This thesis deals with the study of properties of selected exact solutions to Einstein's equations. The first part states the basic properties of the examined solutions. It briefly summarizes basic models of black holes, whose generalizations will be important in the following part of the work. Next, it discusses a simple model of an infinite cosmic string and presents one of its possible constructions. After that, it reviews the properties of a class of Robinson-Trautman solutions, which includes photon rockets. These are used to describe sources accelerated by the emission of null dust. The final section presents the C-metric describing a pair of uniformly accelerating black holes. The second part of the work presents the constructions of cosmic strings, which arise both in C-metric and the simpler model of Schwarzschild black hole, pierced by a cosmic string. The presented construction is based on a sequence of generalized photon rockets and allows to determine the energy-momentum tensor of strings, which exists as a generalized function.