Understanding asteroid collisions is a key part of Solar System science. To interpret observations of more than 100 asteroid families, various numerical simulations are used. In this work, we prefer the smoothed particle hydrodynamics (SPH), which allows a detailed description of impact mechanics, shock wave propagation, fragmentation of the target, ejection, or reaccumulation controlled by self-gravity and secondary collisions. Since the respective time scale may reach the orbital time scale, the SPH is often complemented by efficient N-body integrators and collisional handlers. In the review part of the thesis, we describe details of numerical methods and their implementation in the new OpenSPH code. We also thoroughly test the code, using analytical solutions and laboratory experiments as references, and discuss its stability and convergence with respect to spatial resolution. In the refereed papers, included in the thesis, we focus on collisions with targets of particular sizes ($D = 10$ and $100\,\mathrm{km}$). We explore the dependence of outcomes on the target size, the projectile size, the impact speed, the impact angle, and most importantly, the initial spin rate. We demonstrate that rotation significantly decreases the effective strength of the targets and increases the ejected mass. We self-consistently compute the angular momentum transfer due to sub-catastrophic impacts, which determines the overall evolution of spin rates in asteroid populations. Last but not least, we interpret the fourth largest main-belt asteroid (10) Hygiea and its collisional family. Besides the size-frequency distribution or the velocity field of fragments, we use the shape of Hygiea derived from adaptive-optics observations as a novel constraint for collisional modeling. This allowed us to not only determine the parameters of this major impact event, but to reveal impact-induced material weakening, needed to obtain consistent axial ratios.