The existence of homoclinic orbits is an intrinsic feature of static black hole's space-time. The regularity of the geodesic motion around these black holes can therefore be quickly disrupted due to the small changes in the original space-time. The nature of the dynamics around a perturbed orbit depends on the manner of intersection of the surrounding stable and unstable manifolds. If they intersect transversally, the homoclinic orbit splits into chaotic layers.

In this thesis, the mathematical formulation of chaotic dynamical systems and main properties of the geodesic motion in circular space-times are discussed. Thereupon, the space-time around a static black hole is reproduced by classical approximations by using Paczyński-Wiita and logarithmic pseudo-Newtonian potentials. By means of the effective potential method, the homoclinic orbits are found for these potentials. In addition, the analysis of the general circular space-time is done and the equations of geodesic motion in axially symmetric space-times are examined. Finally, the motion in a Schwarzschild space-time with a static axially symmetric external source is inspected.