We study nonlinear stability of steady state solutions of partial differential equations governing the thermomechanical evolution of viscoelastic fluids; materials that exhibit both viscous as well as elastic response when undergoing deformation. It is well-known that thermodynamical concepts can be gainfully exploited in the construction of Lyapunov functionals for nonlinear stability analysis of spatially homogeneous equilibrium rest states in thermodynamically closed systems. We show that the thermodynamically oriented approach can be utilized in the nonlinear stability analysis of spatially inhomogeneous non-equilibrium steady states in thermodynamically open systems as well. The thesis consists of two parts. In the first part, we revisit the classical construction of Lyapunov functionals in thermodynamically closed systems and we apply the nonlinear stability theory to compressible heat-conducting viscoelastic fluids modeled by a multiscale, as well as a purely macroscopic approach. In the second part, we focus on two special instances of thermodynamically open systems. First, we show that the spatially inhomogeneous non-equilibrium steady state of an incompressible heat-conducting viscoelastic fluid, which occupies a mechanically isolated vessel with walls kept at spatially non-uniform temperature, is globally asymptotically stable. Second, we investigate nonlinear stability of the steady internal flow of an incompressible viscoelastic fluid driven by an inflow of mechanical energy through the system's boundary and we derive bounds on the dimensionless flow parameters which guarantee global asymptotic stability of the steady flow.