

EVALUATION OF A BACHELOR THESIS BY SUPERVISOR

Title: Elliptic Curves and Diophantine Equations
Author: Adam Klepáč

SUMMARY OF THE CONTENTS

The thesis focuses on methods of solving Diophantine equations. After discussing the easy cases of polynomial equations with rational coefficients in one variable and linear and quadratic equations in two variables, the author focuses on cubic equations in two variables. This leads to the theory of rational points on elliptic curves and their group structure. The author shows that each rational cubic equation in two variables is birationally equivalent either to a line or to a rational cubic equation in the Weierstraß normal form. Then he describes and proves (leaving out some degenerate cases) the existence of the usual group structure in the latter case and finally he establishes the Nagel-Lutz Theorem, which among others says that the torsion part of the group of rational points is finite. The theory is complemented by a number computed examples.

EVALUATION OF THE THESIS

Topic of the thesis. The topic is on one hand accessible after finishing the basic Bachelor course in algebra, but on the other hand goes well beyond other courses taught within the Bachelor program. I consider it adequate and it was in fact suggested by the author himself.

Contribution of the student. The author wrote this rather long thesis completely independently and using several sources. He worked out his own proofs of the birational equivalence to the Weierstrass normal form (this is a little weaker result than the usually stated isomorphism of projective varieties) and the associativity of the group operation. He also came up himself with the computations in the examples.

Mathematical quality. The thesis is to my best knowledge correct and the author proved his ability to write a rigorous mathematical text.

The only part I find a little unclear is the treatment of the two cases $P + (Q + R) = (P + Q) + R$ and $P + (Q + R) = -((P + Q) + R)$ in the first half of page 38. If the second case hypothetically occurred, why could we find an instance of it with $P = R = -Q$, as one claims there?

Usage of the literature. The literature is properly cited and the list of references is extensive.

Formal aspects. The thesis meets usual standards from the formal point of view.

CONCLUSION

I recommend to recognize this nice piece work as a Bachelor thesis.

The suggested grading will be communicated directly to the head of the examination (sub)committee.

doc. RNDr. Jan Šťovíček, Ph.D.

Department of Algebra, Faculty of Math.-Phys, Charles University

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