j=1}^{N} \beta_j x_{jt}, \tag{3.10}$$

where x_j represents j-th macroeconomic indicator and β are constant coefficients. Simple linear relation for value of threshold is considered. Macroeconomic condition change affects the value of threshold for default at time. This value is probably higher in good time and lower in bad time. Generally, recession decreases the value of threshold for default events. The default probability of firm i at time t is given by equation (3.11) in case of the constant default threshold at time.

$$p_i = P(Y_{it} = 1) = P(R_{it} < T) = P(\sqrt{\rho}F_t + \sqrt{1-\rho}U_{it} < \beta_0) = \phi(\beta_0), \tag{3.11}$$

where ϕ is function of cumulative standard normal distribution. In general, other distribution function can be used, for example logistic distribution can be assumed (Hamerle, Liebig, Scheule 2004). Conditional default probability on realization f_t of random factor at time t can be described by following formula.

$$p_i(f_t) = P(U_{it} < \frac{\beta_0 - \sqrt{\rho}f_t}{\sqrt{1-\rho}}) = \phi(\frac{\beta_0 - \sqrt{\rho}f_t}{\sqrt{1-\rho}}) \tag{3.12}$$

Default probability of firm i at time t is given by equation (3.13) in the case when change of the threshold is considered according equation (3.10).

$$p_{it} = P(Y_{it} = 1) = P(R_{it} < T) = P(\sqrt{\rho}F_t + \sqrt{1-\rho}U_{it} < \beta_0 + \sum_{j=1}^{N} \beta_j x_{jt}) = \phi(\beta_0 + \sum_{j=1}^{N} \beta_j x_{jt}), \tag{3.13}$$

The conditional default probability on realization f_t of random factor and macroeconomic indicators x_t at time t can be obtained in this case from formula (3.14).

$$p_i(f_t) = P(U_{it} < \frac{\beta_0 + \sum_{j=1}^{N} \beta_j x_{jt} - \sqrt{\rho}f_t}{\sqrt{1-\rho}}) = \phi(\frac{\beta_0 + \sum_{j=1}^{N} \beta_j x_{jt} - \sqrt{\rho}f_t}{\sqrt{1-\rho}}) \tag{3.14}$$

The same result is obtained under the assumption that macroeconomic indicators are considered as a part of the factor of assets return independent on firm i at time t. This concept is used for example in (Hamerle, Liebig, Scheule 2004). Formally,

$$R_{it} = \alpha F_t + \beta_0 + \sum_{j=1}^{N} \beta_j x_{jt} + \omega U_{it}. \tag{3.15}$$

If very high number of borrowers in portfolio is assumed, all counterparties have the same individual probability p_i and all default events are independent, then according the "law of large numbers" default rate on the portfolio can be estimated as individual default probability.

$$P(p(f_t) = p_i(f_t)|F_t = f_t) = 1 \tag{3.16}$$

Unconditional default probability can be obtained by

$$p = P(Y_t = 1) = \int_{-\infty}^{\infty} P(Y_t = 1|F_t = f_t)\psi(f_t)df_t = \int_{-\infty}^{\infty} p(f_t)\psi(f_t)df_t, \tag{3.17}$$

where ψ is function of standard normal distribution.

Random factor is assumed independent between borrowers. Number of defaults $D_t(f_t)$ at time t have binomial distribution with conditional default probability $p(f_t)$ and given number of companies N_t.

$$D(f_t)\tilde{\ }\mathrm{Bi}(N_t, p(f_t)) \tag{3.18}$$

Conditional probability of having exactly d_t default at time t can be expressed as

$$P(D_t = d_t|F_t = f_t) = \binom{n_t}{d_t} p(f_t)^{d_t}(1 - p(f_t))^{n_t - d_t}. \tag{3.19}$$

Unconditional probability of having exactly d_t default at time t can be expressed as

$$P(D_t = d_t) = \int_{-\infty}^{+\infty} \binom{n_t}{d_t} p(f_t)^{d_t}(1 - p(f_t))^{n_t - d_t}\psi(f_t)df_t. \tag{3.20}$$

3.2.1 One-Factor Model Estimation

Parameters of model (3.12) or (3.14) can be estimated whereby log-likelihood function. Number of defaults D_t is conditional binomial distributed random variable with number of borrowers N_t and conditional probability $p(f_t)$ according equation (3.18). Data of the defaults numbers d_t are observed. Realization d_t and n_t of random variables D_t and N_t are known.

$$d_t = \sum_{i=1}^{n_t} d_{it}$$

Unconditional number of defaults can be computed by integral over the random effect (3.17). Log-likelihood function depends only on parameters β and ρ. Formally for model (3.12)

$$l(\beta,\rho) = \sum_{t=1}^{T} \ln \left\{ \int_{-\infty}^{\infty} \binom{n_t}{d_t} \phi \left(\frac{\beta_0 - \sqrt{\rho} f_t}{\sqrt{1-\rho}} \right)^{d_t} \left[1 - \phi \left(\frac{\beta_0 - \sqrt{\rho} f_t}{\sqrt{1-\rho}} \right) \right]^{n_t - d_t} \psi(f_t) df_t \right\}. \tag{3.21}$$

Log-likelihood function for model (3.14) can be expressed similarly by equation (3.22).

$$l(\beta_0, \cdots, \beta_N, \rho) = \sum_{t=1}^{T} \ln \{ \int_{-\infty}^{\infty} \binom{n_t}{d_t} \phi \left(\frac{\beta_0 + \sum_{j=1}^{N} \beta_j x_{jt} - \sqrt{\rho} f_t}{\sqrt{1-\rho}} \right)^{d_t}$$
$$\left[1 - \phi \left(\frac{\beta_0 + \sum_{j=1}^{N} \beta_j x_{jt} - \sqrt{\rho} f_t}{\sqrt{1-\rho}} \right) \right]^{n_t - d_t} \psi(f_t) df_t \}. \tag{3.22}$$

3.3 Multi-Factor Model

These type of models are generalized version of the one-factor model. Multi-factor models assumed M correlated factors in the economy. Multi-factor model framework can be interpreted as a world of the M economies or countries where factor is common for all firms of the appropriate economy or country. These M economies are related, because there is correlation between factors. A two-factor model is discussed for example in (Céspedes, Martín 2002). A continuous version of three-factor model can be found in (Hui, Lo, Huang 2003).

In case of model 3.5 you can generalize to multi-factor models by the following equations.

$$R_{it}^1 = \sqrt{\rho_1} F_t^1 + \sqrt{1-\rho_1} U_{it}^1$$
$$\cdots$$
$$R_{it}^M = \sqrt{\rho_M} F_t^M + \sqrt{1-\rho_M} U_{it}^M \tag{3.23}$$

$$f_i = \rho_{ij} f_j + \sqrt{1-\rho_{ij}^2}\eta_{ij} \quad \forall i,j \in \{1,\cdots,M\},\ i \neq j$$

$$\rho_{ij} = \mathrm{corr}(f_i, f_j) \quad \forall i,j \in \{1,\cdots,M\},\ i \neq j \tag{3.24}$$

where $f_1 \cdots f_M,\ \eta_{ij}\ \forall i,j \in \{1,\cdots,M\},\ i \neq j$ are N(0,1) i.i.d.

Conditional default probability can be derived for each country similarly as in case of one-factor model. Conditional default probability satisfies following equations.

$$p_i^1(f_t^1) = \phi(\frac{T_1 - \sqrt{\rho_1} f_t^1}{\sqrt{1-\rho_1}})$$
$$\cdots$$
$$p_i^M(f_t^M) = \phi(\frac{T_M - \sqrt{\rho_M} f_t^M}{\sqrt{1-\rho_M}}) \tag{3.25}$$

where $T_1 \cdots T_N,$ is value of threshold which can be modeled as constant in the time or random variable as in the case of one-factor model. $\rho_1,\cdots,\rho_M$ are constants represents correlation between firm assets in the each economy or country. Due to independent of all default events, portfolio default probability can be modeled by weighted sum of default in each of segment. "Law of large number" can be applied. Default rate on the each segment can be estimated as individual probability of the firm in the specific segment. Default rate on the portfolio is estimated by default rates in the segments weighted by fraction of the segments in the portfolio.

Formally,

$$P(p(f_t) = w_t^1 p_i^1(f_t^1) + \cdots + w_t^M p_i^M(f_t^M)) = 1, \tag{3.26}$$

where $w_t^1,\cdots,w_t^N$ represent fraction of the specific segment in the time t in the portfolio. Formally,

$$w_t^i = N_t^i / N_t, \tag{3.27}$$

where N_t^i denotes numbers of firms in the i-th specific economy in the time t and N_t denotes number of firms in the portfolio in the time t.

Number of defaults $D^i(f_t^i)$ is binomial distributed within the specific segment of economy.

$$D^1(f_t^1)\tilde{\ } Bi(N_t^1, p^1(f_t^1))$$

$$\cdots$$

$$D^M(f_t^M)\tilde{}Bi(N_t^M, p^M(f_t^M)) \tag{3.28}$$

Conditional probability of having exactly d_t default at time t in whole economy can be expressed as product of conditional probabilities for the industry specific sector due to independent of random events within segments as well as between segments.

$$P(D_t = d_t|F_t = f_t) = \sum_{s_1=0}^{d_t} \binom{n_t^1}{s_1} p^1(f_t)^{s_1}(1-p^1(f_t))^{n_t^1-s_1}$$

$$\cdot \sum_{s_2=0}^{d_t-s_1} \binom{n_t^2}{s_2} p^2(f_t)^{s_2}(1-p^2(f_t))^{n_t^2-s_2} \cdots \sum_{s_M=0}^{d_t-s_{M-1}-s_{M-2}-\cdots-s_1} \binom{n_t^M}{s_M} p^1(f_t)^{s_M}(1-p^M(f_t))^{n_t^M-s_M} \tag{3.29}$$

Equation (3.29) is valid for $d_t \leq n_i \quad \forall i \in \{1, \cdots, M\}$. For other case equation (3.29) should be adjusted. This assumption is very realistic in our case. We want to model default for industry specific economy. Number of defaults in the whole economic is very small compare to number of firms in the industry specific economy sector in case of considered five segments (AGR, CON, MAN, TRD, TRN).

Unconditional probability of having exactly d_t default at time t can be expressed as

$$P(D_t = d_t) = \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} P(D_t = d_t|F_t = f_t)\psi(f_t^1, \cdots, f_t^M)df_t^1 \cdots df_t^M. \tag{3.30}$$

3.3.1 Multi-Factor Model Estimation

Parameters of model (3.25) can be estimated similarly as for the one-factor model. However, likelihood function is more complicated in case of multi-factor model.

$$l(\beta^1, \cdots, \beta^M, \rho^1, \cdots, \rho^M) = \sum_{t=1}^{T} \ln\{\int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} \sum_{s_1=0}^{d_t} \binom{n_t^1}{s_1} p^1(f_t)^{s_1}(1-p^1(f_t))^{n_t^1-s_1}$$

$$\cdot \sum_{s_2=0}^{d_t-s_1} \binom{n_t^2}{s_2} p^2(f_t)^{s_2}(1-p^2(f_t))^{n_t^2-s_2}$$

$$\cdots \sum_{s_M=0}^{d_t-s_{M-1}-s_{M-2}-\cdots-s_1} \binom{n_t^M}{s_M} p^1(f_t)^{s_M}(1-p^M(f_t))^{n_t^M-s_M}\psi(f_t^1, \cdots, f_t^M)df_t^1 \cdots df_t^M\} \tag{3.31}$$

Multi-factor models assumed, that data of defaults numbers d_t^i and numbers of firms n_t^i $\forall i \in \{1, \cdots, M\}$ are observed in the each specific sector of the economy separately.

Chapter 4

Results of A Latent Factor Model for The Finnish Economy

4.1 Used Data

Data on bankruptcies are used to estimate a one-factor model. This was a monthly time series of firms' bankruptcies and yearly time series of firms' numbers. Data about numbers of Firms were disaggregated from annual data.[1] GDP, interest rates, debt ratio and exchange rates were used as macroeconomic indicators in models (3.14). Despite also lagged macroeconomic variables were tested only lagged exchange rate was significant in the case of latent one-factor model. The other macroeconomic indicators were significant only as non-lagged variables. All calculations were based on monthly data.

4.2 Used Model

We started with estimation of one-factor model for aggregate economy. Constant correlation between normalized assets returns of the firms is assumed. This model can provide better results for relatively homogenous portfolio. Due to this fact, industry specific sectors were considered. We estimated one-factor model separately for each industry specific sectors (AGR, MAN, CON, TRD, TRN). Unfortunately, this model is not able to give sufficient results of relationships between industry specific sectors. Multi-factor model could be better for providing some results about interaction between industry specific sectors. This kind of model follows the mutual relationship of sectors by correlation parameters of the industry specific factors. However, estimation of multi-factor models is numerically fairly complicated. We

[1]Number of firms were disaggregated from annual data with EKTA (Bank of Finland software)

had available data of five industry specific sector, it means five-factor model would have to be used. Only estimation of one-sector model separately for each industry specific sector has been done. Model (3.14) was estimated for aggregate economy and also for each industry specific sector. This model follows the relationship between default rate and macroeconomic indicators and can be use for stress testing as well.

4.3 Aggregate Economy

Models (3.12) and (3.14) were estimated for the Finnish economy for used data. Both of the models were also re-estimated for data started by 1/1993 due to change in bankruptcy law in 1993. Obtain results were compared.

Table 4.1 shows estimation of model (3.12) for data started by 1/1988. Constant parameter β_0 was estimated as -2.9528. It corresponds to default probability about 0.16%. Estimated correlation between normalized return assets of the borrowers is about 1.7%. It corresponds to 12-month correlation between normalized return assets of the borrowers about 5.7%. Both coefficients were highly significant. 12-month default probability corresponds to estimated monthly default probability about 1.89% under assumption of constant default development.

Parameter	**Estimate**	**Standard error**	**Pr>|t|**
β_0	-2.9528	0.009731	<.0001
ρ	0.01659	0.001701	<.0001

Table 4.1: Estimation of model (3.12) for data started by 1/1988 (aggregate ec.)

Table 4.2 shows estimation of model (3.12) for data started by 1/1993. Constant parameter β_0 was estimated as -2.9699. It corresponds to default probability about 0.15%. Estimated correlation between normalized return assets of the borrowers is about 1.5%. It corresponds to 12-month correlation between normalized return assets of the borrowers about 5.7%. Both coefficients were highly significant. 12-month default probability corresponds to estimated monthly default probability about 1.79% under assumption of constant default development. You can see very similar results in the both cases. We can conclude, that model is fairly robust due to change in bankruptcy law in 1993.

Table 4.3 shows estimation of models (3.14) for data started by 1/1988. GDP (β_1), interest rate (β_2) and exchange rate (β_3) were used as a macroeconomic indicators in this calculation. These estimations confirmed theory of negative relationship between GDP and default probability and positive relationship of default probability with interest rates. Dummy variable (β_4) was used to allow for the bankruptcy

Parameter	Estimate	Standard error	Pr>|t|
β_0	-2.9699	0.01118	<.0001
ρ	0.01518	0.001877	<.0001

Table 4.2: Estimation of model (3.12) for data started by 1/1993 (aggregate ec.)

law change in 1993. Values of this variable are zero till end of 1992 and one from the beginning of 1993. Difference of real GDP computed according to equation (2.1) was considered. Interest rates (R) were represented by real 12-months interest rate computed according to equation (2.2). Exchange rate (ER) is represented by nominal US/EURO exchange rate. Finnish markka was used before introducing of euro in Finland. According to this model there is a positive relationship between default rate and US/EURO nominal exchange rate. Four month lagged variable of exchange rate was used. Estimated unobservable factor coefficient is about 0.7%. All coefficients were significant at 5% confidence level. Figure 4.1 shows performance of estimated model (3.14) for data started by 1/1988.

Parameter	Estimate	Standard error	Pr>|t|
β_0	-3.5085	0.06804	<.0001
β_1 (GDP)	-0.04348	0.005699	<.0001
β_2 (R)	0.05427	0.004450	<.0001
β_3 (ER_{t-4})	0.1171	0.05064	0.0219
β_4 (DUMMY)	0.2426	0.02590	<.0001
ρ	0.006827	0.000735	<.0001

Table 4.3: Estimation of model (3.14) for data started by 1/1988 (aggregate ec.)

We tried to re-estimate the model for data started by 1/1993. Table 4.4 shows that the results as regards relationship between default rate, GDP and interest rate were fairly similar, but relationship between default rate and exchange rate was different. Because of the exchange rate coefficient (β_3) insignificancy in the case of model estimation for data started by 1/1993, we can conclude weak or unstable relationship between exchange rate and default rate at time. Further we can conclude, that relationship between default rate, GDP and interest rates is quite stable at time.

Furthermore we tried to add some indicators of debt to the model due to Merton concept of default event. We constructed debt indicator as ratio between outstanding loans to corporations and entrepreneurs and GDP according to equation (2.3). It

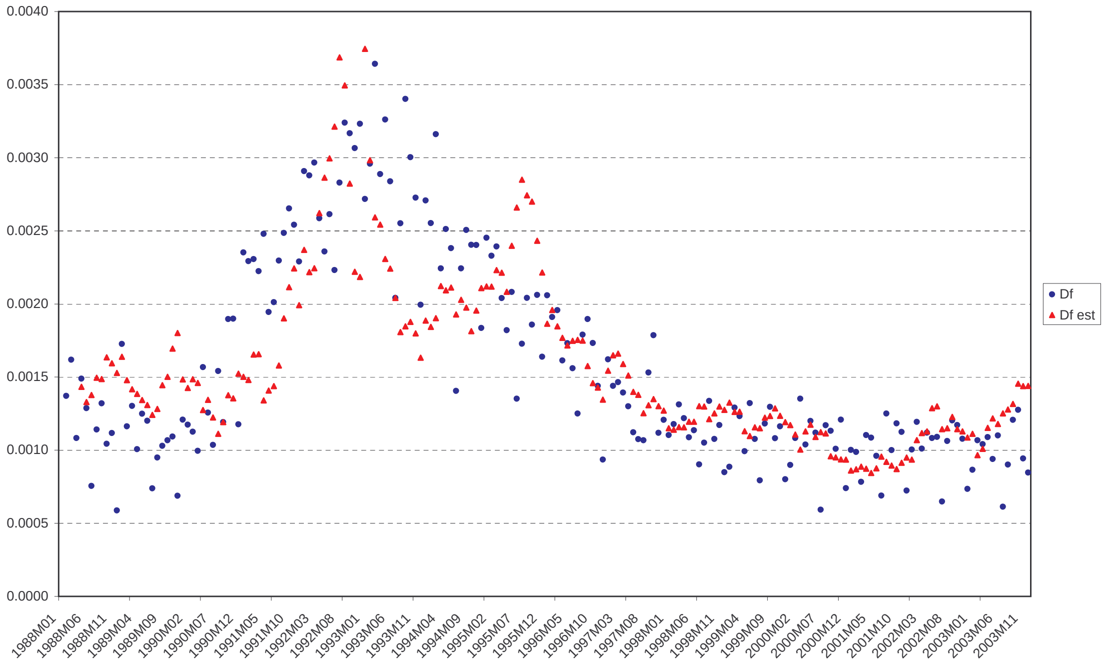

Figure 4.1: Performance of the one-factor model for the Finnish economy

was available from 1/1990 after disaggregation to monthly data. We had to restrict beginning of all our time series to 1/1990 due to the limited debt indicator time series. Following table 4.5 demonstrates estimated model (3.14) with debt indicator (DEBT).

Debt indicator is highly significant in the estimated model. This model can better explain default rate than model without debt indicator. Estimation proved positive relationship between default rate and debt indicator. Exchange rate is not significant on the 5% of confidence model. We can re-estimate this model with debt indicator and without exchange rate. Table 4.6 shows result of re-estimated model. All coefficients are highly significant.

Figure 4.2 shows performance of estimated model (3.14) for data started by 1/1990 with debt indicator and without exchange rate (table 4.6).

One-factor model assumes constant correlation of normalized return assets of borrowers. This assumption can be satisfy in the case of the homogenous portfolio. For this reason, the following analysis was focused on the industry specific sectors.

Parameter	Estimate	Standard error	Pr>|t|
β_0	-3.0971	0.05112	<.0001
β_1 (GDP)	-0.05478	0.007379	<.0001
β_2 (R)	0.06537	0.004971	<.0001
β_3 (ER$_{t-4}$)	-0.06831	0.05256	0.1960
ρ	0.004806	0.000632	<.0001

Table 4.4: Estimation of model (3.14) for data started by 1/1993 (aggregate ec.)

Parameter	Estimate	Standard error	Pr>|t|
β_0	-3.3969	0.04896	<.0001
β_1 (GDP)	-0.04114	0.004281	<.0001
β_2 (R)	0.01587	0.004527	0.0006
β_3 (ER$_{t-4}$)	0.06670	0.03612	0.0666
β_4 (DEBT)	0.1767	0.01629	<.0001
β_5 (DUMMY)	0.1187	0.02154	<.0001
ρ	0.003097	0.000374	<.0001

Table 4.5: Estimation of model (3.14) with debt indicator for data started by 1/1990 (aggregate economy)

4.4 Agriculture

Result of the one-factor model (3.14) for agriculture (table 4.7) shows a significant influence of the latent factor in the model. Coefficient ρ is significant on the 1% confidence level. Contrary to the empirical model (chapter 3), results of the one-factor model show negative relationship between default rate and GDP on the 5% confidence level. Exchange rates and interest rates are probably insignificancy for default events in the sector of agriculture. Due to insignificant coefficient β_4, there was not probably impact of bankruptcy law change on default level in Agriculture sector.

Following table 4.8 shows result of one-factor model (3.14), where debt ratio indicator were considered. Due to this fact, only time series started by 1/1990 was accounted. All macroeconomic indicators are insignificant in contrast with result for model which was estimated for data started by 1/1998. Default rate in agriculture can be explained only by unobservable factors in this case, because coefficient ρ was significant on the 1% of confidence level.

Parameter	Estimate	Standard error	Pr>|t|
β_0	-3.3222	0.02794	<.0001
β_1 (GDP)	-0.04027	0.004301	<.0001
β_2 (R)	0.01802	0.004424	<.0001
β_4 (DEBT)	0.1795	0.01639	<.0001
β_5 (DUMMY)	0.1092	0.02113	<.0001
ρ	0.003170	0.000382	<.0001

Table 4.6: Estimation of model (3.14) with debt indicator for data started by 1/1990 (aggregate economy)

Parameter	Estimate	Standard error	Pr > |t|
β_0	-3.4311	0.1300	<.0001
β_1 (GDP)	-0.02653	0.01097	0.0165
β_2 (R)	-0.00319	0.008534	0.7089
β_3 (ER_{t-4})	0.1354	0.09641	0.1617
β_4 (DUMMY)	0.04148	0.04937	0.4019
ρ	0.008009	0.002649	0.0029

Table 4.7: Estimation of model (3.14) for agriculture

4.5 Manufacturing

Results of the one-factor model (3.14) demonstrates similar behavioral of the manufacturing sector as the aggregate economy (see table 4.9). However our results show insignificancy of exchange rate for default rate prediction. Model proved dependence of default rate on GDP and interest rate. Both coefficients (β_1, β_2) were highly significant. Change of the bankruptcy law in 1993 was important for level of default rate in this sector according achieved results (coefficient β_4). Unobserved factor is still highly significant.

Table 4.10 shows result of Model (3.14) for manufacturing, where debt is took into account. Coefficient of dummy variable (β_4) is insignificant in this model. Change of the bankruptcy law is not important in the model, when debt indicator is considered. All the others coefficients are significant on the 5% of confidence level.

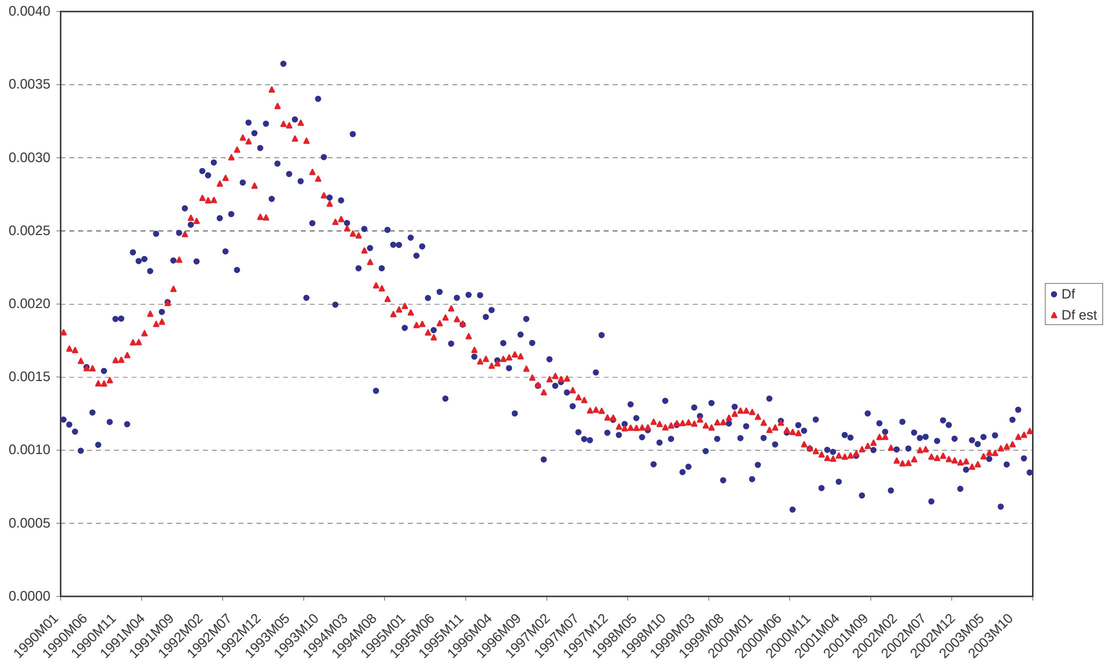

Figure 4.2: Performance of the one-factor model with debt indicator for the Finnish economy

4.6 Construction

Results of the one-factor model for construction are similar to sector of manufacturing (see table 4.11). Except exchange rates, all variables included in the model are significant. Exchange rate probably do not play important role for default event of firms.

Table 4.12 summarizes obtain estimation of (3.14) model for construction sector with inclusion of debt indicator. All coefficients are significant on the 5% confidence level. Results proved positive correlation between default events and indebtedness of corporate and entrepreneurs. GDP, interest rates and change of bankruptcy law were still important for explaining of default rate.

4.7 Trade

Table 4.13 shows results of one-factor model (3.14) for sector Trade. Level of default rate depends on GDP, interest rate and exchange rate in the economy. Coefficient of exchange rate (β_3) is significant on 5% confidence level. Al the other coefficients

Parameter	Estimate	Standard error	Pr > |t|
β_0	-3.5185	0.1754	<.0001
β_1 (GDP)	-0.01081	0.01292	0.4036
β_2 (R)	-0.00428	0.01025	0.6769
β_3 (DEBT)	0.2080	0.1456	0.1548
β_4 (DUMMY)	0.07308	0.06341	0.2507
ρ	0.007383	0.002756	0.0081

Table 4.8: Estimation of model (3.14) for agriculture (with debt indicator) and data started by 1/1990

Parameter	Estimate	Standard error	Pr > |t|
β_0	-3.3654	0.08968	<.0001
β_1 (GDP)	-0.04865	0.007380	<.0001
β_2 (R)	0.06016	0.005745	<.0001
β_3 (ER_{t-4})	0.09880	0.06638	0.1383
β_4 (DUMMY)	0.1747	0.03364	<.0001
ρ	0.01012	0.001237	<.0001

Table 4.9: Estimation of model (3.14) for manufacturing

are highly significant. Exchange rate plays important role in the sector of trade due to international business. This model proved this intuitive exception. Change of bankruptcy law was important for the default rate level in trade according this model (coefficient β_4). Unobserved factor is still significant.

Further we estimated model, where exchange rate was replaced by debt indicator (see table 4.14). All coefficients are significant on the 5% confidence level.

4.8 Transport

The following table 4.15 demonstrates similar result for transport as we obtained for manufacturing and construction. Default rate depends negatively on GDP and positively on interest rates. Exchange rates are not important for default rate in transport. All coefficients except exchange rate are highly significant.

Table 4.16 shows estimated model (3.14) for transport, where debt indicator was considered. In this case only debt indicator and change in the bankruptcy law are important macro indicators for explaining of default rate. Unobservable factor is

Parameter	**Estimate**	**Standard error**	**Pr > |t|**
β_0	-3.1738	0.03683	<.0001
β_1 (GDP)	-0.04184	0.005695	<.0001
β_2 (R)	0.01334	0.005632	0.0190
β_3 (DEBT)	0.05686	0.005316	0.0001
β_4 (DUMMY)	0.04120	0.02726	0.1326
ρ	0.004158	0.000684	<.0001

Table 4.10: Estimation of model (3.14) for manufacturing (with debt indicator) and data started by 1/1990

Parameter	**Estimate**	**Standard error**	**Pr > |t|**
β_0	-3.3014	0.07853	<.0001
β_1 (GDP)	-0.04505	0.006531	<.0001
β_2 (R)	0.04938	0.005086	<.0001
β_3 (ER_{t-4})	0.05533	0.05832	0.3440
β_4 (DUMMY)	0.1766	0.02986	<.0001
ρ	0.007381	0.000986	<.0001

Table 4.11: Estimation of model (3.14) for construction

still highly significant.

Parameter	Estimate	Standard error	Pr > |t|
β_0	-3.2043	0.03452	<.0001
β_1 (GDP)	-0.02573	0.005705	<.0001
β_2 (R)	0.01118	0.005346	<.0381
β_3 (DEBT)	0.4956	0.05446	<.0001
β_4 (DUMMY)	0.06308	0.02571	0.0152
ρ	0.003339	0.000603	<.0001

Table 4.12: Estimation of model (3.14) for construction (with debt indicator) and data started by 1/1990

Parameter	Estimate	Standard error	Pr > |t|
β_0	-3.5832	0.08157	<.0001
β_1 (GDP)	-0.04550	0.006812	<.0001
β_2 (R)	0.06406	0.005301	<.0001
β_3 (ER_{t-4})	0.1480	0.06057	0.0155
β_4 (DUMMY)	0.2909	0.03093	<.0001
ρ	0.009184	0.001050	<.0001

Table 4.13: Estimation of model (3.14) for trade

4.9 Comparison of Results for Industry Specific Sectors

Table 4.17 compares the estimation of model (3.14) for industry specific sectors. Marks * and ** denote significancy of estimation (1% confidence level, 5% confidence level). Only significant coefficients on the 5% confidence level are introduced in the table.

The obtained results have proved negative relationship between default rate and GDP for all investigated sectors of the economy. The estimated coefficients for GDP were quite similar for manufacturing, construction and trade, but default rate for sector of manufacturing is probably the strongest related to GDP. Similar coefficients were obtained for construction and trade. Both of them were about -0.045. The weakest relationship between default rate and GDP was estimated for the sector of transport and agriculture. However these relations were still proved against empirical models, where relationship was not proved for agriculture. All of the estimated coefficients for GDP were significant on the 5% confidence level. Except agriculture,

Parameter	Estimate	Standard error	Pr > |t|
β_0	-3.3635	0.03419	<.0001
β_1 (GDP)	-0.01695	0.006046	0.0057
β_2 (R)	0.01400	0.005696	0.0150
β_3 (DEBT)	0.2546	0.02351	<.0001
β_4 (DUMMY)	0.1282	0.02519	<.0001
ρ	0.004104	0.000567	<.0001

Table 4.14: Estimation of model (3.14) for trade (with debt indicator) and data started by 1/1990

Parameter	Estimate	Standard error	Pr > |t|
β_0	-3.4396	0.07669	<.0001
β_1 (GDP)	-0.02305	0.0066671	0.0007
β_2 (R)	0.02356	0.005120	<.0001
β_3 (ER_{t-4})	0.04380	0.05758	0.4478
β_4 (DUMMY)	0.1957	0.03017	<.0001
ρ	0.004561	0.000962	<.0001

Table 4.15: Estimation of model (3.14) for transport

they were significant even on 1% confidence level.

Interest rates (R) play important role for default events in all examined sectors except agriculture. The sector of agriculture is not probably sensitive on the change of the interest rate. Coefficients of interest rates were significant on the 1% of confidence level for all the others sectors. There were proved positive relationship between default rates and interest rates. The most dependent sector on the interest rate is probably trade and also manufacturing. Conversely, the weakest relation was obtained for the transport. However, the estimated coefficients for interest rate were fairly similar except transport.

Exchange rate (ER) was important for default event only in the sector of trade. Value of exchange rate play probably important role in this sector due to international trade. US/EURO nominal exchange rate was considered. However, we can not reject the exchange rate as an important indicator of default event in others sectors due to high correlation with interest rate. In the case of trade, positive relationship between default rates and exchange rates was proved. This result means less default events with stronger currency. This obtained result is not so clear according the economy theory. The value of the four lagged exchange rate was the

Parameter	Estimate	Standard error	Pr > |t|
β_0	-3.4470	0.06308	<.0001
β_1 (GDP)	-0.01348	0.007194	0.0627
β_2 (R)	0.009504	0.006553	0.1489
β_3 (DEBT)	0.04651	0.1840	0.0124
β_4 (DUMMY)	0.1586	0.03218	<.0001
ρ	0.003202	0.000822	0.0001

Table 4.16: Estimation of model (3.14) for transport (with debt indicator) and data started by 1/1990

most significant.

Change of bankruptcy law (DUMMY) probably affects level of default rates in all sectors except agriculture. Coefficients of the used dummy variable were significant on the 5% of confidence level in the cases of manufacturing, construction, trade and transport. It seems that change of this law does not influence on the agriculture sector. Sector of construction and trade were affected very similar according the alike values of estimated coefficients for dummy variables.

Unobserved factor was significant in all cases. Coefficients ρ were significant for all industry specific sectors. The value of this coefficients were fairly similar.

Sector of Economy	GDP	R	ER_{t-4}	DUMMY	ρ
Aggregate Economy	-0.04348**	0.05427**	0.1171*	0.2426**	0.006827**
Agriculture	-0.02653*	–	–	–	0.008009**
Manufacturing	-0.04865**	0.06016**	–	0.1747**	0.010120**
Construction	-0.04505**	0.04938**	–	0.2986**	0.007381**
Trade	-0.04550**	0.06406**	0.1480*	0.2909**	0.009184**
Transport	-0.02305**	0.02356**	–	0.1957**	0.000962**

Table 4.17: Comparison of models (3.14) for the industry specific sector of economy

Slightly different results are showed in the table 4.18, which demonstrates results of the one-factor models for aggregate and industry specific economy. Data started by 1/1990 were used for model estimation. Debt indicator was considered. This models contains GDP, interest rates, debt indicator and dummy variable as proxy for change of the bankruptcy law. Marks * and ** have the same meaning as in the previous case. Only significant coefficients are introduced in the table 4.18.

The obtained results have confirmed negative relationship between GDP and default rate only in the case of manufacturing, construction and trade. However in the case of transport coefficient was significant on the 6.27% confidence level. These results show time instability of this relationship in the case of transport and mainly in agriculture, where estimated coefficient was highly insignificant. The strongest relation was obtained in manufacturing. Default events is probably most affected by recession in manufacturing. This result corresponds with previous results (tab. 4.17).

Similar results were obtained for interest rate (R). A positive relationship between interest rates and default rate was proved in the case of manufacturing, construction and trade. The strongest relation was obtained in trade. This result corresponds with previous results (tab. 4.17).

Debt indicator (DEBT) was considered as ratio between gross debt of industry (outstanding loans to corporate and entrepreneurs) and value added of that industry. Coefficients of indebtedness indicator were significant in the all considered sectors except agriculture. Our hypothesis, that indebtedness is important determinant of default rate has been proved. Positive relationship between indebtedness and default rate in economy has been showed in all sectors except agriculture. Sector of agriculture seems to be independent or only slightly depend on macroeconomic environment.

Coefficients of change of bankruptcy law (DUMMY) are significant in the case of construction, trade and transport. Coefficient is insignificant in the case of manufacturing when debt indicator was be included to the model estimated on data started by 1/1990.

Very similar value of ρ coefficients were obtained in all cases. These coefficients represent unobservable factors. Slightly different result was estimated for agriculture, where value of this coefficient is higher due to insignificancy of macroeconomic variables in the model.

Sector of Economy	**GDP**	**R**	**DEBT**	**DUMMY**	ρ
Aggregate Economy	-0.04027**	0.01802**	0.1795**	0.1092**	0.003170**
Agriculture	–	–	–	–	0.007383**
Manufacturing	-0.04184**	0.01334*	0.05686**	–	0.004158**
Construction	-0.02573**	0.01118*	0.4956**	0.06308*	0.003339**
Trade	-0.01695**	0.01400**	0.2546**	0.1282**	0.004104**
Transport	–	–	0.04651*	0.1586**	0.003202**

Table 4.18: Comparison of models (3.14) for the industry specific sector of economy

The relationship between the respective sectors of the economy is apparent in the results of one-factor models. The relationship can be described by the correlation matrix for default rates (df) of industry specific economy (agriculture - AGR, manufacturing - MAN, trade - TRD, construction - CON, transport - TRN). Significance of the each coefficient is introduced in parenthesis. The correlation matrix demonstrates high correlation between manufacturing, trade and construction. Default rate of transport is less correlated with others. You can see also very low correlation of agriculture with all others industry specific sectors.

	$\mathbf{df_{AGR}}$	$\mathbf{df_{MAN}}$	$\mathbf{df_{TRD}}$	$\mathbf{df_{CON}}$	$\mathbf{df_{TRN}}$
$\mathbf{df_{AGR}}$	1.00000	0.14754 (0.0445)	0.17953 (0.0142)	0.20101 (0.0059)	0.29201 (<.0001)
$\mathbf{df_{MAN}}$	0.14754 (0.0445)	1.00000	0.91775 (<.0001)	0.88748 (<.0001)	0.45449 (<.0001)
$\mathbf{df_{TRD}}$	0.17953 (0.0142)	0.91775 (<.0001)	1.00000	0.90995 (<.0001)	0.52673 (<.0001)
$\mathbf{df_{CON}}$	0.20101 (0.0059)	0.88748 (<.0001)	0.90995 (<.0001)	1.00000	0.50152 (<.0001)
$\mathbf{df_{TRN}}$	0.29201 (0.0059)	0.45449 (<.0001)	0.52673 (<.0001)	0.50152 (<.0001)	1.00000

Table 4.19: Pearson correlation coefficients for the industry specific default rate

Chapter 5

Macroeconomic Credit Risk Model of the Czech Economy

This chapter focuses on the development of a macroeconomic model of default rate in the Czech economy (Jakubík 2006). The model is able to estimate the expected ratio of non-performing loans on the total credit portfolio as a function of key macroeconomic indicators. The non-performing ratio indicator is one of the Czech National Bank stress test model inputs (Čihák, Heřmánek 2005). This ratio was considered as a constant parameter in the past. The value of the parameter was based on the historically observed negative value. This new approach used in the Czech National Bank (CNB), described in this chapter, is able to model the effects of different macroeconomic shocks on portfolio quality and together with stress test also on capital adequacy ratio of the whole banking sector. These shocks can be set up on the bases of historical experiences or designed as an output of internal macroeconomic prediction model.

5.1 Credit Risk Model

Two basic group of models are usually used for credit risk modeling. The first group of models try to estimated individual risk of debtors. They are involved in credit risk assessment of the commercial banks and are called individual credit risk models. Nevertheless, banks can also incorporate some macroeconomic indicators into a model in an effort to avoid problem of credit risk assessment procyclicality.[1] Outputs of the individual credit risk models can provide inputs for capital adequancy

[1] That is, the problem where the credit risk of a single entity is assesed in positive terms during a period of economic growth and in negative terms during a period of economic slowdown. Credit risk models which fail to address the issue of pro-cyclicality might result in a further strengthening of the economic downturn.

ratio calculation as well – Internal Ratings-Based Approach (IRB) – New Basel Capital Accord (Gordy 2003), (Finger 2001).[2] The estimated model in this chapter belongs to group of macro credit risk models. These group of models try to estimate aggregate credit risk, therefore fit to financial stability purposes.

The structural model was chosen for stress test purpose in CNB. The aim of the model is prediction of the possible future development of non-performing loans as a function of the negative changes in macroeconomic environment. Selected approach follows one-factor model which was introduced in chapter 3.2.

5.2 Credit Risk Models in Central Banks

Most of the central banks use some kind of sensitivity analysis or stress test. However, only some of them employ a macroeconomic credit risk model. When central banks use macroeconomic credit risk models, they usually employ empirical type models[3] as in the case of Great Britain, Germany, Belgium or Finland. The Bank of England employs empirical model estimated bankruptcy ratio of non-financial corporation and default rate on mortgages and credit cards portfolio (Bunn, Cunningham, Drehmann 2004). These estimated results enter into model of credit losses as explanatory variables. Default rates are estimated by real GDP, real interest rates, unemployment rate, corporation indebtedness and other aggregates indicators. In the case of Finland, the macro credit risk model is based on the logistic regression. The model explains industry specific default rate by macroeconomic indicators (Virolainen 2004). This model consider as explanatory variables real GDP, nominal interest rates and indebtedness indicators of the industry specific sectors. Default rates are estimated as ratio of numbers of firms bankruptcies on total numbers of firms in given industry specific sector. The Hungarian central bank also prepares a macro credit risk model, which employ numbers of bankruptcies for given industry specific sectors as well. The model came from the Finnish central bank used methodology. In the case of Germany, a panel regression model of German commercial banks was used (Deutsche Bundesbank 2005). Logit transformation of the ratio of provisions on credit portfolio was considered as explanation variable. This model employs change of risk free interest rate, GDP growth rate, credit growth rate as explanatory variables. The Belgian central bank employs logistic regression model estimated aggregate corporate default rate (National Bank of Belgium 2005). Output gap, long-term nominal interest rate and lagged corporate default rate are considered as explanatory variables in the model. Generally speaking, the central

[2] A One-factor model was used to calibrate risk weights for the purposes of Basel II framework (default probability, assets corelation of borrowers within risk classes).

[3] Types of the credit risk models are discussed in the chapter Introduction.

banks are just at the beginning of their efforts to monitor relations between credit portfolio quality and macroeconomic environment.

5.3 Used Data

Quarterly data for the Czech economy were employed in all calculations. The estimated model is based on the time series of bad loans and selected macroeconomic indicators.

5.3.1 Bad loans

The aggregate default rate was estimated as the ratio of new bad loans on total amount of loans in the economy.[4] The quarterly time series of the new bad loans were available from Q1/1997 to Q3/2005. Nevertheless, their development was affected by one-off measures which meant change of classification of delinquent loans secured by realproperty during years 1999-2001.[5] Significant deviation of observed ratio of new classified loans on total banks loans potfolio occured during this period. However, this administrative change of classification does not mean change of real portfolio quality.

Influence of classification change was adjusted by introduction of dummy variable with values 1 for quarter when significant deviation from long-term trend of this indicator occurred and 0 otherwise. The value 1 was considered in Q3/1999, Q4/1999, Q4/2000, Q2/2002.

The alternative approach for default rate approximation is to use bankruptcy data. This approach was used for example for the macro credit risk model of the Finnish economy (Virolainen 2004), (Jakubík 2006). This kind of data is available for the Czech economy from the beginning of the transformation process. However this time series can be probably used for the model estimation only since the end of the nineties.[6] Figure 5.1 demonstrates monthly time series of bankruptcies numbers. It was defined as number of bankruptcy proposals. There is probably some time lag

[4] That is, loans which became "bad" in the given quarter. The moment of default means the time when the loan was classified as substandard or worse for the first time. Shifts within the "bad" loans category (for example, a further downgrading of the loan from doubtful to loss) will not affect the default rate according to this definition. This variable does not correspond to the proportion of total non-performing loans, which are not an optimum measure of credit risk as they may include loans which were first classified a very long time ago and which remain in the loan portfolio, for example, for accounting purposes and are not related to the current economic situation.

[5] CNB Provision of 17 September 1997 stipulating the principles for classifying loan receivables and for provisioning for these receivables, as amended.

[6] The time series of bankruptcies shows that the number of bankruptcies at the start of the 1990s was very low, probably as a result of inadequate legislation.

Figure 5.1: Default rates development in the Czech economy

between starting of bankruptcy proceedings and proposals. The default event also can happen earlier before bankruptcy proceeding starting. Finally, bankruptcy data was not used for the macro credit risk model estimation of the Czech economy due to quite frequent legal changes in the past. However chart 5.1 confirms a similar development of the new bad loans on total loans portfolio and firms bankruptcies.

5.3.2 Considered Macroeconomic Indicators

Broadly considered macroeconomic indicators in literature are gross domestic product (GDP) and interest rates.[7] GDP is a basic cyclical indicator of the economy. Downturn or slowdown affects firms profit, unemployment rate, value of assets (eg. real estate), etc. It is exhibited by an increase in firms' credit risk. Increase of interest rates have similar effects on credit portfolio. Higher interest rates increase financing cost of firms and households, decrease market value of assets, etc.

We considered the annual real GDP growth rate. 1M and 1Y interbank rate (PRIBOR[8]) were considered as nominal interest rates. Real interest rates were used

[7] e.g. (Virolainen 2004), (Deutsche Bundesbank 2005), (Rösch 2003), (Jakubík 2004)
[8] Prague Interbank Offered Interest Rate

as ex-post real interest rate deflated by consumer price index. Further real effective exchange rate and nominal exchange rate CZK/EUR and CZK/USD were considered due to high dependence of the Czech corporate sector on exchange rate and a small open economy.[9] The last considered indicator was indicator of indebtedness of the aggregate economy measured as a ratio of total outstanding clients loans on GDP.

The view of economic interpretation was considered during the final selection. We focused mainly on macroeconomic indicators which are actually used for the stress test scenarios of the Czech National Bank.[10] We also based the final selection on the connection our macroeconomic credit risk model with official macroeconomic prediction model of the Czech National Bank.

5.4 Model Estimation

We employed the concept of the one-factor model. We used bad loans data in place of bankruptcies data, therefore we made following additional assumptions. Each crown[11] of loans was assumed as a separate one-crown loan of one client. Total bad loans at an appropriate moment correspond to numbers of new default clients. Furthermore, total loans correspond to total numbers of the clients. Estimation of the one factor-model described in the chapter 3.2 can be used under this consideration.[12]

From a statistical point of view, the best model included GDP, nominal interest rate, inflation and dummy variable for adjustment of the one-shot methodical changes of the loans classification rules. The selected model is in line with macroeconomic stress test scenarios and outputs of the macroeconomic prediction model of the Czech National Bank. The annual growth rate was used in the case of GDP. The most significant interest rate was four quarter lagged nominal annual interbank rate (1Y-PRIBOR). Year on year two quarter lagged consumer price index (CPI) growth rate was selected. The model without including the dummy variable was also considered. However, the performance of the model was quite similar. Nevertheless the results were a little overvalued in at the end of the observed time period.

Table 5.1 demonstrates results of the estimated model of aggregate default rate in the Czech economy. All coefficients were significant on the 5% confidence level. The negative relationship between gross domestic product and default rate was con-

[9] An internal CNB calculation based on CPIs and continuous weights corresponding to the average previous annual trade turnover was used to calculate the real exchange rate.

[10] These indicators thus affect the resulting capital adequacy in the stress testing through two channels. The first acts directly via their effect on banks' balance sheet, while the other operates indirectly via the estimate of credit risk.

[11] crown=koruna

[12] The assumtion regarding koruna loans is somewhat simplified, as koruna loans are not in fact independent.

firmed, therefore higher GDP growth leads to credit risk decreasing. Conversely, positive relationship between interest rate and default rate was estimated. Both these results are in line with our results for the Finnish economy as well as general economic theory. The effect of the nominal interest rate is dampened by including the inflation growth rate into the model. For this reason the estimated inflation coefficient has negative sign. Combination of nominal interest rate and inflation shows dependence of credit risk on the real interest rate. However, estimated coefficients have a slightly different absolute value and different lagging. High significance of the ρ coefficient demonstrates, that latent factor is still important despite of the macroeconomic indicators included into the model. These results signal the importance of the other factors except macroeconomic indicators for explanation of the aggregate credit default rate.

Variable description corresponding to estimated coefficient		**Notation**	**Estimation**	**Standard error**	**Pr>|t|**
Constant	(β_0)	c	-2.0731	0.1019	<0.0001
Gross domestic product	(β_1)	gdp	-4.9947	1.9613	0.0162
Nominal interest rate	(β_2)	R_{t-4}	2.7839	0.9076	0.0045
Inflation	(β_3)	π_{t-2}	-2.4364	1.0994	0.0344
Dummy	(β_4)	dum	0.3296	0.06629	<0.0001
Influence of latent factor	(ρ)	ρ	0.01211	0.003243	0.0008

Table 5.1: Macro credit risk model (3.14) of the Czech Economy

The following equation (5.1) of one-factor model (3.14) express estimated relation for the aggregate default rate in the Czech economy.[13]

$$df_t = \phi(-2.0731 - 4.9947gdp_t + 2.7839R_{t-4} - 2.4364\pi_{t-2} + 0.3296dum_t) \qquad (5.1)$$

Figure 5.2 shows performance of the estimated macro credit risk model for the Czech economy.

Equation 5.1 can be simplified due to zero value of dummy variable in the future. Macroeconomic credit risk model is described by equation 5.2, which can be used for prediction of quarterly default rate.

$$df_t = \phi(-2.0731 - 4.9947gdp_t + 2.7839R_{t-4} - 2.4364\pi_{t-2}) \qquad (5.2)$$

[13] ϕ denotes function of cumulative standard normal distribution, df denotes quarter default rate, index t denotes appropriate time.

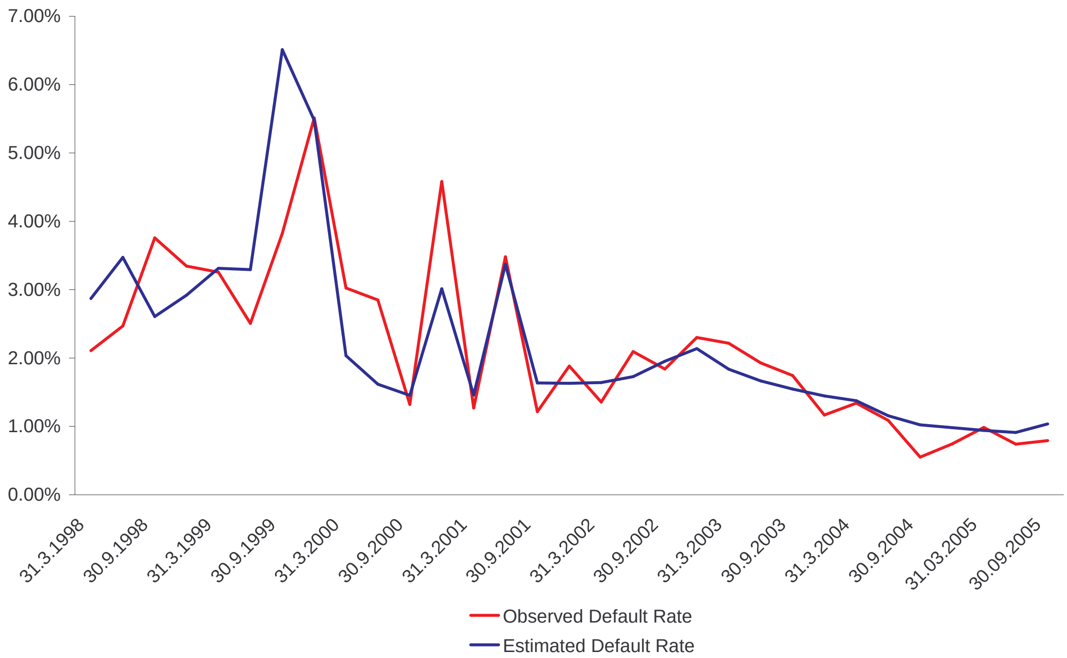

Figure 5.2: Performance of the one-factor model for the Czech economy

The estimated model is version of the binary choice model, therefore standard approaches for measurement of the model quality can be applied. However, a lot less common indicators exist. The test of the hypothesis that all coefficients β_j except constant coefficient are zero (H_0: $\beta_1 = \beta_2 = \cdots = \beta_K = 0$) is one of the model quality tests. This hypothesis can by tested by likelihood ratio $\lambda = \frac{L_C}{L_U}$, where L_C denotes likelihood function of constrained model and L_U likelihood function of unconstrained model. If tested hypothesis satisfies, then $-2ln\lambda$ is asymptotic chi-squared distributed variable with K degrees of freedom.[14] The result of this test was highly significant.

Further observed criteria of pseudo coefficients of determination based on the likelihood function confirmed good model quality. These coefficients should stay in the interval [0,1]. The result close to one confirmed good model quality.

$$R_E^2 = 1 - \frac{\ln L_U}{\ln L_C}^{\frac{-2}{n}\ln L_C} = 0.97 \qquad \text{Estrella (1988)} \quad (5.3)$$

[14]The known result of the distribution is mentioned, for example, by Roa (1973).

$$R^2_{CU1} = 1-(\frac{L_C}{L_U})^{\frac{2}{n}} = 0.95 \qquad \text{Cragg-Uhler (1970)} \quad (5.4)$$

$$R^2_{CU2} = \frac{1-(\frac{L_C}{L_U})^{\frac{2}{n}}}{1-L_C^{\frac{2}{n}}} = 0.95 \qquad \text{Cragg-Uhler (1970)} \quad (5.5)$$

$$R^2_{VZ} = \frac{\ln L_U - \ln L_C}{2(\ln L_U - \ln L_C)+n}\frac{2\ln L_C - n}{2\ln L_C} = 0.80 \qquad \text{Veall-Zimmermann (1992)} \quad (5.6)$$

5.5 Using of the model for stress test

The impact of the macroeconomic shocks on the aggregate default rate can be tested by estimated macroeconomic credit risk model for the Czech economy. The estimated model employs quarterly time series, therefore estimated defalut rate is on the quarterly base as well. It is necessary to annualize the quarterly default rate for the stress test purpose. Two approaches are possible for solving this problem. First, multiply quarterly default rate by four, which is the upper estimate of the annual default rate. Second, calculation of the four quarter default rates and their sum under the assumption that observed portfolio does not change. Inputs of the macroeconomic credit risk model have to be set up for default rate prediction. These inputs are simultaneous parameters of the stress testing. There are non-lagged annual real GDP growth rate, four quarter lagged nominal interest rate and two quarter lagged annual inflation rate. These parameters can be set up expertly or as a percentage deviation from macroeconomic model prediction or as an output of the the Czech National Bank prediction model under realization of the hardly likely, but not completely impossible negative macroeconomic shocks.

Following table 5.2 shows results of the macroeconomic credit risk model for different values of the GDP growth rate, nominal interest rate and inflation rate. These examples are only of illustrative value for demonstration of the model sensitivity and does not mean real using values in CNB.

Table 5.2 demonstrates for example credit risk sensitivity on change 1% of the GDP growth rate. This sensitivity differs for different level of the GDP growth rate ceteris paribus. Impact of the slowdown is lower for the higher initial level of the GDP growth rate. The reason is cumulative distribution function in the model 5.2.

The results of the macroeconomic credit risk model are employed in the current version of the CNB stress test for estimation of the ratio of the bad loans on total

		GDP Growth Rate							
CPI	**R**	**-1%**	**0%**	**1%**	**2%**	**3%**	**4%**	**5%**	**6%**
1%	**2%**	2.3%	2.1%	1.8%	1.6%	1.4%	1.2%	1.1%	1.0%
	3%	2.5%	2.2%	2.0%	1.7%	1.5%	1.3%	1.2%	1.0%
	4%	2.6%	2.4%	2.1%	1.8%	1.6%	1.4%	1.3%	1.1%
	5%	2.8%	2.5%	2.2%	2.0%	1.8%	1.5%	1.4%	1.2%
	8%	3.4%	3.0%	2.7%	2.4%	2.2%	1.9%	1.7%	1.5%
2%	**3%**	2.3%	2.1%	1.8%	1.6%	1.4%	1.3%	1.1%	1.0%
	4%	2.5%	2.2%	2.0%	1.7%	1.5%	1.4%	1.2%	1.0%
	5%	2.7%	2.4%	2.1%	1.9%	1.6%	1.5%	1.3%	1.1%
	8%	3.2%	2.9%	2.6%	2.3%	2.0%	1.8%	1.6%	1.4%
3%	**4%**	2.4%	2.1%	1.9%	1.6%	1.4%	1.3%	1.1%	1.0%
	5%	2.5%	2.2%	2.0%	1.8%	1.6%	1.4%	1.2%	1.1%
	8%	3.1%	2.7%	2.4%	2.2%	1.9%	1.7%	1.5%	1.3%
4%	**5%**	2.4%	2.1%	1.9%	1.7%	1.5%	1.3%	1.1%	1.0%
	8%	2.9%	2.6%	2.3%	2.0%	1.8%	1.6%	1.4%	1.2%

Table 5.2: Sensitivity of the macro credit risk model outputs

loans portfolio. This ratio is used as a input parameter of the stress test. Credit risk model is able to generate bad loans on banking portfolio as a result of the macroeconomic shocks - change of the real GDP growth rate, nominal interest rate or inflation growth rate.

Conclusion

We have investigated macroeconomic models of default rate estimation. We followed two possible approaches. First, empirical models were researched. Second, latent factor models were examined. All the models used are derived form individual risk models. Empirical models are based on the idea of traditional models. This approach assumes estimation of empirical function. Linear, logit or probit functions are usually used. Latent factor models are derived from the Merton idea. These models were originally employed in individual risk modeling. Unobservable factors are used by latent models in the credit risk modeling. Normal distribution of these unobservable factor is usually assumed. A static version of this model was considered for estimation in this paper. Coefficients can be estimated by likelihood function. Solution of a maximization problem leads to the integral over the random effects.

We employed monthly data of the Finnish economy. Bankruptcy data and time series of the firm's number were key time series used. A lot of macroeconomic indicators were considered. Finally GDP, interest rates, exchange rate and firm's indebtedness were employed in default rate modeling. Times series starting 1/1988 and finishing 12/2003, were available for all considered data except indebtedness. Outstanding loans to corporate and entrepreneurs were available only from 1/1990. Due to shorter time series of indebtedness part of the analysis were restricted to the period 1/1990 - 12/2003. Yearly or quarterly time series were disaggregated. The whole aggregate economy as well as industry specific sectors - agriculture, manufacturing, construction, trade and transport were investigated.

Firstly, linear vector autoregressive models were researched in the case of empirical dynamic models. Industry specific default rates were investigated. Any relationship with macroeconomic indicators were not proved in the sector of agriculture. A negative relationship of default rate with GDP was proved in other sectors except trade. A positive relationship of default rate with interest rate was proved in all cases except agriculture and transport.

Furthermore a one-factor model was used for default rate estimation of aggregate economy and also industry specific sectors. A multi-factor model was also considered. But only, a one-factor model was estimated due to the fairly numerical complication of multi-factor models. Unobservable factor of this model was significant in all cases. One-factor model signaled different behavior of the agriculture sector. This sector is probably independent or poorly dependent on the macroeconomic environment. A negative relationship between GDP and default rate was solidly proved in case of manufacturing, construction and trade. Weak negative relations is probably between default rates and GDP in transport. Very similar conclusion with positive relations was proved for interest rate, but any relations between interest rate and default rate in agriculture was rejected. A significant indicator of the default rate is firm's indebtedness. Positive relations was proved in all case except agriculture. The exchange rate probably affects the default rate only in the case of trade exposed to international business.

This research is connected to study of Virolainen (2004). We tried to improved suggested model of default rate. (Virolainen 2004) study is based on the logit empirical model. Estimated one-factor model offers alternative to empirical model without any microeconomic foundation. We used very similar indicators as in previous research. However some slight differences can be observed. The previous study did not find any role of the real interest rates. Over against real interest rates were employed in our model and significant strong relation was proved at least in case of manufacturing, construction and trade. The agriculture sector is less affected by macroeconomic indicators according to our study than in the previous study. This problem can relate to seeming regression, because time series stationarity was not investigated in the previous study. However all significant relations in the both studies has the same sign.

Some aspects of latent factor model would be further elaborated. The different assumption on default distribution can be considered. Performance of the one-factor models used can be improved by using dynamic factor latent model. In this case correlation of assets return is not constant as in the case of static factor model. This type of model lead to very complicated likelihood function. More advanced numerical technics are necessary for their estimation. Elaboration of stress scenario would be used to analyse the influence on the default rate in the Finnish economy.

Although the Finnish economy was affected by a strong recession and the structural changes in the begining of nineties, performance of the estimated model was fairly good. Our study proved important influence of the macroeconomic variables on the default rates in the economy. Differences between industrial sectors were showed. Our study investigated two possible approach for credit risk modeling and their comparison. Latent factor model was found as more powerful in macroeconomic modeling of default rate. We estimated one-factors model for aggregate economy and also industry specific sectors. These models can be used for stress testing or default rate prediction.

Further, we employed one-factor model for the macroeconomic credit risk model estimation of the Czech aggregate economy. This model proved very strong relation between banks' portfolio quality and macroeconomic environment. Impact of the the macroeconomic environment changes on the aggregate default rate can be tested by estimated macroeconomic credit risk model for the Czech economy. The model was incorporated into current version of the Czech National Bank stress test for financial stability purpose.

Macro credit risk model for the Czech economy would be further elaborated. Microeconomic data can be incorporated into the model due to chosen approach. Industry-specific data would be employed as well. The problem of the default probability estimation is related to loss given default. The worst possible scenario 100% loss is assumed for incorporation of the macroeconomic credit risk model into the current version of the CNB stress test. Loss given default probably depends on the default probability. Model of the loss given default could be developed for stress test improvement.

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Appendix

Name of Variable	Short Name of Variable	Order of Stationarity
Default Rate	$\texttt{df}$	I(1)
Default Rate in Agriculture	$\texttt{df}_{\texttt{AGR}}$	I(0)
Default Rate in Construction	$\texttt{df}_{\texttt{CON}}$	I(1)
Default Rate in Manufacturing	$\texttt{df}_{\texttt{MAN}}$	I(1)
Default Rate in Trade	$\texttt{df}_{\texttt{TRD}}$	I(1)
Default Rate in Transport	$\texttt{df}_{\texttt{TRN}}$	I(1)

Table 5.3: The order of stationarity of default rates

Name of Variable	Short Name of Variable	Order of Stationarity
real GDP difference	$\texttt{dGDP}$	I(1)
real GDP difference from trend	$\texttt{GDPdif}$	I(1)
nominal interest rate 1M	$\texttt{r1M}$	I(1)
nominal interest rate 3M	$\texttt{r3M}$	I(1)
nominal interest rate 12M	$\texttt{r12M}$	I(1)
real interest rate 1M (CPI)	$\texttt{r1M}_{\texttt{CPI}}$	I(1)
real interest rate 3M (CPI)	$\texttt{r3M}_{\texttt{CPI}}$	I(1)
real interest rate 12M (CPI)	$\texttt{r12M}_{\texttt{CPI}}$	I(1)
real interest rate 1M (PPI)	$\texttt{r1M}_{\texttt{PPI}}$	I(1)
real interest rate 3M (PPI)	$\texttt{r3M}_{\texttt{PPI}}$	I(1)
real interest rate 12M (PPI)	$\texttt{r12M}_{\texttt{PPI}}$	I(1)

Table 5.4: The order of stationarity of macroeconomic indicators

Name of Variable	**Short Name of Variable**	**Order of Stationarity**
difference of real value added in agriculture	$\mathtt{dGDP}_{\mathtt{AGR}}$	I(0)
difference of real value added in construction	$\mathtt{dGDP}_{\mathtt{CON}}$	I(1)
difference of real value added in manufacturing	$\mathtt{dGDP}_{\mathtt{MAN}}$	I(1)
difference of real value added in trade	$\mathtt{dGDP}_{\mathtt{TRD}}$	I(1)
difference of real value added in transport	$\mathtt{dGDP}_{\mathtt{TRN}}$	I(1)

Table 5.5: The order of stationarity of values added

Teze diplomové práce

Název: Kreditní rizikové modely a jejich vztah k ekonomickému cyklu
Předkladatel: Ing. Ing. Petr Jakubík, Ph.D.
Konsultant diplomové práce: PhDr. Petr Teplý

Charakteristika tématu

Význam kreditních rizikových modelů roste zejména se zaváděním nových pravidel pro výpočet kapitálové přiměřenosti známých jako Basel II. Významnou komponentou pro výpočet je pravděpodobnost defaultu. Teorie rozlišuje tři základní přístupy. Tradiční modely jsou založeny na porovnání dostupných informací o klientech a předpovídání jejich kvality. Strukturální modely vychází z Mertonova modelu, kde hodnota firmy je získána jako cena opce a default je specifikován jako hodnota opce ve vztahu k firemní zadluženosti. Třetím přístupem jsou tzv. redukované modely, které vychází z tržní ceny dluhopisů. Všechny tyto tři modely byly původně určeny pro výpočet individuální pravděpodobnosti defaultu, ale v poslední době jsou rozšiřovány i na agregátní míru defaultu. Zájem o tyto modely roste ze strany centrálních, ale i komerčních bank. V souvislosti s měnovou politikou hraje významnou roli vztah míry defaultu k ekonomickým indikátorům.

Hypotézy

- Pomocí empirických a latentních faktorových modelů je možné modelovat agregátní míru defaultu
- Míra defaultu v ekonomice závisí na makroekonomických indikátorech
- Hrubý domácí produkt, úrokové sazby a míra zadluženosti patří ke klíčovým indikátorům míry defaultu v ekonomice
- Latentní jedno-faktorový model pro agregatní míru defaultu je možno odhadnout pouze z dat o bankrotech firem a makroekonomických indikátorů
- Míra defaultu v jednotlivých sektorech ekonomiky se vyvíjí odlišně

- Mezi mírou defaultu a makroekonomickými indikátory je odlišný vztah pro různé sektory ekonomiky

Osnova

- Úvod
- Související práce s tématem
- Makroekonomické kreditní rizikové modely
  - Empirické modely
    - Statické modely
    - Dynamické modely
  - Jedno-faktorový model
  - Multi-faktorový model
- Výsledky latentního faktorového modelu pro finskou ekonomiku
  - Užitá data
  - Užité modely
  - Agregátní ekonomika, zemědělství, výroba, stavebnictví, obchod, doprava
- Závěr
- Literatura

Metody práce

- Matematická statistika
- Teorie časových řad
- Použití statistického software pro odhad modelů

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Ing. Ing. Petr Jakubík, Ph.D.	PhDr. Petr Teplý
(Předkladatel)	(Konsultant)