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Report on PhD thesis “Point processes as a mathematical tool to describe black hole accretion disc stochastical variability” by Tomáš Pecháček

The main subject of Mgr. T. Pecháček’s thesis is Fourier analysis of time series generated by so called point processes. The astrophysical context here is the random variability of X-ray radiation emitted from black hole accretion discs. The X-ray emission is thought to originate in events localized in time and space, each of them causing a short burst (a flare) of X-ray photons. The underlying physical process is currently uncertain but is usually assumed to be related to magnetic activity inside the accretion disc, in analogy with X-ray flares occurring on the surface of the Sun. The main purpose of the thesis is derivation of general formulae for power spectral density of the time series generated by the processes. The thesis is very “mathematical” in character: the motivation from observational data is formulated as a general shape of a power spectrum (no discussion of the latter on, e.g., spectral states of the sources), the final results are presented as shapes and slopes of PSD, and there is very little references to observational data in between.

The thesis contains four chapters: Introduction, description of relativistic effects on radiation in Schwarzschild metric; the main results are presented in the third chapter “Spot models and the accretion disc variability” and the last chapter contains conclusions and future prospects. Some additional material is presented in four Appendices, and there is also a bibliography of the subject. What would certainly be helpful to have is a collected explanation of all mathematical symbols used – this seems to be standard in theses of mathematical character.

The second chapter, on relativistic effects, is not directly related to the general problem of point processes, but is motivated by the specific situation considered in the thesis: X-ray emission from spots rotating on an accretion disc is modulated by the varying relativistic effects (energy shifts, time delay) and the signatures of the modulation should be observable in X-ray light curves and power spectra. Mgr. Pecháček uses techniques of spherical trigonometry to derive formulae necessary for further derivation of a number of analytical approximations for the energy shift, light delay and light magnification. These are then meant to be used to compute the modulation of the primary signal from active spots on the disc. Having accurate analytical formulae for the relativistic effects is very useful indeed, since direct integration of geodesics is too time consuming to be performed in real time while fitting data. The author demonstrates that his approximations are better than those published earlier in the literature, and they are very accurate in absolute terms. As such, they are of general use and will undoubtedly be used by other researchers in this field.

I would like to emphasize here that the problem of relativistic effects is a separate one from the problem of the signal from flares. That is, one can imagine the same point process describing the production of X-rays but with a different velocity field (even non-Keplerian) of the emitting regions. Moreover, even for the model considered in this thesis, i.e. active regions attached to a Keplerian disc, there is a problem of mapping the time-scales of individual flares with radial position on the disc. There is an uncertainty here even though there exists a natural mapping of flare time scale with the Keplerian time scale. The formulae derived in Chapter 2 assume Keplerian velocity field. The formalism developed in Chapter 3 is more general but it also seems to assume that a single flare is associated to an active region following a circular orbit. I will return to this point later.

Chapter 3 presents the main results of the thesis. It begins with a very formal introduction of concepts underlying the variability, i.e. probability space, random processes and point processes. Interestingly, in the very first subsection of the chapter the author makes an important remark, that only a Gaussian random process is uniquely determined by its power spectrum. This is not always realized by people working on data analysis and power spectrum modeling, even though it is obvious from mathematical point of view. It is thus very useful to have such a remark made and illustrated by concrete examples of light curves, the more so that the real data are certainly non-Gaussian.

The author then defines a general point process. This is defined by giving a description of the distribution of points in a multidimensional space of parameters of the model, like ignition time of flares, flare duration, location on the disc, etc. Mgr. Pecháček derives formulae for the power spectrum of the point process, which is illustrated by a special case of the Poisson process, i.e. a process where the events are independent. Relations between events are introduced next, and a cluster process is defined, consisting of related events. The author derives a general formula for the power spectrum of a cluster process. A number of specific models are defined: Chinese process, Hawkes process and the flare avalanche model. These differ in the way the relations between events are defined. Generally speaking, there are two classes of events: spontaneous and stimulated ("parent" and "offspring"), and each event can give rise to an entire avalanche of events. The author derives formulae for power spectra of these specific processes and shows dependencies of the power spectra on model parameters. An important next step is to introduce marks, as a way of relating model parameters between them (for example, location on the disc with flare time scale). This is expected in all physically realistic model. In the final section of Chapter 3 the author discusses high-frequency limit of PSD and its dependence on the profile function of individual flare.

Undoubtedly, Mgr. Pecháček demonstrated impressive working knowledge of mathematical analysis, in particular in complex domain. He feels at ease with both abstract concepts and complicated practical aspects of the mathematical problem.

He is also well versed in the broader context of point processes, not only in astrophysics.

The analysis of the shapes of power spectra on various parameters is thorough and it is certainly an important first step to application of his result to real data. Wisely, in most cases Mgr. Pecháček ignored the relativistic effects while plotting the various PSD in Chapter 3, concentrating on the PSD from point processes as such. Observational data gathered and analyzed so far seems to suggest that relativistic effects are rather unimportant in shaping the observed power spectra from accreting black holes. Therefore it makes sense to understand the underlying random process first, and only then look for evidences of relativistic effects.

The main benefit from analytical studies of any kind (as opposed to simulations) is the possibility of deriving analytical formulae to study general properties of results. In this sense the author could do more focused investigations of the shapes of power spectra. He could attempt to derive general answer to questions like, when the PSD can be described as a (broken) power law, what are the slopes of the different parts, for what parameters one can obtain a long part with f^1 slope, what slope of the high frequency part can one obtain, etc. These would make the discussion closer to the observed data.

Some more detailed comments :

1. In Figs. 3.12 and 3.13 the results of simulations (red) should be binned (e.g. logarithmically). Without that the individual points have large uncertainties, which prevents meaningful comparison of the results obtained by different methods.

2. In Fig. 3.13 it would make sense to show the results with and without relativistic effects, otherwise their strength is impossible to assess.

3. In Sec. 3.5 on high-frequency limit of PSD there is, I think, a problem with what k means. Formula 3.206 implies that for, for example, a simple exponential profile, k should be 0 (PSD $\propto\omega^{-2}$), while for the double exponential profile (exp(t) for t<0, exp(-t) for t>=0), k=1 (PSD $\propto\omega^{-4}$). But in the latter case only the function itself is continuous, while its first derivative is not, implying k=0 rather than 1. What would then k be for a simple exponential profile?

In my opinion, the value of such mathematical considerations as presented by Mgr. Pecháček is in enabling us to understand some real (astro)physical phenomena. From this perspective, the thesis is undoubtedly important, as it treats in a general way a broad class of models. At the same time, I think it is not possible to gain real insight into physical processes responsible for X-ray variability by purely mathematical studies of power spectra, however careful they are. I am not sure if it is possible to, for example, distinguish between the various kinds of processes discussed here (Hawkes, Chinese, avalanches, etc) based on the appearance of their power spectra. There is a lot more known about the X-ray variability of accreting black hole systems. Relevant properties of single light curves include: the rms-flux relation (McHardy, Uttley), and higher order moments of the light curves (Coppi, Maccarone). These two aspects can be checked within the framework considered by the author (although non analytically, I suppose). Going into cross-correlations analysis we have cross-spectra (including time delays) and Fourier-frequency resolved spectra. These require further physical input to the models discussed in this thesis, since they involve the energy spectral properties. Also, a given model would have to applicable to the whole range of luminosities covered by a given source. These remarks are, of course, not meant to undermine Mgr. Pecháček’s work but to point out the richness of the problem.

Finally, the form of the thesis is correct, division into chapters logical, additional material appears in appendices. Typesetting of difficult mathematical equations is flawless. The English is correct, I could find only a few minor language problems.

Summarizing, in my opinion the presented dissertation demonstrates the ability of Mgr. Pecháček to solve difficult mathematical problems. The presented results are new, interesting and applicable to the astrophysical problem of X-ray variability from accreting compact objects.