Posudek oponenta bakalářské práce
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Název práce: Algebraické vlastnosti barevnosti grafu
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Navrhovaná známka: 1-2

The thesis deals with the Alon-Tarsi theorem, which is a powerful algebraic tool to deduce upper bounds on list chromatic number of graphs.

In the first section, the author summarizes basic terminology and some simple observations related to list coloring. In the next section, he states and proves the Alon-Tarsi theorem. In the third section, the author presents his original research contribution: a proof that the square of a cycle of length $3k$ has list chromatic number equal to 3. The last section is devoted to the statement and proof of a result of Fleishner and Stiebitz, which is another application of the Alon-Tarsi theorem.

Let me comment on the five sections separately, and then conclude by some general remarks.

The purpose of the first two sections is to introduce the main tools, which are used in the rest of the thesis. These two sections are well organized and written in a very clear manner. Unfortunately, there are several mistakes and inaccurate expressions, which hamper the understanding of the mathematical content:

- Proposition 1 should end with '... at least $k+1$' instead of '... at least k.' and on the first line of its proof, '$k-1$ colors' should be 'k colors'.
- On the second line of the same proof, 'complete graph $K_{\binom{2k}{k}-1}$' should be 'complete bipartite graph $K_{\binom{2k}{k}-1,\binom{2k}{k}-1}$'.
- In the paragraph before Proposition 2, it is stressed that all the faces of a triangulation are triangles, including the outer face. Unfortunately, the statement and proof of Proposition 2 does not work for this definition of a triangulation. It is necessary to assume that the outer face is bounded by a cycle of arbitrary length, while the inner faces are all triangles.
- In condition (iii) of Proposition 2, the expression 'w in the inner face of G' should be rephrased as 'w not on the outer face of G'.
- In the statement of Lemma 1, '$1 \leq i \leq n-1$' should be '$1 < i < n$'.
- Line 4 of page 14: S_n should be S_{n-1}
- The equation at the bottom of page 15 is strange. For one thing, on the left-hand side, $d_1, \ldots, d_n$ should probably be $x_1, \ldots, x_n$. Even then, the equation does not really make much sense without specifying a D for each $(d_1, \ldots, d_n)$. It would be much less confusing to omit this whole equation and just state Lemma 4.

Despite these inaccuracies, the first two sections of the thesis are easy to follow and provide a good introduction to Alon-Tarsi theorem and related concepts.

Section 3 of the thesis presents the author's original research. It deals with the list chromatic number of the square of a cycle. Although the method applied here is nice and original, the presentation should be improved. First of all, the term 'square of cycle' should be defined explicitly, since there are several different notions of graph products, and the term 'square of a graph' is ambiguous. It should also be specified which orientation of the underlying graph is considered.

To make the presentation even more confusing, the author does not specify which result he is about to prove in this section. He should mention explicitly before the beginning of the proof that he is only going to deal with the case when the length of the cycle is a multiple of three, and that in the remaining cases, it is possible to apply simpler, elementary methods to deduce that the list-chromatic number is equal to the chromatic number. Instead, the author never mentions the elementary cases at all.

Another flaw that complicates the understanding of the text is the lack of precision in the definitions. For instance, in Definition 2 the author says that o_n^0 is 'the number of sequences of length n containing an odd number of ones that end with one' but in fact, it later turns out from the context that o_n^0 only counts the sequences without two consecutive zeros. Similarly, N_e is defined as the number of Eulerian subgraphs of G with even number of arcs, but it later becomes clear that N_e is meant to only count the graphs with less than ℓ_0 arcs. Essentially, the reader needs to reverse-engineer the proper definitions from equations (3.1) and (3.2). To make matters worse, there is apparently a '+1' term missing from equation (3.1), and on line 6 of the next paragraph, the phrase 'σ stands for an even sequence...' should read '$\sigma 0$ stands for an even sequence...'.

Apart from the above-mentioned errors, there seems to be a mathematical flaw in the argument: when the author encodes Eulerian subgraphs from the set $\mathcal{S}$ by 01-sequences, he fails to take into account the fact that a sequence whose all terms are equal to 1 does not encode a Eulerian subgraph from $\mathcal{S}$. Fortunately, this oversight may be easily fixed and does not cause any substantial gap in the argument.

Despite these flaws in the presentation, I would like to stress that the mathematical arguments involved in the proof of the result are nontrivial and the obtained result is nice.

The last section of the thesis presents the statement and proof of a theorem of Fleischner and Stiebitz, which is an elegant application the Alon-Tarsi method. Even though the result is somewhat technical, the author has managed to present it in a clear manner.

To summarize, the thesis shows that the author has a very good understanding of the topic and the ability to come up with original ideas. The mathematical arguments presented in the thesis are elegant and mostly correct. I also appreciate the author's brave decision to write the thesis in English.

On the other hand, the presentation of the work could be improved. Apart of the specific flaws pointed out above, the text has numerous spelling errors, which should have been detected by a spell-checker (or by better proof-reading).