Evaluation of a Bachelor Thesis by Supervisor

Title: The classical McKay correspondence
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Summary of the contents

The thesis is concerned with finite subgroups $G < \mathrm{SL}(2, \mathbb{C})$, their classification in terms of simply laced Dynkin diagrams, and the singularity obtained at the origin of the corresponding quotient variety $\mathbb{C}^2 /\!\!/ G$. The interesting phenomenon is that the Dynkin diagrams classifying the finite subgroups G as above appear naturally both in the complex representation theory of G and in the resolution of the singularity of $\mathbb{C}^2 /\!\!/ G$. The author explains this theory in 3 sections, where he treats the classification of finite subgroups $G < \mathrm{SL}(2, \mathbb{C})$, relevant aspects of the representation theory of G, and the geometry of the quotient singularity (in this order).

The thesis was written very independently. In particular, the choice of the presentation and proofs should be attributed to the author. As far as I am aware of, he considered various and quite different proofs of the key facts.

Evaluation of the thesis

Topic of the thesis. The topic was involved from the point of view of this Bachelor program. It includes topics (such as representation theory of groups or algebraic geometry) which are taught only in a limited way within the program, and the main point was to combine aspects of these theories together in order to see something interesting. The author has chosen the topic knowing possible difficulties and I was convinced he was able to cope with them.

Contribution of the student. He presented the main results in a very logical way based on diverse sources and when possible, he included elementary but clever proofs crafted by himself.

For instance, the classification of finite subgroups of $G < \mathrm{SL}(2, \mathbb{C})$ is a well-known result, which is usually proved by first classifying finite subgroups of $\mathrm{SO}(3, \mathbb{R})$ and then using a relation between these two groups (via a well-known double cover of $\mathrm{SO}(3, \mathbb{R})$ by the special unitary group, which is a subgroup of $\mathrm{SL}(2, \mathbb{C})$). The proof presented here is direct and uses only basic linear algebra in order to find enough constraints on the structure of the finite subgroups in question, which in the end allow to classify them.

Mathematical quality. The text is rigorous and presented in a logical way. The first two sections (which would themselves suffice for a thesis) are written in a very nice way, with good feeling as to what is important to say and prove and what is sufficient to cite. The third section on geometric aspects is too sketchy, which is a result of time pressure before the deadline.

Usage of the literature. The literature is used and cited appropriately.

Formal aspects. The thesis meets usual standards from the formal point of view. The time pressure when finishing the thesis, however, resulted in a higher amount of misprints then it was necessary in my opinion. None of these is critical nor it affects the mathematical quality, but it makes the resulting text less comfortable to read.

Comments

Here I list a few particular misprints and also places in §3 which deserve to be improved in my opinion.

- Page 4, statement of Prop. 1.1.5: $a \in A$ (instead of $a \in N(A) \setminus A$).
- Pages 8/9, proof of Prop. 1.1.15: The eigenvalues are $e^{\frac{i\pi}{4}}$ and $e^{-\frac{i\pi}{4}}$.
- Page 11, proof of Prop. 1.2.9: It seems that the inequalities should be $|\lambda_1| \leq \cdots \leq |\lambda_n|$ and $\sum a_i^2 \lambda_i^2 \leq \sum a_i^2 \lambda_n^2$.
- Page 12, proof of Prop. 1.2.14: I suppose that the edge e also must not be contained in G'.
- Page 19, proof of Prop. 2.2.9: The triangle inequality says that

$$\sum a_k |\lambda_k| \dim(\rho_k) \geq \left| \sum a_k \lambda_k \dim(\rho_k) \right|.$$

- Page 23, first paragraph of §3.3: It would be nice to introduce the concept of singular variety (perhaps this is what Def. 3.3.1 was meant to be).
- Page 24, Example: Several misprints in the notation of coordinates. More importantly, however, the algorithm for finding a cover of a blowup of a hypersurface (not hyperplane!), albeit correct, should have been justified.
- Page 25, Thm. 3.3.9: It is a pity that it is not explained anywhere what the "non-extended version of the extended Dynkin diagram" precisely is, i.e. which vertex we should remove. A good place to include this would be Rem. 1.2.18, for example. Moreover, the proof of the theorem is written up only for cyclic groups and a citation is given for the other cases. This is all right, but I would prefer to have that declared at the beginning of the proof and included a more precise reference (which part of/theorem in Hemelsoet's thesis).

Conclusion

I recommend to recognize this nice piece work as a Bachelor thesis.

The suggested grading will be communicated directly to the head of the examination (sub)committee.

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July 30, 2021